Answers:

Relay 1

1. 6
2. 59
3. 6
4. 15080
5. 32
6. 45

Relay 2

1. 240
2. 2017
3. 2
4. 6
5. 4
6. 75

Relay 3

1. 25
2. 12
3. 34
4. -386
5. 13
6. 11

Relay 4

1. 17
2. 15
3. 22
4. 96
5. 4
6. 2

Relay 5

1. 5
2. 713
3. -40
4. 4
5. 6
6. 7

Solutions:

Relay 1

1. $x=2\sqrt{3+x}\Rightarrow x^2=12+4x\Rightarrow x^2-4x-12=0\Rightarrow (x-6)(x+2)=0\Rightarrow x=6$ (since the value must be positive)

2. $1110T+7=6667=59\cdot 113$, so the smallest prime factor is 59

3. $0.1(T+1)=0.1(59+1)=0.1\cdot 60=6$

4. The roots are ± 6 and ± 2, so the polynomial is $f(x)=(x^2-36)(x^2-4)$
$=x^4-40x^2+144$. That makes $A=1,\ B=0,\ C=-40,\ D=0,$ and $E=144$
$\Rightarrow C+E-B=-40+144-0=104$ and $E+A-D=144+1-0=145$. Since these two numbers do not share any prime factors, the least common multiple is
$104\cdot 145=15080$.

5. $A=5,\ B=1+5+0+8+0=14,$ and $C=15,$ so $(\langle A,B,C\rangle\times\langle 1,2,3\rangle)\cdot\langle 3,2,1\rangle=$
$$\begin{vmatrix}5 & 14 & 15\\ 1 & 2 & 3\\ 3 & 2 & 1\end{vmatrix}=10+126+30-90-30-14=32$$

6. $\lim\limits_{n\to\infty}P(n)=\lim\limits_{n\to\infty}32\left(1+\dfrac{13}{n}\right)^n=32e^{13}$, so $32+13=45$

Relay 2

1. $\int_1^2\left(-6+\sum\limits_{n=2}^{7}\left(nx^{n-1}\right)\right)dx=\left(-6x+\sum\limits_{n=2}^{7}x^n\right)\Bigg|_1^2=(-12+4+8+16+32+64+128)-(-6+1+1$
$+1+1+1+1)=240$

2. $2^{\frac{T}{20}-1}-37=2^{\frac{240}{20}-1}-37=2^{11}-37=2048-37=2011$.

$2012 = 2^2 \cdot 503$ (wasteful)

$2013 = 3 \cdot 11 \cdot 61$ (wasteful)

$2014 = 2 \cdot 19 \cdot 53$ (wasteful)

$2015 = 5 \cdot 13 \cdot 31$ (wasteful)

$2016 = 2^5 \cdot 3^2 \cdot 7$ (wasteful)

2017 is prime, so it is not wasteful

3. $2+0+1+7=10$, which leaves a remainder of 1 when divided by 9, so $T=1$

$$B+1 = 1+\frac{1}{1-0}\int_0^1 xe^x\,dx = 1+\left((x-1)e^x\right)\Big|_0^1 = 1+\left(0-(-1)\right) = 1+1 = 2$$

4. $d = \dfrac{|8\cdot 3+4\cdot 2+8\cdot 4+8|}{\sqrt{8^2+4^2+8^2}} = \dfrac{72}{12} = 6$

5. $A^{-1} = \dfrac{1}{-4}\begin{bmatrix} -6 & -4 \\ -4 & -2 \end{bmatrix} = \begin{bmatrix} 1.5 & 1 \\ 1 & 0.5 \end{bmatrix} \Rightarrow 1.5+1+1+0.5=4$

6. $V = \pi(5)^2\left(\dfrac{9}{\pi}\right) = 225$, so at 3 cubic meters per second, it would take 75 seconds

Relay 3

1. The area enclosed by the ellipse is $A = 4\cdot 2\pi = 8\pi \approx 25.12$, so the answer is 25

2. 1337^{25} ends in 7 and 25^{1337} ends in 5, so $7+5=12$

3. $V = \left|\dfrac{1}{6}\begin{vmatrix} 3 & 1 & 4 \\ 1 & 12 & 9 \\ 2 & 6 & 12 \end{vmatrix}\right| = \left|\dfrac{1}{6}(432+18+24-96-162-12)\right| = \dfrac{1}{6}(204) = 34$

4. $P(\text{win}) = \dfrac{\binom{20}{2}+\binom{26}{2}-\binom{10}{2}}{\binom{52}{2}} = \dfrac{190+325-45}{1326} = \dfrac{470}{1326} = \dfrac{235}{663}$, so $P(\text{lose}) = \dfrac{428}{663}$;

expected value is $39\cdot 34\cdot\dfrac{235}{663} + (-39\cdot 34)\cdot\dfrac{428}{663} = 470-856 = -386$

5. $A \equiv -386 \pmod{197}$, so $A = -386 + 2 \cdot 197 = 8$

$6x^2 + 2x^2 \dfrac{dy}{dx} + 4xy - 4xy\dfrac{dy}{dx} - 2y^2 + x\dfrac{dy}{dx} + y = 0 \Rightarrow 6 + 2\dfrac{dy}{dx} + 8 - 8\dfrac{dy}{dx} - 8 + \dfrac{dy}{dx} + 2 = 0 \Rightarrow$

$-5\dfrac{dy}{dx} = -8 \Rightarrow \dfrac{dy}{dx} = \dfrac{8}{5}$. Since $A = 8,\ B = 5,$ and $A + B = 8 + 5 = 13$

6. $V = \pi\int_0^{\pi}\left(e^x \sin 3x\right)^2 dx = \pi\int_0^{\pi}\left(e^{2x}\sin^2 3x\right)dx = \pi\int_0^{\pi} e^{2x}\left(\dfrac{1-\cos 6x}{2}\right)dx$

$= \dfrac{\pi}{2}\int_0^{\pi}\left(e^{2x} - e^{2x}\cos 6x\right)dx$. Use integration by parts on the second part of the integrand to get the antiderivative $\Rightarrow \dfrac{\pi}{2}\left(\dfrac{1}{2}e^{2x} - \dfrac{3}{20}e^{2x}\sin 6x - \dfrac{1}{20}e^{2x}\cos 6x\right)\Bigg|_0^{\pi}$

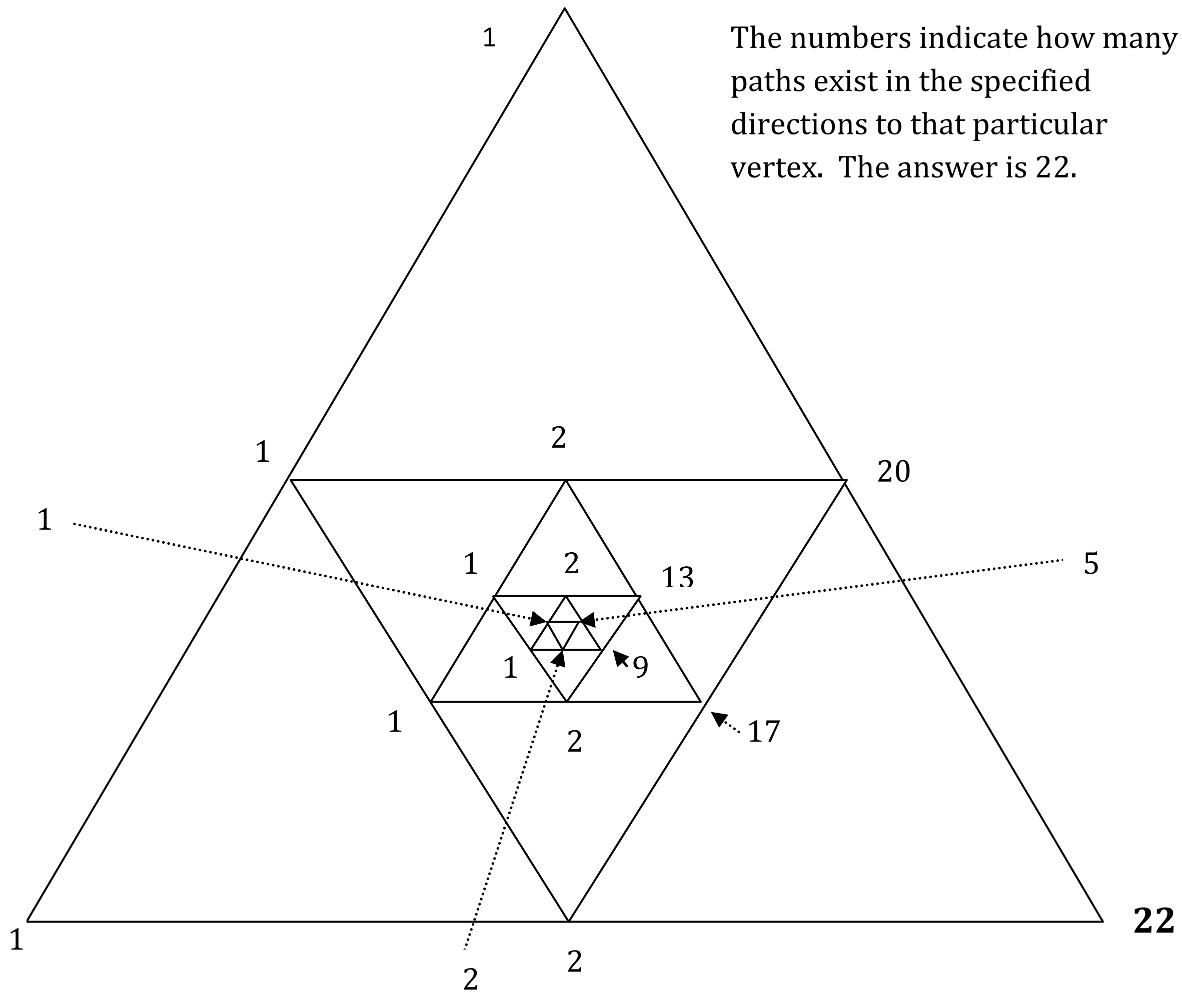
$= \dfrac{\pi}{2}\left(\left(\dfrac{9}{20}e^{2\pi}\right) - \left(\dfrac{9}{20}\right)\right) = \dfrac{9\pi}{40}\left(e^{2\pi} - 1\right) \Rightarrow A = 9,\ B = 40,\ C = 2 \Rightarrow \dfrac{B - 2A}{C} = \dfrac{40 - 2\cdot 9}{2} = 11$

Relay 4

1. $\dfrac{\pi}{A} = \int_0^1 \sqrt{1 - x^2}\,dx = \dfrac{1}{4}\pi(1)^2 = \dfrac{\pi}{4} \Rightarrow A = 4 \Rightarrow A^2 + 1 = 4^2 + 1 = 17$

2. $2011 - 674 = 1337 = A^B + 2\cdot 17 + 7 = A^B + 41 \Rightarrow A^B = 1296 = 6^4 \Rightarrow A = 6$ and $B = 4 \Rightarrow$ $\dbinom{6}{4} = \dfrac{6!}{2!4!} = \dfrac{720}{48} = 15$ (the other possible expressions for 1296 would be 36^2 and 1296^1, which give larger values when performing the combination)

3.



The numbers indicate how many paths exist in the specified directions to that particular vertex. The answer is 22.

4. $(20\times3)(3\times6)=(20\times6)$; right inverse would be (6×20), then transpose is (20×6), multipled by (6×4) gives (20×4), so $B=20\Rightarrow\begin{vmatrix}1 & 3 & 5\\ 2 & -20 & 6\\ 3 & -6 & 9\end{vmatrix}$

$=-180+54-60+300+36-54=96$

5. $A=9+6=15$ and $B=1+5=6$; $\displaystyle\sum_{n=0}^{\infty}\left(\frac{15}{27}\left(\frac{15}{3}\right)^{n-1}6^{1-n}\right)=\frac{5}{9}\sum_{n=0}^{\infty}\left(\frac{5}{6}\right)^{n-1}=\frac{5}{9}\cdot\frac{6/5}{1-5/6}$

$=\frac{5}{9}\cdot\frac{6}{5}\cdot6=4$

6. $f(x)=\frac{d}{dx}\left(\int_0^{\sqrt{x}}t\sin\left(t^2\right)dt\right)=\sqrt{x}\sin\left(\sqrt{x}\right)^2\cdot\frac{1}{2\sqrt{x}}=\frac{1}{2}\sin x$; $\left(4f\left(\pi/4\right)\right)^2=\left(4\cdot\frac{\sqrt{2}}{4}\right)^2=2$

Relay 5

1. $A=2\cdot\frac{1}{2}\int_0^{\pi}\theta^4 d\theta=\frac{1}{5}\theta^5\Big|_0^{\pi}=\frac{\pi^5}{5}$, so $B=5$

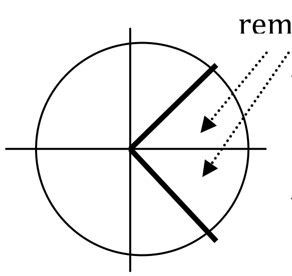
2. Mr. Topry starts on p. 625. Reading until the end of the three-digit pages takes $375\cdot 3=1125$ digits, so there are 1352 digits left, which when divided by 4 equals 338. So the total number of pages is $375+338=713$.

3. $T=11$; $\begin{vmatrix}11&2&3&4\\11&6&7&8\\9&10&11&12\\11&14&15&11\end{vmatrix}=\begin{vmatrix}11&2&3&4\\0&4&4&4\\-2&4&4&4\\11&14&15&11\end{vmatrix}=\begin{vmatrix}11&2&3&4\\2&0&0&0\\-2&4&4&4\\11&14&15&11\end{vmatrix}$

$=-2\begin{vmatrix}2&3&4\\4&4&4\\14&15&11\end{vmatrix}=-2(88+168+240-224-120-132)=-2(20)=-40$

4. $V=2\pi^2(60)(20)^2=48000\pi^2$, so $A=48000$; positive difference between 4 and 8 is 4

5. Cone has radius 1 and height 1. The fly can cover an entire sphere except for the part cut out from the cone and cap of the cone. A side view of the sphere and cone looks like:



$$V=\frac{1}{2}\cdot\frac{4}{3}\pi(1)^3+\pi\int_0^{\sqrt{2}/2}\left(\left(1-x^2\right)-x^2\right)dx=\frac{2}{3}\pi+\pi\left(x-\frac{2}{3}x^3\right)\Bigg|_0^{\sqrt{2}/2}=\frac{2}{3}\pi$$
$$+\pi\left(\frac{\sqrt{2}}{2}-\frac{2}{3}\cdot\frac{1}{2}\cdot\frac{\sqrt{2}}{2}\right)=\pi\left(\frac{2}{3}+\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{6}\right)=\left(\frac{2+\sqrt{2}}{3}\right)\pi$$

Therefore, $A=2$, $B=2$, and $C=3\Rightarrow A^C-B=2^3-2=6$

6. $\sum_{n=1}^{\infty}\frac{24}{n^2+6n+8}=\sum_{n=1}^{\infty}\left(\frac{12}{n+2}-\frac{12}{n+4}\right)=\left(\frac{12}{3}-\frac{12}{5}\right)+\left(\frac{12}{4}-\frac{12}{6}\right)+\left(\frac{12}{5}-\frac{12}{7}\right)+\left(\frac{12}{6}-\frac{12}{8}\right)+\ldots$
Every negative term in a set of parentheses cancels with the positive term in the second set of parentheses afterward. Therefore, the only terms that don't cancel are $\frac{12}{3}$=4 and $\frac{12}{4}=3$, and $4+3=7$.