This test consists of five relays of six questions each. “TAFTPQITR” stands for “the answer from the previous question in the relay”, so if question 3 in a relay references TAFTPQITR, that is the answer from question 2 in that relay.

All answers on this test are integers.

Relay 1

1. Find the value of $2\sqrt{3+2\sqrt{3+2\sqrt{3+\ldots}}}$.

2. Let $T=$ TAFTPQITR. Find the smallest positive prime factor of $1110T+7$.

3. Let $T=$ TAFTPQITR. Find the value of $0.1(T+1)$.

4. Let $T=$ TAFTPQITR. There exists a polynomial of the form $y=Ax^4+Bx^3+Cx^2+Dx+E$, where $A>0$ and A, B, C, D, and E are relatively prime integers, whose roots are $\pm T$ and $\pm(T-4)$. What is the least common multiple of $C+E-B$ and $E+A-D$?

5. Let $T=$ TAFTPQITR. If T is an A-digit number, $B=$ the sum of the digits of T, and $C=\left\lfloor \frac{T}{1000} \right\rfloor$ (the greatest integer less than or equal to $\frac{T}{1000}$), what is the value of $\left(\langle A,B,C\rangle \times \langle 1,2,3\rangle\right)\cdot\langle 3,2,1\rangle$?

6. Let $T=$ TAFTPQITR. If $P(n)=T\left(1+\frac{13}{n}\right)^n$ represents the population of an ant colony after n weeks, and $\lim_{n\to\infty} P(n)=Ae^B$, find the value of $A+B$.

Relay 2

1. Evaluate the integral $\int_1^2\left(-6+\sum_{n=2}^{7}\left(nx^{n-1}\right)\right)dx$.

2. Let $T=$ TAFTPQITR. A wasteful number is a positive integer whose prime factorization uses more digits (including exponents) than the number itself uses. For example, the number 6 uses 1 digit, but the prime factorization of 6 is $2\cdot 3$, which uses 2 digits, making 6 a wasteful number (do not write the number 1 as an exponent when writing the prime factorization). The number 44 is also wasteful as it uses 2 digits while its prime

factorization, $2^2 \cdot 11$, uses 4 digits. What is the smallest non-wasteful number greater than $2^{\frac{T}{20}-1} - 37$?

3. Let $T =$ the remainder when the sum of the digits of TAFTPQITR is divided by 9. If B is the average value of the function $f(x) = xe^{Tx}$ over the interval $[0,1]$, find the value of $B+1$.

4. Let $T =$ TAFTPQITR. What is the shortest distance between the point $(3, T, 2T)$ and the plane with equation $8x + 4y + 8z + 8 = 0$?

5. Let $T =$ TAFTPQITR. If $A = \begin{bmatrix} -2 & 4 \\ 4 & -T \end{bmatrix}$, find the sum of the elements of A^{-1}.

6. Let $T =$ TAFTPQITR. A cylindrical tank with radius of length $T+1$ meters is initially empty. Water begins filling the tank at a rate of 3 cubic meters per second. After how many seconds has the water level risen $9/\pi$ meters?

Relay 3

1. Find the area enclosed by the graph of $2^{-4}\left(x^2 + (2y)^2\right) = 1$, rounded to the nearest whole number.

2. Let $T =$ TAFTPQITR. What is the sum of the unit's digits of 1337^T and T^{1337}?

3. Let $T =$ TAFTPQITR. Find the volume of the tetrahedron with vertices at the points $(3,1,4)$, $(1,T,9)$, $(2,6,T)$, and the origin.

4. Let $T =$ TAFTPQITR. Mr. Lucent Topry is playing a game called "39", where he draws two cards from a standard deck of 52 cards. He gains $39T$ points if both cards are even or if both cards are red. He loses $39T$ points otherwise. What is Mr. Topry's expected value of points after playing this game one time?

5. Let $T =$ TAFTPQITR. If A is the smallest positive integer satisfying $A \equiv T \pmod{197}$, and if the slope of the line tangent to the curve $2x^3 + 2x^2y - 2xy^2 + xy = 0$ at the point $(1,2)$ is given as A/B, find the value of $A+B$.

6. Let $T =$ TAFTPQITR. Mr. Topry is making a new-fangled laser device. The device's shape is that of a solid formed by rotating the region bounded by the function $f(x) = e^x \sin\left((T-10)x\right)$ and the lines $x = 0$, $x = \pi$, and $y = 0$ about the x-axis. The volume of the solid can be written in the form $\frac{A\pi}{B}\left(e^{C\pi} - 1\right)$, where A and B are relatively prime positive integers. What is the value of $\frac{B-2A}{C}$?

Relay 4

1. If $\frac{\pi}{A} = \int_0^1 \sqrt{1-x^2}\,dx$, find the value of $A^2 + 1$.

2. Let $T =$ TAFTPQITR. The year 674 years ago can be represented as $A^B + 2T + 7$, where A and B are positive integers. Find the smallest value of $\binom{A}{B}$.

3. Let $T =$ TAFTPQITR. Equilateral triangle ABC is positioned such that side BC is horizontal, with vertex B to the left of vertex C and with vertex A above side BC. A smaller equilateral triangle DEF is inscribed in triangle ABC, dividing its interior into four non-overlapping equilateral triangles. Triangle DEF is then inscribed with an even smaller equilateral triangle in the same way, and this process repeats until the interior of triangle ABC has been divided into $T+1$ parts. Starting at vertex B, how many paths are there to vertex C, if one can only move along edges of the triangles that go up and to the right, down and to the right, or horizontally to the right, stopping only at a vertex of any of the triangles?

4. Let $T =$ TAFTPQITR. A $(T-2)\times 3$ matrix is multiplied by a 3×6 matrix. The right inverse matrix of the resulting matrix is found, then that matrix is transposed. The resulting matrix is finally multiplied by a 6×4 matrix. Let $B =$ the number of rows in this final matrix. What is the determinant of the matrix $\begin{bmatrix} 1 & 3 & 5 \\ 2 & -B & 6 \\ 3 & -6 & 9 \end{bmatrix}$?

5. Let $T =$ TAFTPQITR. Let $A =$ the sum of the digits of T, and let $B =$ the sum of the digits of A. Evaluate the sum $\sum_{n=0}^{\infty}\left(\frac{A}{27}\left(\frac{A}{3}\right)^{n-1} B^{1-n}\right)$.

6. Let $T=$ TAFTPQITR. If $f(x)=\frac{d}{dx}\left(\int_0^{\sqrt{x}} t\sin\left(t^2\right)dt\right)$, what is the value of $\left(T\cdot f\left(\pi/T\right)\right)^2$?

Relay 5

1. The area enclosed by the graph of $r=\theta^2$ from $\theta=-\pi$ to $\theta=\pi$ can be written as $\frac{\pi^B}{B}$. Find the value of B.

2. Let $T=$ TAFTPQITR. Mr. Topry was reading a book entitled *Codes and Ciphers for the Less Statistically-Inclined*, but he started reading on page T^4, not on page 1, and read consecutively forward in the book. He noticed it took exactly 2477 digits to number all the pages he read. How many pages did he read?

3. Let $T=$ the sum of the digits of TAFTPQITR. Find the value of the determinant
$$\begin{vmatrix} T & 2 & 3 & 4 \\ T & 6 & 7 & 8 \\ 9 & 10 & T & 12 \\ T & 14 & 15 & T \end{vmatrix}.$$

4. Let $T=$ TAFTPQITR. Lucent was eating his breakfast and came across a peculiar donut. The donut's shape was an unusually perfect torus. The length from the center of the hole in the donut to the center of a circular cross-section of the donut had length $100+T$ inches, and the circular cross-section had radius $60+T$ inches. If the volume of the donut before it was promptly eaten could be written as $A\pi^2$ square inches, where A is a positive integer, what is the positive difference between the nonzero digits of A?

5. Let $T=$ TAFTPQITR. A fly is attached to the top vertex of a solid cone with radius of length $T-3$ and height 1 by a flexible rope of length 1. Neither the fly nor the rope can cross into the cone. The total volume the fly can cover can be written as $\frac{A+\sqrt{B}}{C}\pi$, where A, B, and C are relatively prime positive integers. Find the value of A^C-B.

6. Let $T=$ TAFTPQITR. Evaluate the sum $\sum_{n=1}^{\infty}\frac{24}{n^2+Tn+8}$.