Answers:

0. -20
1. 33
2. 360
3. 2
4. $6/7$
5. $\pi/4$
6. 18
7. $4022/447$
8. BAEDFCGH (listed in that order)
9. 47
10. $41/57$
11. $\left(\frac{1}{2},\frac{\sqrt{10}}{5}\right]$
12. 82.6875
13. $24\pi\sqrt{10}$
14. 6

Solutions:

0. $$f(x)=\frac{(x+3)(x-3)}{(x+5)(x-3)}$$

$$A=\frac{x^2}{x^2}=1,\ B=-5,\ C=\frac{3+3}{3+5}=\frac{3}{4},\text{ and } D=3$$

$$\frac{ABD}{C}=\frac{(1)(-5)(3)}{3/4}=-20$$

1. $A_n=$ the largest integer x such that $x!\le 7^n$.

For $n=1$, $x!\le 7^1=7\Rightarrow x=3$. For $n=2$, $x!\le 7^2=49\Rightarrow x=4$. For $n=3$, $x!\le 7^3=343$ $\Rightarrow x=5$. For $n=4$, $x!\le 7^4=2401\Rightarrow x=6$. For $n=5$, $x!\le 7^5=16807\Rightarrow x=7$. For $n=6$, $x!\le 7^6=117659\Rightarrow x=8$. $3+4+5+6+7+8=33$.

2. $5=A(2)^B$ and $25=A(4)^B\Rightarrow A=1$ and $B=\log_2 5$

$5=C(D)^2$ and $25=C(D)^4\Rightarrow C=1$ and $D=\sqrt{5}$ $(D>0)$

$y=\dfrac{5}{1+3(5)^x}$ contains the points $(2,E)$ and $(4,F)$, implying $E=\dfrac{5}{76}$ and $F=\dfrac{5}{1876}$

$$\frac{E-F}{EF}=\frac{1}{F}-\frac{1}{E}=\frac{1876}{5}-\frac{76}{5}=360$$

3. $$\begin{bmatrix}1&1&0\\0&2&-2\\4&2&3\end{bmatrix}^{-1}=\frac{1}{2}\begin{bmatrix}10&-3&-2\\-8&3&2\\-8&2&2\end{bmatrix}$$

$$\begin{bmatrix}A&B&C\\D&E&F\\G&H&I\end{bmatrix}=\frac{1}{2}\begin{bmatrix}10&-3&-2\\-8&3&2\\-8&2&2\end{bmatrix}\begin{bmatrix}-3&5&-1\\-10&6&-6\\0&15&2\end{bmatrix}=\begin{bmatrix}0&1&2\\-3&4&-3\\2&1&0\end{bmatrix}$$

$$(B+C+D+E)^A+(FGH)^I=(1+2-3+4)^0+(-3\cdot 2\cdot 1)^0=1+1=2$$

4. $$A=\left|-\frac{120}{35}+\frac{119}{35}i\right|=\frac{169}{35}$$

$$\left(\frac{450\text{mi}}{\text{hr}}\right)\left(\frac{5280\text{ft}}{\text{mi}}\right)\left(\frac{1\text{hr}}{3600\text{s}}\right)=660\ \text{ft}/\text{s},\ \text{so } B=660$$

$$C=\binom{4+10-1}{10}=\binom{13}{10}=286$$

$$\text{Area} = \frac{1}{2}(5)^2\left(\frac{187.2}{180}\pi\right) = 13\pi, \text{ so } D = 13$$

$$\frac{AB}{CD} = \frac{\left(169/35\right)(660)}{(286)(13)} = \frac{6}{7}$$

5. $$A = \frac{\left|\vec{a}\times\vec{b}\right|}{\vec{a}\cdot\vec{b}} = \frac{\left|\vec{a}\right|\left|\vec{b}\right|\sin\theta}{\left|\vec{a}\right|\left|\vec{b}\right|\cos\theta} = \tan\theta = \tan 75^\circ = 2+\sqrt{3}$$

$$B = \frac{\sqrt{6}-\sqrt{2}}{4}\cos 75^\circ = \frac{\left(\sqrt{6}-\sqrt{2}\right)^2}{16} = \frac{2-\sqrt{3}}{4}$$

$$C = \frac{\sqrt{6}+\sqrt{2}}{4}\cos 75^\circ = \frac{\left(\sqrt{6}+\sqrt{2}\right)\left(\sqrt{6}-\sqrt{2}\right)}{16} = \frac{1}{4}$$

$\left(A\cdot B\right)^C = \left(\frac{\left(2+\sqrt{3}\right)\left(2-\sqrt{3}\right)}{4}\right)^{\frac{1}{4}} = \left(\frac{1}{4}\right)^{\frac{1}{4}} = \frac{\sqrt{2}}{2}$, and the smallest radian angle θ such that

$\sin\theta = \frac{\sqrt{2}}{2}$ is $\theta = \pi/4$

6. $\left(-3,297^\circ\right) = \left(3,117^\circ\right)$, and by Law of Cosines, $A^2 = 4^2+3^2-2(4)(3)\cos(117-57)^\circ$

$$= 16+9-12 = 13 \Rightarrow A = \sqrt{13}$$

$$2700 = 100e^{3B} \Rightarrow e^{3B} = 27 \Rightarrow 3B = \ln 27 \Rightarrow B = \ln 3$$

$C = \text{trace of matrix} = -11$ and $D = \text{determinant of matrix} = 4$

$$\left(A^2+C+D\right)e^B = (13-11+4)e^{\ln 3} = 6\cdot 3 = 18$$

7. $A = 2011$, $B = \frac{2011\cdot 2012}{2} = 2011\cdot 1006$, $C = \frac{2011\cdot 2012\cdot 4023}{6} = 2011\cdot 1006\cdot 1341$

$$S = \frac{1}{2}+\frac{4}{4}+\frac{9}{8}+\frac{16}{16}+\frac{25}{32}+\ldots$$

$$-\frac{1}{2}S = -\frac{1}{4}-\frac{4}{8}-\frac{9}{16}-\frac{16}{32}-\ldots$$

$$\frac{1}{2}S = \frac{1}{2}+\frac{3}{4}+\frac{5}{8}+\frac{7}{16}+\frac{9}{32}+\ldots$$

$$-\frac{1}{4}S = -\frac{1}{4}-\frac{3}{8}-\frac{5}{16}-\frac{7}{32}-\ldots$$

$$\frac{1}{4}S = \frac{1}{2}+\frac{2}{4}+\frac{2}{8}+\frac{2}{16}+\frac{2}{32}+\ldots = \frac{1}{2}+1 = \frac{3}{2} \Rightarrow S = 6$$

Therefore, $6=\frac{2011\cdot 1006\cdot 1341\cdot D}{2011\cdot 2011\cdot 1006}\Rightarrow D=\frac{6\cdot 2011}{1341}=\frac{4022}{447}$.

8. The frequencies of each type of 4-card poker hand are:

Straight flush: $\binom{4}{1}\binom{11}{1}=44$

Four of a kind: $\binom{13}{1}=13$

Flush: $\binom{4}{1}\binom{13}{4}-44=2816$

Straight: $\binom{11}{1}\binom{4}{1}^4-44=2772$

Three of a kind: $\binom{13}{1}\binom{4}{3}\binom{48}{1}=2496$

Two pair: $\binom{13}{2}\binom{4}{2}^2=2808$

One pair: $\binom{13}{1}\binom{4}{2}\binom{12}{2}\binom{4}{1}^2=82368$

High card: $\left(\binom{13}{4}-11\right)\left(\binom{4}{1}^4-4\right)=177408$

The hands are listed from smallest frequency to largest, so the order is BAEDFCGH.

9. $0=x-2\sqrt{x}-15=\left(\sqrt{x}-5\right)\left(\sqrt{x}+3\right)\Rightarrow A=25$

$0=x-5\sqrt{x}+6=\left(\sqrt{x}-3\right)\left(\sqrt{x}-2\right)\Rightarrow B=9+4=13$

$0=x-3\sqrt{x}-4=\left(\sqrt{x}-4\right)\left(\sqrt{x}+1\right)\Rightarrow C=16$

$0=2x+\sqrt{x}-1=\left(2\sqrt{x}-1\right)\left(\sqrt{x}+1\right)\Rightarrow D=1/4$

$A\cdot B+C\cdot D=25\cdot 13+16\cdot 1/4=329=7\cdot 47$, so the largest prime factor is 47.

10. $\tan\left(\tan^{-1}\left(\frac{1}{4}\right)+\tan^{-1}\left(\frac{2}{5}\right)\right)=\frac{\frac{1}{4}+\frac{2}{5}}{1-\frac{1}{4}\cdot\frac{2}{5}}=\frac{13}{18}$. Therefore, $\tan^{-1}X=\tan^{-1}3-\tan^{-1}\left(\frac{13}{18}\right)$

$\Rightarrow X=\tan\left(\tan^{-1}3-\tan^{-1}\left(\frac{13}{18}\right)\right)=\frac{3-\frac{13}{18}}{1+3\cdot\frac{13}{18}}=\frac{41}{57}$

11. For $x\left(2-3x\right)^{-\frac{1}{2}}$, $2-3x>0\Rightarrow x<\frac{2}{3}$.

For $\sqrt{3x+1}$, $3x+1\geq 0\Rightarrow x\geq -\frac{1}{3}$.

For $\ln(2x-1)$, $2x-1>0 \Rightarrow x>\frac{1}{2}$.

For $\sin^{-1}\left(5x^2-1\right)$, $-1\le 5x^2-1\le 1 \Rightarrow 0\le 5x^2\le 2 \Rightarrow 0\le x^2\le \frac{2}{5} \Rightarrow -\frac{\sqrt{10}}{5}\le x\le \frac{\sqrt{10}}{5}$.

The intersection of all domains is $\frac{1}{2}<x\le\frac{\sqrt{10}}{5}$, so the interval is $\left(\frac{1}{2},\frac{\sqrt{10}}{5}\right]$.

12. $0=-4.9A^2+19.6A+156.8=-4.9\left(A^2-4A-32\right)=-4.9(A-8)(A+4)$, so $A=8$.

$0=-4.9(15)^2+29.4(15)+B \Rightarrow B=661.5$

$\frac{B}{A}=\frac{661.5}{8}=82.6875$

13. $0=5x^2-50x+125+6y^2+12y+6-30=5(x-5)^2+6(y+1)^2-30$, which is equivalent to the equation $\frac{(x-5)^2}{6}+\frac{(y+1)^2}{5}=1$, which encloses an area of $\pi\sqrt{30}$.

$4x=y^2+6y+9+4 \Rightarrow x=\frac{1}{4}(y+3)^2+1$, and the area enclosed by its graph and the graph of $x=4$ is $\frac{2}{3}bh$, where b is the length of the line segment portion of $x=4$ between the points where it intersects the parabola, and h is the length between the vertex of the parabola and the line. The two graphs intersect when $y=-3\pm2\sqrt{3}$, so $b=2\cdot2\sqrt{3}=4\sqrt{3}$, and $h=4-1=3$, so the area enclosed is $\frac{2}{3}\left(4\sqrt{3}\right)(3)=8\sqrt{3}$. $A\cdot B=\pi\sqrt{30}\cdot8\sqrt{3}=24\pi\sqrt{10}$.

14. $$\frac{x^6+x^5+2x^4+x^3+2x^2+2x+3}{(x+1)\left(x^2+1\right)\left(x^4+1\right)}=\frac{A\left(x^2+1\right)\left(x^4+1\right)+B(x+1)\left(x^4+1\right)+C(x+1)\left(x^2+1\right)}{(x+1)\left(x^2+1\right)\left(x^4+1\right)}$$

Plugging $x=-1$ into the numerators gives $4=4A \Rightarrow A=1$.

Plugging $x=i$ into the numerators gives $2+2i=B(1+i)(2) \Rightarrow B=1$.

Plugging $x=\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}i$ into the numerators gives $1+\left(1+\frac{2}{\sqrt{2}}\right)i$

$=C\left(1+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}i\right)(1+i)=C\left(1+\left(1+\frac{2}{\sqrt{2}}\right)i\right) \Rightarrow C=1$

$A+2B+3C=1+2+3=6$