

Answers:

0. 266

1. 17.5 or  $\frac{35}{2}$

2. 52

3. 127

4. 52

5. SSNASA

6. 512

7.  $x(x+y)$  or  $x^2 + xy$

8. 5561

9. 12

10.  $64\sqrt{5}$

11.  $\frac{879}{5}$  or 175.8

12. 4

13. 14

14. 520

Solutions:

0.  $A = 126/17$ ,  $B = -14$ ,  $C = 17/9$ , and  $D = 20$

$$ACD + B = \left(\frac{126}{17}\right)\left(\frac{17}{9}\right)(20) + (-14) = 280 - 14 = 266$$

1. The lines through the points  $(3/2, 0)$  and  $(0, -6)$  is  $4x - y = 6$  ( $A = 4, B = -1, C = 6$ ).

$$F = 4(-2) + 3(3) = 1, \text{ and that line has slope } -4/3, \text{ so other line has slope } 3/4.$$

$$-\frac{D}{2} = \frac{3}{4} \Rightarrow D = -\frac{3}{2}. \quad E = -\frac{3}{2}(-2) + 2 \cdot 3 = 3 + 6 = 9$$

$$A + B + C + D + E + F = 4 - 1 + 6 - \frac{3}{2} + 9 + 1 = \frac{35}{2}.$$

2.  $A = \binom{7}{3} + \binom{7}{4} + \binom{7}{5} + \binom{7}{6} + \binom{7}{7} = 35 + 35 + 21 + 7 + 1 = 99$

$$1000N - N = 1256.7 - 1.2 \Rightarrow 999N = 1255.5 \Rightarrow N = \frac{1255.5}{999} = \frac{2511}{1998} = \frac{93}{74} \Rightarrow B = 93 - 74 = 19$$

$$(3 - 2i)x = 5 - 5i \Rightarrow x = \frac{5 - 5i}{3 - 2i} = \frac{25}{13} - \frac{5}{13}i \Rightarrow C = a + b = \frac{25}{13} - \frac{5}{13} = \frac{20}{13}$$

$$\frac{A - B}{C} = \frac{99 - 19}{20/13} = 80 \cdot \frac{13}{20} = 52$$

3. Each interior angle in a regular decagon has interior angle of measure  $144^\circ$ .

Therefore,  $\angle KBC = \angle KCB = 36^\circ$ , making  $A = 180 - 2 \cdot 36 = 108$ .

$$\sqrt{27 \cdot 7 \cdot 14 \cdot 6} = \frac{1}{2} \cdot 21 \cdot B \Rightarrow B = 12.$$

$$C = \frac{6 \cdot 12}{6 + 12} = \frac{72}{18} = 4$$

The first triangle can be proven by AAS, the second triangle can be proven by AAS, and the third triangle can be proven by SSS. The fourth cannot be proven, so  $D = 3$ .

$$A + B + C + D = 108 + 12 + 4 + 3 = 127$$

4. 
$$A = \begin{vmatrix} 2 & 1 & -5 & -6 \\ -1 & 4 & 3 & 5 \\ 1 & 0 & -2 & -3 \\ 3 & -2 & -8 & -10 \end{vmatrix} = \begin{vmatrix} 1 & -5 & -6 \\ 4 & 3 & 5 \\ -2 & -8 & -10 \end{vmatrix} - 2 \begin{vmatrix} 2 & 1 & -6 \\ -1 & 4 & 5 \\ 3 & -2 & -10 \end{vmatrix} + 3 \begin{vmatrix} 2 & 1 & -5 \\ -1 & 4 & 3 \\ 3 & -2 & -8 \end{vmatrix}$$

$$= 16 - 2(5) + 3(-1) = 16 - 10 - 3 = 3$$

Since  $\left\lfloor \frac{44}{5} \right\rfloor + \left\lfloor \frac{44}{25} \right\rfloor = 8 + 1 = 9$ ,  $\left\lfloor \frac{45}{5} \right\rfloor + \left\lfloor \frac{45}{25} \right\rfloor = 9 + 1 = 10$ ,  $\left\lfloor \frac{49}{5} \right\rfloor + \left\lfloor \frac{49}{25} \right\rfloor = 9 + 1 = 10$ , and  $\left\lfloor \frac{50}{5} \right\rfloor + \left\lfloor \frac{50}{25} \right\rfloor = 10 + 2 = 12$ ,  $B$  must be between 45 and 49, inclusive. Since  $B$  is a perfect square,  $B = 49$ .  
 $A + B = 3 + 49 = 52$

5. S: transversal intersects both lines, but two lines could be skew  
 S: three axes in 3-D coordinate system aren't coplanar, two axes and  $y = x$  are coplanar  
 N: if two lines intersected, they'd have a point in common, which when taken into consideration with another point on each of the two lines would make them coplanar, a contradiction  
 A: a postulate of geometry  
 S: they could be parallel or skew  
 A: if the two planes intersected, then one of them would intersect the plane they are parallel to, a contradiction; thus, the planes are parallel  
 SSNASA is the answer

6. 
$$\frac{1}{(x-2)(x-3)} = \frac{1}{2x} \Rightarrow x = 6 = A$$

$$\frac{8B}{8+B} = 5 \Rightarrow B = \frac{40}{3}$$

In 12 seconds, the two machines will make 3 copies each and the other machine will make 2 copies, so 8 in total. Thus 40 copies will be made per minute, yielding 480 copies in 12 minutes.  $C = 12$

$$.4D + .06(175) = .3(D + 175) \Rightarrow .1D = .24(175) = 42 \Rightarrow D = 420$$

$$AB + C + D = 6\left(\frac{40}{3}\right) + 12 + 420 = 80 + 12 + 420 = 512$$

7. 
$$A = \left( x \left( x - \frac{y^2}{x} \right) \div \left( \frac{1}{y} - \frac{1}{x} \right) \right) \div \left( \left( \frac{x^2}{y} - y \right) \left( \frac{x}{y} - 1 \right) \right)^{-2} = x \frac{x^2 - y^2}{x} \frac{xy}{x - y} \frac{y}{x^2 - y^2} \frac{(x - y)^2}{y^2}$$

$$= \frac{x^2(x - y)^3(x + y)y^2}{x(x - y)^2(x + y)y^2} = x(x - y)$$

$$B = \left( \frac{x^3}{y^3} - 1 \right) \left( \frac{x^3 - 3x^2y + 3xy^2 - y^3}{x^3 - y^3} \right) \left( \frac{x}{y} - 1 \right)^{-1} \div \left( \frac{x^2}{y^2} - 1 \right) = \frac{x^3 - y^3}{y^3} \frac{(x - y)^3}{x^3 - y^3} \left( \frac{y}{x - y} \right) \frac{y^2}{x^2 - y^2}$$

$$= \frac{(x - y)^4(x^2 + xy + y^2)y^3}{y^3(x - y)^3(x + y)(x^2 + xy + y^2)} = \frac{x - y}{x + y}$$

$$\frac{A}{B} = \frac{x(x-y)}{\frac{(x-y)}{(x+y)}} = x(x+y)$$

$$8. \quad A = \sum_{n=1}^9 (-2)^{n-1} = \frac{1 - (-2)^9}{1 - (-2)} = \frac{1 + 512}{1 + 2} = \frac{513}{3} = 171$$

$$B = \sum_{n=0}^{\infty} \left(\frac{1}{10}\right)^n = \frac{1}{1 - 1/10} = \frac{1}{9/10} = \frac{10}{9}$$

$$C = \sum_{n=1}^{100} (n+3) = \frac{100}{2}(4+103) = 50(107) = 5350$$

$$-143 = -3 + (D-1)(-7) \Rightarrow -140 = (D-1)(-7) \Rightarrow D-1 = 20 \Rightarrow D = 21$$

$$AB + C + D = 171\left(\frac{10}{9}\right) + 5350 + 21 = 190 + 5350 + 21 = 5561$$

$$9. \quad 4 = h(0) = h(A) = 15A - \frac{3}{4}A^2 + 4 \Rightarrow A\left(15 - \frac{3}{4}A\right) = 0 \Rightarrow A = 20$$

$$B = (x+5)(x-5) - (x+15)(x-15) = (x^2 - 25) - (x^2 - 225) = -25 + 225 = 200$$

$$C = -3^2 + 0! + 6x^0 = -9 + 1 + 6 = -2$$

$$\frac{B}{A} - C = \frac{200}{20} - (-2) = 10 + 2 = 12$$

10. The sum of the solutions is 6, so 2 must be a root.

$$2^3 - 6 \cdot 2^2 - 24 \cdot 2 + A = 0 \Rightarrow A = 48 + 24 - 8 = 64$$

$$k^2 + 2 + \frac{1}{k^2} = 9 \text{ and } k^2 - 2 + \frac{1}{k^2} = B^2 \Rightarrow B^2 + 2 = 7 \Rightarrow B = \sqrt{5}$$

$$AB = 64\sqrt{5}$$

$$11. \quad A = \frac{180 \cdot 38}{40} = 171$$

$$6 \cdot 8 = 10B \Rightarrow B = \frac{24}{5}$$

$$C = \sqrt{10^2 - 4^2} = \sqrt{100 - 16} = \sqrt{84} = 2\sqrt{21}$$

$$D = 2\sqrt{5^2 - 2^2} = 2\sqrt{25 - 4} = 2\sqrt{21}$$

$$A + B + C - D = 171 + \frac{24}{5} + 2\sqrt{21} - 2\sqrt{21} = \frac{879}{5}$$

12. Slope through origin and  $(5,3)$  is  $\frac{3}{5}$ , so first line has slope  $-\frac{5}{3}$ . Since it's through  $(5,3)$ , first line is  $y = -\frac{5}{3}x + \frac{34}{3}$ . Slope through origin and  $(-4,1)$  is  $-\frac{1}{4}$ , so second line has slope 4. Since it's through  $(-4,1)$ , second line is  $y = 4x + 17$ .
- $$-\frac{5}{3}x + \frac{34}{3} = 4x + 17 \Rightarrow \frac{17}{3}x = -\frac{17}{3} \Rightarrow x = -1 \Rightarrow y = 13 \Rightarrow A = -1 + 13 = 12$$
- $$10x + 20y + 50z = 1000, x = 10y \Rightarrow 120y + 50z = 1000 \Rightarrow \text{Since } y \text{ and } z \text{ are positive integers, } y = 5 \text{ and } z = 8 = B$$
- $$A - B = 12 - 8 = 4$$
13.  $A - 2 + 4(A - 7) = 2A \Rightarrow 5A - 30 = 2A \Rightarrow 3A = 30 \Rightarrow A = 10$
- $$Y = -\frac{1}{6} \begin{bmatrix} 3 & -9 \\ -2 & 4 \end{bmatrix} \begin{bmatrix} -4 & 5 \\ -2 & 7 \end{bmatrix} = -\frac{1}{6} \begin{bmatrix} 6 & -48 \\ 0 & 18 \end{bmatrix} = \begin{bmatrix} -1 & 8 \\ 0 & -3 \end{bmatrix} \Rightarrow B = -1 + 8 + 0 - 3 = 4$$
- $$A + B = 10 + 4 = 14$$
14.  $\frac{12}{A-2} + \frac{12}{A+2} = \frac{25}{A} \Rightarrow \frac{24A}{A^2-4} = \frac{25}{A} \Rightarrow 24A^2 = 25A^2 - 100 \Rightarrow A^2 = 100 \Rightarrow A = 10$
- $$\frac{260}{B} = \frac{260}{B+13} + 1 \Rightarrow \frac{260}{B} = \frac{B+273}{B+13} \Rightarrow B^2 + 273B = 260B + 3380 \Rightarrow B^2 + 13B - 3380 = 0$$
- $$\Rightarrow (B+65)(B-52) = 0 \Rightarrow B = 52$$
- $$AB = 10 \cdot 52 = 520$$