Answers:

1. C
2. D
3. B
4. B
5. A
6. B
7. C
8. D
9. A
10. D
11. A
12. A
13. B
14. D
15. A
16. C
17. E
18. B
19. D
20. B
21. D
22. C
23. D
24. B
25. C
26. C
27. A
28. A
29. D
30. C

Solutions:

1. $z \cdot \bar{z} = (3+4i)(3-4i) = 9-16i^2 = 9+16 = 25$

2. $\sqrt{-2} \cdot \sqrt{-8} = i\sqrt{2} \cdot 2i\sqrt{2} = 4i^2 = -4$

3. $i^{-2011} = i^{1-2012} = i^1 = i$

4. $f(1-i) = (1-i)^2 - \dfrac{4}{1-i} = -2i - 2(1+i) = -2-4i$

5. $\dfrac{(5+3i)(1-i)}{(1+i)(1-i)} = \dfrac{5-5i+3i+3}{1+1} = \dfrac{8-2i}{2} = 4-i$

6. Both inequalities represent open discs with radius 5 in the complex plane whose centers lie on the boundary of the other open disc. Therefore, the overlap of the two regions is $2\left(\dfrac{5^2\sqrt{3}}{4}\right) + 4\left(\dfrac{1}{6}\pi(5)^2 - \dfrac{5^2\sqrt{3}}{4}\right) = \dfrac{25\sqrt{3}}{2} + \dfrac{50\pi}{3} - 25\sqrt{3} = \dfrac{50\pi}{3} - \dfrac{25\sqrt{3}}{2}$ $= \dfrac{100\pi - 75\sqrt{3}}{6}$. Divide this by the area enclosed by the original circle, which is 25π, to get the probability $\dfrac{4\pi - 3\sqrt{3}}{6\pi} \approx 0.39$, which is closest to 0.4.

7. Since $a+bi$ is on the vertical line halfway between the first two complex numbers, we must have $a=7$. Also, $a+bi$ must be on the horizontal line halfway between the first and third complex numbers, meaning we must have $b=-4$. Therefore, $a+b = 7-4 = 3$.

8. $\sqrt{-\dfrac{5}{8}} + \sqrt{-\dfrac{1}{40}} = \dfrac{\sqrt{10}}{4}i + \dfrac{\sqrt{10}}{20}i = \dfrac{6\sqrt{10}}{20}i = \dfrac{3\sqrt{10}}{10}i$

9. $\left(1+\sqrt{3}i\right)^{2011} = 2^{2011}\left(\mathrm{c}is60^\circ\right)^{2011} = 2^{2011}\,\mathrm{c}is60^\circ = 2^{2011}\left(\dfrac{1}{2} + \dfrac{\sqrt{3}}{2}i\right) = 2^{2010}\left(1+\sqrt{3}i\right)$

10. Such complex numbers $a+bi$ must satisfy $a^2+b^2 = 625$. The only integer values that would satisfy this equation would be when a and b are 7 and 24, 15 and 20, 0 and 25, or their negatives. For the first two pairs, there are 8 ordered pairs each (each one positive or negative and reversed), and for the last pair, there are 4

ordered pairs (positive or negative 25 and reversed). Therefore, there are 20 total combinations.

11. $(2+3i)(4-i)-(3+2i)(3+i)=8-2i+12i+3-9-3i-6i+2=4+i$

12. $\sin 5x$ is the imaginary part of $(\cos x+i\sin x)^5$, which is $5\cos^4 x\sin x-10\cos^2 x\sin^3 x$ $+\sin^5 x=5\left(1-\sin^2 x\right)^2\sin x-10\left(1-\sin^2 x\right)\sin^3 x+\sin^5 x$. Collecting only the $\sin^3 x$ terms would give a coefficient of $5(-2)-10(1)=-20$

13. First, the common x should be factored out, which gives a real zero of 0. By Descartes' Rule of Signs, $\frac{f(x)}{x}$ has 1 sign change. Additionally, $\frac{f(-x)}{x}=x^6-ax^5$ $+bx^3-cx^2-d$, which has 3 sign changes. Therefore, there are at least 2 imaginary zeros.

14. All of the exponents are divisible by 4, so $i^{2011!}+i^{2010!}+i^{2009!}+i^{2008!}=1+1+1+1=4$.

15. The summation includes 397 of the 398 398th roots of 1, the only one not included is 1. Since the sum of all of the 398th roots is 0, the sought summation is -1.

16. $7+24i=(a+bi)^2=a^2-b^2+2abi\Rightarrow(a,b)=(4,3)$ or $(-4,-3)\Rightarrow|a|+|b|=4+3=7$

17. Using the Law of Cosines, $x^2=4^2+8^2-2(4)(8)\cos\left(\frac{5\pi}{12}-\frac{\pi}{12}\right)=16+64-32=48$ $\Rightarrow x=4\sqrt{3}$

18. $3\sin\left(\omega t+\frac{\pi}{3}\right)=3\cos\left(\frac{\pi}{2}-\left(\omega t+\frac{\pi}{3}\right)\right)=3\cos\left(-\omega t+\frac{\pi}{6}\right)=3\cos\left(\omega t-\frac{\pi}{6}\right)=3e^{-\frac{i\pi}{6}}$

19. The only possible values for the sums that create an integer magnitude would be 5 and 9. There are 4 possible ways to roll each sum, so the probability is $8/36=2/9$.

20. This is the same as the area enclosed by the triangle with coordinates $(2,-1)$, $(-4,2)$, and $(-2,-5)$. Using the Shoelace method, the enclosed area is

$$\begin{array}{r|rr|l} & 2 & -1 & \\ 4 & -4 & 2 & 4 \\ -4 & -2 & -5 & 20 \\ -10 & 2 & -1 & 2 \\ -10 & & & 26 \end{array} \Rightarrow A = \frac{1}{2}\left|-10-26\right| = \frac{1}{2}(36) = 18$$

21. $\ln(-5)+\ln 2=\ln(-1)+\ln 10=i\pi+\ln 10$

22. Multiplying the first equation by $-i$ and adding that to the second equation gives $(5-5i)y=-10i \Rightarrow y=\frac{-2i}{1-i}=1-i$. Plugging this back into the first equation gives $(1+3i)x=-5+5i \Rightarrow x=\frac{-5+5i}{1+3i}=1+2i$. Therefore, $x+2y=1+2i+2(1-i)=3$.

23. $\sinh(-ix)=\frac{e^{-ix}-e^{ix}}{2}=-\sinh(ix)$, and $\sinh(ix)=\frac{e^{ix}-e^{-ix}}{2}=i\sin x$

$\Rightarrow \sinh(-ix)=-i\sin x \Rightarrow \sin x=\sin x=\frac{\sinh(-ix)}{-i}=i\sinh(-ix)$

24. $\left|\frac{(4+2i)(8-6i)}{(3-i)(3+4i)}\right|=\frac{2\sqrt{5}\cdot 10}{\sqrt{10}\cdot 5}=2\sqrt{2}$

25. For I, all the conjugates are the same entries, and the transpose is the same, so it is a Hermitian matrix. For II & III, the imaginary entries would be the negatives of each other, which when transposed do not equal the original matrix. For IV, the imaginary entries are each others' conjugates, so taking the conjugates, then transposing would give the same matrix, so it is a Hermitian matrix. The entries on the main diagonal are not their own conjugates, so V is not. Therefore, only 2 of the matrices are Hermitian matrices.

26. The standard position for $100+99i$ in the complex plane is just slightly less than ${}^{\pi}\!/_{4}$, so raised to the 14th power, the complex number is just slightly less than $14\cdot {}^{\pi}\!/_{4}={}^{7\pi}\!/_{2}$. Therefore, the complex number is in quadrant III.

27. The common ratio is $r=\frac{-14+2i}{18-26i}=\frac{1}{5}-\frac{2}{5}i$, which has a magnitude less than 1.

Therefore, the sum is $\frac{-18-26i}{1-\left(\frac{1}{5}-\frac{2}{5}i\right)}=\frac{-18-26i}{\frac{4}{5}+\frac{2}{5}i}=\frac{-45-65i}{2+i}=(-9-13i)(2-i)$

$=-31-17i$

28. The only real seventh root of unity is 1, and the sum of all the seventh roots of unity is 0, so the sum of the imaginary seventh roots of unity is -1.

29. $\binom{6}{3}x^3i^3=-20ix^3$, so the coefficient is $-20i$

30. The expression $\left||z-z_1|-|z-z_2|\right|=1$ means all complex numbers whose difference between the distances from the two fixed complex numbers is 1. This is the definition of a hyperbola.