

For all questions, answer choice "E) NOTA" means none of the above answers is correct.

1. If $z = 3 + 4i$, find the value of $z \cdot \bar{z}$, where \bar{z} is the complex conjugate of z .

- A) -5 B) 5 C) 25 D) $-7 + 24i$ E) NOTA

2. Evaluate: $\sqrt{-2} \cdot \sqrt{-8}$

- A) $4i$ B) $-4i$ C) 4 D) -4 E) NOTA

3. Evaluate: i^{-2011}

- A) 1 B) i C) -1 D) $-i$ E) NOTA

4. If $f(z) = z^2 - \frac{4}{z}$, find the value of $f(1 - i)$.

- A) $-2i$ B) $-2 - 4i$ C) $-2 + 4i$ D) $-4 + 2i$ E) NOTA

5. Write in $a + bi$ form: $\frac{5 + 3i}{1 + i}$

- A) $4 - i$ B) $4 + i$ C) $1 - 4i$ D) $1 + 4i$ E) NOTA

6. A random complex number z is selected such that $|z - (5 + 2i)| < 5$. Which of the following is closest to the probability that $|z - (2 - 2i)| < 5$ as well?

- A) 0.2 B) 0.4 C) 0.6 D) 0.8 E) 1

7. In the complex plane, $a + bi$ is equidistant from $4 + 2i$, $10 + 2i$, and $4 - 10i$. Evaluate $a + b$.

- A) -2 B) 2 C) 3 D) 6 E) NOTA

8. Simplify: $\sqrt{-\frac{5}{8}} + \sqrt{-\frac{1}{40}}$

- A) $-\frac{3\sqrt{10}}{10}i$ B) $-\frac{\sqrt{10}}{5}i$ C) $\frac{\sqrt{10}}{5}i$ D) $\frac{3\sqrt{10}}{10}i$ E) NOTA

9. Evaluate: $(1 + \sqrt{3}i)^{2011}$

- A) $2^{2010}(1 + \sqrt{3}i)$ B) $2^{2010}(1 - \sqrt{3}i)$ C) $2^{2011}(1 + \sqrt{3}i)$ D) $2^{2011}(1 - \sqrt{3}i)$ E) NOTA

10. How many complex numbers whose real and imaginary parts are integers have a magnitude of 25?

- A) 8 B) 12 C) 16 D) 20 E) NOTA

11. Evaluate: $(2 + 3i)(4 - i) - (3 + 2i)(3 + i)$

- A) $4 + i$ B) $-4 + i$ C) $6 + i$ D) $-6 + i$ E) NOTA

12. If $\sin 5x$ is written only in terms of $\sin x$, what is the coefficient of the $\sin^3 x$ term?

- A) -20 B) -10 C) 10 D) 20 E) NOTA

13. If a , b , c , and d are natural numbers, what is the minimum number of imaginary zeros for the polynomial $f(x) = x^7 + ax^6 - bx^4 - cx^3 - dx$?

- A) 0 B) 2 C) 4 D) 6 E) NOTA

14. Evaluate: $i^{2011!} + i^{2010!} + i^{2009!} + i^{2008!}$

- A) 0 B) $2 + 2i$ C) $2 - 2i$ D) 4 E) NOTA

15. Evaluate: $\sum_{n=1}^{397} cis\left(\frac{n\pi}{199}\right)$

- A) -1 B) 0 C) 1 D) 198 E) NOTA

16. If $(a + bi)^2 = 7 + 24i$, where a and b are real numbers, find the value of $|a| + |b|$.

- A) 1 B) 5 C) 7 D) 11 E) NOTA

17. In the complex plane, what is the distance between $4cis\left(\frac{\pi}{12}\right)$ and $8cis\left(\frac{5\pi}{12}\right)$?

- A) $4\sqrt{22}$ B) $6\sqrt{2}$ C) $2\sqrt{22}$ D) $4\sqrt{7}$ E) NOTA

18. In engineering, phasors are often used to represent sinusoidal signals. A phasor is a complex number representing the phase and magnitude of the sinusoidal signal. In general, the phasor representation of $A\cos(\omega t + \theta)$ is $Ae^{i\theta}$. Which of the following is a

correct phasor representation of $3\sin\left(\omega t + \frac{\pi}{3}\right)$?

- A) $3e^{-\frac{i\pi}{3}}$ B) $3e^{-\frac{i\pi}{6}}$ C) $3e^{\frac{i\pi}{6}}$ D) $3e^{\frac{i\pi}{3}}$ E) NOTA

19. Two fair six-sided dice are rolled, resulting in a sum of a . What is the probability that the magnitude of $a + 12i$ is an integer?

- A) $\frac{1}{12}$ B) $\frac{1}{9}$ C) $\frac{1}{6}$ D) $\frac{2}{9}$ E) NOTA

20. Three points in the complex plane are defined by $2 - i$, $-4 + 2i$, and $-2 - 5i$. If these three points are connected, what is the area enclosed by the triangle?

- A) 9 B) 18 C) 27 D) 36 E) NOTA

21. Evaluate over complex numbers: $\ln(-5) + \ln 2$

- A) $\ln 10$ B) $i + \ln 10$ C) $\frac{i\pi}{2} + \ln 10$ D) $i\pi + \ln 10$ E) NOTA

22. Let x and y be complex numbers. Evaluate $x + 2y$ if $(1 + 3i)x + (2 + i)y = -2 + 4i$ and $(-3 + i)x + (4 - 3i)y = -4 - 12i$.

- A) $3 + 4i$ B) $3 + 3i$ C) 3 D) $3 - 5i$ E) NOTA

23. If $\sinh x = \frac{e^x - e^{-x}}{2}$, which of the following expressions is equivalent to $\sin x$?

- A) $\sinh(ix)$ B) $i\sinh(ix)$ C) $\sinh(-ix)$ D) $i\sinh(-ix)$ E) NOTA

24. Evaluate: $\left| \frac{(4 + 2i)(8 - 6i)}{(3 - i)(3 + 4i)} \right|$

- A) 2 B) $2\sqrt{2}$ C) 4 D) $4\sqrt{2}$ E) NOTA

25. A Hermitian matrix is a square matrix that is equal to its conjugate transpose (that is, take the conjugate of each element, then take the transpose of the resulting matrix).

How many of the following matrices are Hermitian matrices?

I) $\begin{bmatrix} 2 & 1 \\ 1 & -3 \end{bmatrix}$ II) $\begin{bmatrix} 2 & i \\ i & 3 \end{bmatrix}$ III) $\begin{bmatrix} 2i & 1 \\ 1 & 3i \end{bmatrix}$ IV) $\begin{bmatrix} -2 & 1+i \\ 1-i & 3 \end{bmatrix}$ V) $\begin{bmatrix} 2+i & 1-i \\ 1+i & 2-i \end{bmatrix}$

A) 0 B) 1 C) 2 D) 3 E) NOTA

26. If $(100 + 99i)^{14}$ is plotted in the complex plane, in which quadrant would it be?

A) I B) II C) III D) IV E) NOTA

27. Find the sum of the infinite geometric series $(-18 - 26i) + (-14 + 2i) + (-2 + 6i) + \dots$. The sum uses the same formula as that for an infinite geometric series with real numbers as terms as long as the magnitude of the common ratio is less than 1.

A) $-31 - 17i$ B) $-32 - 16i$ C) $-33 - 15i$ D) the sum does not converge
E) NOTA

28. Find the sum of the seventh roots of unity that are imaginary.

A) -1 B) 0 C) 1 D) 2 E) NOTA

29. What is the coefficient of the x^3 term in the expansion of $(x + i)^6$?

A) -5 B) -20 C) $-5i$ D) $-20i$ E) NOTA

30. Let $z_1 = 3 - i$ and $z_2 = 7 + 2i$. In the complex plane, the locus of points z that satisfy the equation $\|z - z_1\| - \|z - z_2\| = 1$ is best described by what shape?

A) a line B) an ellipse C) a hyperbola D) a parabola E) NOTA