Answers:

1. C
2. E
3. A
4. D
5. B
6. A
7. B
8. A
9. E
10. D
11. A
12. C
13. C
14. D
15. B
16. D
17. B
18. C
19. B
20. E
21. B
22. A
23. C
24. D
25. C
26. E
27. C
28. C
29. A
30. D

Solutions:

1. Since $e < 5 < e^2$, $1 < \ln 5 < 2$.

2. Each expression can be written as the twelfth root of some number. In order, those numbers are 240, 900, 960, and 300, so when written from least to greatest, the order would be I, IV, II, III.

3. $f(x) = x^3 - 2x^2 - 24x = x(x-6)(x+4)$, so $0 = f(w^3+1) = (w^3+1)(w^3-5)(w^3+5)$. Therefore, the three real solutions are -1 and $\pm\sqrt[3]{5}$, the sum of which is -1.

4. $\dfrac{4}{A^2} = \dfrac{9B}{A} - 2B^2 \Rightarrow 4 = 9BA - 2B^2A^2 \Rightarrow 0 = 2(AB)^2 - 9AB + 4 = (2AB-1)(AB-4)$
$\Rightarrow AB = \dfrac{1}{2}$ or $AB = 4 \Rightarrow A = \dfrac{1}{2}B^{-1}$ or $A = 4B^{-1}$. $\dfrac{1}{2} - 1 + 4 - 1 = \dfrac{5}{2}$

5. $\left(x + \dfrac{1}{x}\right)^2 = x^2 + \dfrac{1}{x^2} + 2 = \dfrac{6}{7} + \dfrac{7}{6} + 2 = \dfrac{36+49+84}{42} = \dfrac{169}{42}$

6. $3 = \log_2\left|x^2+2x-3\right| - \log_2\left|x+3\right| = \log_2\left|x-1\right| \Rightarrow \left|x-1\right| = 2^3 = 8 \Rightarrow x = -7$ or $x = 9$, and the sum of those value is $-7+9=2$.

7. Let s be the side length of the two shapes. Then $\dfrac{s^2\sqrt{3}}{4} = \dfrac{1}{2}\log b^4 = 2\log b$ and $s^2 = \log a^3 = 3\log a$. Therefore, $\dfrac{8}{\sqrt{3}}\log b = 3\log a \Rightarrow \log_a b = \dfrac{\log b}{\log a} = \dfrac{3\sqrt{3}}{8}$.

8. $x^2 = x+6 \Rightarrow 0 = x^2 - x - 6 = (x-3)(x+2) \Rightarrow x = 3$ or $x = -2$, and both expressions are defined for both values, so the sum of the y-values is $\log_6(3)^2 + \log_6(-2)^2$
$= \log_6 9 + \log_6 4 = \log_6 36 = 2$.

9. $0 = 4x^{4/3} - 5x^{2/3} - 9 = \left(4x^{2/3} - 9\right)\left(x^{2/3}+1\right) \Rightarrow x^{2/3} = \dfrac{9}{4} \Rightarrow x = \pm\dfrac{27}{8}$ (there is no real solution to the second factor), so the sum of the real solutions is $-\dfrac{27}{8} + \dfrac{27}{8} = 0$.

10. $\displaystyle\sum_{n=2}^{2011} \frac{1}{\log_n 2011!} = \sum_{n=2}^{2011} \log_{2011!} n = \log_{2011!} 2011! = 1$

11. I is false, but would be true if the two parts on the right side of the equal sign were added instead. II is true by a simple property of logarithms. III is false, but would be true if only the x and y were divided on the right, and the logarithm were taken of that quotient. IV is false, but would be true if the -3 on the right side of the equal sign were a 3. Thus, only 1 statement is true.

12. $5^{4/9} \approx \left(3^{1.5}\right)^{4/9} = 3^{2/3}$, which would be the x-value on the graph where the y-value is $2/3$. Using the graph, that value is approximately 2.1.

13. $\log 6 = \log 3 + \log 2 = \log 3 + (1 - \log 5) = .4771 + 1 - .6990 = .7781$

14. $\frac{1}{2} = \sum_{i=3}^{\infty}\left(4^x\right)^{i-1} = \frac{4^{2x}}{1-4^x} \Rightarrow 2\left(4^{2x}\right) = 1 - 4^x \Rightarrow 0 = 2\left(4^{2x}\right) + 4^x - 1 = \left(2\cdot 4^x - 1\right)\left(4^x + 1\right)$, and because no real value satisfies the second factor, we must have $4^x = 1/2 \Rightarrow x = -1/2$.

15. $\Psi\left(\Phi\left(\Omega(2,262144),2\right),\Phi(8,27)\right) = \Psi\left(\Phi\left(\sqrt{262144},2\right),\Phi(8,27)\right)$
$= \Psi\left(\Phi(512,2),\Phi(8,27)\right) = \Psi\left(\log_2 512, \log_{27} 8\right) = \Psi\left(9, \log_3 2\right) = 9^{\log_3 2} = 2^{\log_3 9} = 2^2 = 4$

16. $\sum_{k=1}^{5} 5^{k+\log_5 k} = \sum_{k=1}^{5} k\cdot 5^k = 1\cdot 5^1 + 2\cdot 5^2 + 3\cdot 5^3 + 4\cdot 5^4 + 5\cdot 5^5 = 1\cdot 5^6 + 0\cdot 5^5 + 4\cdot 5^4 + 3\cdot 5^3 + 2\cdot 5^2$ $+1\cdot 5^1 + 0\cdot 5^0$, so the base-5 representation is 1043210, and the sum of those digits is $1+0+4+3+2+1+0 = 11$.

17. $\frac{3ab}{3ab+1} = \frac{3(\log_8 16)(\log_{16} 5)}{3(\log_8 16)(\log_{16} 5)+1} = \frac{3\log_8 5}{3\log_8 5 + 1} = \frac{\log_2 5}{\log_2 5 + 1} = \frac{\log_2 5}{\log_2 10} = \log 5$

18. $\log(\log A) + \log B = \log(\log C) \Rightarrow \log(B\log A) = \log(\log C) \Rightarrow B\log A = \log C$
$\Rightarrow \log A^B = \log C \Rightarrow A^B = C \Rightarrow A = \sqrt[B]{C}$

19. Take the base-3 logarithm of both sides of the second equation to get $\log_3 x^{\log_y x} = \log_3 3 \Rightarrow \left(\log_y x\right)\left(\log_3 x\right) = 1 \Rightarrow \log_3 x = \log_x y \Rightarrow y = x^{\log_3 x}$. Plugging this into the first equation and taking the base-3 logarithm of both sides gives $8 = \log_3 81^2 = \log_3\left(x^{\log_3 x}\right)^{\log_3 x} = \left(\log_3 x\right)^3 \Rightarrow \log_3 x = 2 \Rightarrow x = 3^2 = 9$.

20. $\ln\left(\log_4\left(x^2-5\right)\right)<0 \Rightarrow 0<\log_4\left(x^2-5\right)<1 \Rightarrow 1<x^2-5<4 \Rightarrow 6<x^2<9$, which contains no integers

21. $2<\dfrac{1}{\log_2 x}+\dfrac{1}{\log_3 x}+\dfrac{1}{\log_4 x}=\log_x 2+\log_x 3+\log_x 4=\log_x 24 \Rightarrow x^2<24$, but x must be positive and not equal to 1, so there are three integer solutions (2, 3, and 4).

22. $2^{4a-b}=\log_7 2401=4 \Rightarrow 4a-b=2$. $\log_4\left(98a+138b\right)=5 \Rightarrow 98a+138b=4^5=1024$. Solving this system gives $a=2$ and $b=6$, so $ab=12$.

23. $$\frac{4\left(\sqrt[3]{9}-\sqrt[3]{15}+\sqrt[3]{25}\right)}{\left(\sqrt[3]{3}+\sqrt[3]{5}\right)\left(\sqrt[3]{9}-\sqrt[3]{15}+\sqrt[3]{25}\right)}=\frac{4\left(\sqrt[3]{9}-\sqrt[3]{15}+\sqrt[3]{25}\right)}{8}=\frac{\sqrt[3]{9}-\sqrt[3]{15}+\sqrt[3]{25}}{2}$$

$=\sqrt[3]{\dfrac{9}{8}}-\sqrt[3]{\dfrac{15}{8}}+\sqrt[3]{\dfrac{25}{8}}$, so $\dfrac{AB}{C}=\dfrac{\left(9/8\right)\left(25/8\right)}{15/8}=\dfrac{15}{8}$

24. $x^{2/3}+15=8x^{1/3} \Rightarrow 0=x^{2/3}-8x^{1/3}+15=\left(x^{1/3}-3\right)\left(x^{1/3}-5\right) \Rightarrow x=27$ or $x=125$. The product of these solutions is $27\cdot125=3375$

25. $\displaystyle\prod_{k=1}^{2011}\log_{k+1}\left(k+3\right)=\frac{(\log 2013)(\log 2014)}{(\log 2)(\log 3)}=\left(\log_2 2013\right)\left(\log_3 2014\right)$. The first number in this product is slightly smaller than 11, and the second number in this product is slightly smaller than 7. Therefore, the product is slightly smaller than 77, so the closest number is 75.

26. $\ln\left(f\left(x\right)\right)=ax+b \Rightarrow f\left(x\right)=e^{ax+b}$, which is exponential if $a\neq0$ and is linear if $a=0$. Since the question says the graph MUST be of what type, the answer is linear or exponential, which is none of the choices.

27. $\displaystyle\sum_{k=2}^{2011}\frac{1}{\sqrt{k}+\sqrt{k-1}}=\sum_{k=2}^{2011}\left(\sqrt{k}-\sqrt{k-1}\right)=\sqrt{2011}-\sqrt{1}$, so $A+B=2011+1=2012$

28. Solving the equation for y gives $\left(x+2\right)^{2y}=27-x^3 \Rightarrow 2y=\log_{x+2}\left(27-x^3\right)$

$\Rightarrow y=\dfrac{1}{2}\log_{x+2}\left(27-x^3\right)$. We must have $x+2>0 \Rightarrow x>-2$, $x+2\neq1 \Rightarrow x\neq-1$, and $27-x^3>0 \Rightarrow x<3$. The intersection of these intervals is $\left(-2,-1\right)\cup\left(-1,3\right)$.

29. $5^x = \left(\sqrt[4]{\sqrt[3]{25}}\right)\left(\sqrt{\sqrt{\sqrt{5}}}\right)\left(\sqrt[4]{\sqrt[6]{5}}\right)\left(\sqrt[3]{\sqrt{125}}\right) = 5^{\frac{1}{6}}5^{\frac{1}{8}}5^{\frac{1}{24}}5^{\frac{1}{2}} = 5^{\frac{1}{6}+\frac{1}{8}+\frac{1}{24}+\frac{1}{2}} = 5^{\frac{5}{6}} \Rightarrow x = \frac{5}{6}$

30. The greatest upper bound of the given interval is 2.159112778×10^9, so taking the base-10 logarithm of this number yields $9 < \log(2159112778) < 10$
$\Rightarrow 0 < \log(\log(2159112778)) < 1 \Rightarrow \log(\log(\log(2159112778))) < 0$, meaning that an additional base-10 logarithm could not be taken since it would be the logarithm of a negative number. Therefore, the largest possible integral value of n is 3.