Answers:

1. D
2. B
3. D
4. C
5. B
6. D
7. C
8. B
9. A
10. D
11. C
12. E
13. C
14. E
15. D
16. C
17. D
18. E
19. B
20. C
21. B
22. A
23. D
24. C
25. D
26. A
27. B
28. C
29. D
30. B

Solutions:

1. $g(2)=2f(1)+3=2(3-4+1)+3=2(0)+3=3$

2. $3x-2=2x^2-x-5 \Rightarrow 0=2x^2-4x-3$, so the sum of the x-coordinates is $-\frac{-4}{2}=2$. Since both points lie on f, the sum of the y-coordinates is $3\cdot 2-2\cdot 2=2$. Therefore, the sum of all coordinates is $2+2=4$.

3. For the original case, the number of positive zeros is the number of sign changes of the function, which is 3, and since there are four zeros total and the function has real coefficients, the other zero would be negative. The only other case for the zeros is 1 positive, 1 negative, and 2 imaginary.

4. $2x^3-11x^2+11x-3=(2x-1)(x^2-5x+3)$, so the zeros are $1/2$ or $\frac{5\pm\sqrt{25-12}}{2}$ $=\frac{5\pm\sqrt{13}}{2}$

5. The minimum value occurs at the vertex, which has x-coordinate $\frac{-(-4)}{2\cdot 4}=\frac{1}{2}$. Plugging that value in, the minimum value is $4\left(1/2\right)^2-4\left(1/2\right)-3=1-2-3=-4$.

6. Squaring the first equation and substituting the second gives $\frac{y}{x}=\frac{1}{2}$. Squaring the second equation gives $y^4+\frac{1}{x^4}+2\frac{y^2}{x^2}=9 \Rightarrow y^4+\frac{1}{x^4}=9-2\left(\frac{1}{2}\right)^2=\frac{17}{2}$.

7. Simplifying gives $g(x)=\frac{4x}{2x-1}$. To find the inverse, $x=\frac{4y}{2y-1} \Rightarrow 2xy-x=4y$ $\Rightarrow x=y(2x-4) \Rightarrow y=\frac{x}{2x-4}$.

8. To get points on k, shift points on g 1 to the right and 3 down. Therefore, the translation of the given point is $(-1+1,2-3)=(0,-1)$.

9. $g(5)=\sum_{i=1}^{5}\frac{2i-1}{i}=2(5)-1-\frac{1}{2}-\frac{1}{3}-\frac{1}{4}-\frac{1}{5}=\frac{463}{60}$

10. $\binom{7}{4}(2x)^4(-1)^3 = -35\cdot 16x^4$, so the coefficient is -560

11. The other imaginary zero is $2i$, so the polynomial with the imaginary zeros is x^2+4. The other irrational zero is $2+\sqrt{3}$, so the polynomial with the irrational zeros is x^2-4x+1. Therefore, $f(x)=\left(x^2+4\right)\left(x^2-4x+1\right)=x^4-4x^3+5x^2-16x+4$, making $a+b+c+d=-4+5-16+4=-11$.

12. $g(x)=\dfrac{x^3-2x-4}{x-2}=\dfrac{(x-2)\left(x^2+2x+2\right)}{x-2}$, so the graph has a hole, not an asymptote, at $x=2$. Therefore, the graph behaves just as $y=x^2+2x+2$, just without a point where $x=2$, meaning there are no asymptotes of the graph.

13. $0\ge 1-\dfrac{2}{x-1}-\dfrac{1}{x}=\dfrac{x^2-4x+1}{x(x-1)}$. The zeros of the numerator are $2\pm\sqrt{3}$, so making a sign chart with those two numbers, 0, and 1, the regions that are zero or negative are $\left(0,2-\sqrt{3}\right]\cup\left(1,2+\sqrt{3}\right]$.

14. The list of possible rational roots include all numbers with a divisor of 4 in the numerator and a divisor of 24 in the denominator. Since 1 and 2 are divisors of 4 and 3, 4, and 6 are divisors of 24, all four choices are possible rational roots.

15. Let $f(x)=ax^2+bx+c$. Then $4=4a+2b+c$, $-1=9a-3b+c$, and $5=16a-4b+c$. Subtracting the second equation from the first gives $5=-5a+5b\Rightarrow 1=-a+b$. Subtracting the second equation from the third gives $6=7a-b$. Adding these last two equations together gives $7=6a\Rightarrow a=7/6\Rightarrow b=13/6\Rightarrow c=-5$, so $f(0)=-5$.

16. $3\left(\dfrac{x}{2x-1}\right)-2\left(\dfrac{x^2}{x+1}\right)=\dfrac{3x(x+1)-2x^2(2x-1)}{(2x-1)(x+1)}=\dfrac{-4x^3+5x^2+3x}{2x^2+x-1}$

17. $f(x)=4x^4+8x^2+9=4x^4+12x^2+9-4x^2=\left(2x^2+3\right)^2-4x^2$
$=\left(2x^2+2x+3\right)\left(2x^2-2x+3\right)$, so the zeros of f are the zeros of either polynomial.
The zeros of the two polynomials are $\dfrac{\pm 2\pm\sqrt{4-24}}{4}=\dfrac{\pm 2\pm 2\sqrt{5}i}{4}=\dfrac{\pm 1\pm\sqrt{5}i}{2}$.

18. $f(g(-a))=f(-a-1)=(-a-1)^2+2=a^2+2a+3$, and $g(f(a))=g\left(a^2+2\right)=a^2+1$.

$a^2+2a+3=a^2+1\Rightarrow 2a=-2\Rightarrow a=-1$

19. The two inequalities intersect at the two points $\left(-\tfrac{5}{3},\tfrac{7}{3}\right)$ and $\left(\tfrac{5}{3},\tfrac{13}{3}\right)$, forming a quadrilateral region with the two graphs' vertices, $(1,5)$ and $\left(-\tfrac{1}{2},0\right)$. Using the Shoelace method, the area of the enclosed region is:

$$\begin{array}{c|cc|c} & 1 & 5 & \\ \tfrac{25}{3} & \tfrac{5}{3} & \tfrac{13}{3} & \tfrac{13}{3} \\ -\tfrac{13}{6} & -\tfrac{1}{2} & 0 & 0 \\ 0 & -\tfrac{5}{3} & \tfrac{7}{3} & -\tfrac{7}{6} \\ \tfrac{7}{3} & 1 & 5 & -\tfrac{25}{3} \\ \tfrac{17}{2} & & & -\tfrac{31}{6} \end{array} \Rightarrow A=\frac{1}{2}\left|\tfrac{17}{2}-\left(-\tfrac{31}{6}\right)\right|=\frac{1}{2}\left(\frac{41}{3}\right)=\frac{41}{6}$$

20. Using Descartes' Rule of Signs, the initial case has 4 positive and 1 negative roots. The least real zeros could be 0 positive and 1 negative, making for 4 imaginary roots.

21. Since $f(2)=0$, $8a-8b-2=0\Rightarrow a-b=\tfrac{1}{4}$. Since $f(-3)=1$, $18a+12b-2=1$ $\Rightarrow 3a+2b=\tfrac{1}{2}$. Adding twice the first equation from the second gives $5a=1$ $\Rightarrow a=\tfrac{1}{5}\Rightarrow b=-\tfrac{1}{20}$. Therefore, $2a-3b=2\left(\tfrac{1}{5}\right)-3\left(-\tfrac{1}{20}\right)=\tfrac{2}{5}+\tfrac{3}{20}=\tfrac{11}{20}$.

22. $x^2+2xy+2y^2-10y+3=(x+y)^2+(y-5)^2-22$, which has a minimum value of -22.

23. The sum of terms with x^7 are $x^2\left(\binom{9}{4}x^5(-2)^4\right)-x\left(\binom{9}{3}x^6(-2)^3\right)+4\left(\binom{9}{2}x^7(-2)^2\right)$ $=(2016+672+576)x^7=3264x^7$, so the coefficient is 3264.

24. $f(f(f(f(-4))))=f(f(f(-3)))=f(f(-1))=f(5)=24$

25. $-2x+1\le 3x^2-4x+1\Rightarrow 0\le 3x^2-2x=x(3x-2)$. Making a sign chart with the numbers 0 and $\tfrac{2}{3}$, the nonnegative parts correspond to $(-\infty,0]\cup\left[\tfrac{2}{3},\infty\right)$.

26. $f(3)=3f(2)-f(1)=3\cdot\tfrac{1}{3}-1=0$

$$f(4)=3f(3)-f(2)=3\cdot 0-\tfrac{1}{3}=-\tfrac{1}{3}$$
$$f(5)=3f(4)-f(3)=3\left(-\tfrac{1}{3}\right)-0=-1$$
$$f(6)=3f(5)-f(4)=3(-1)-\left(-\tfrac{1}{3}\right)=-\tfrac{8}{3}$$
$$f(7)=3f(6)-f(5)=3\left(-\tfrac{8}{3}\right)-(-1)=-7$$

27. $3(x+2)^2-2=f(x+2)=g(x)=h(x)=g(x-1)=f(x+1)=3(x+1)^2-2$, so $(x+2)^2$ $=(x+1)^2\Rightarrow x^2+4x+4=x^2+2x+1\Rightarrow 2x=-3\Rightarrow x=-\tfrac{3}{2}$

28. The intersection of the line perpendicular to $y=2x+3$ through the point $(1,3)$ with the line $y=2x+3$ is the sought point. The perpendicular line has equation $y=-\frac{1}{2}x+\frac{7}{2}$, so $2x+3=-\frac{1}{2}x+\frac{7}{2}\Rightarrow\frac{5}{2}x=\frac{1}{2}\Rightarrow x=\frac{1}{5}$. Plugging into either line equation gives $y=\frac{17}{5}$, so the point is $\left(\tfrac{1}{5},\tfrac{17}{5}\right)$.

29. Let $a=x-3$ and $b=x+4$. Then the equation becomes $a^3+b^3=(a+b)^3$ $=a^3+b^3+3ab(a+b)\Rightarrow 3ab(a+b)=0\Rightarrow x-3=0$ or $x+4=0$ or $2x+1=0$, making the solutions to the equation $x=3,\ -4,$ or $-\tfrac{1}{2}$, the least of which is -4.

30. $\dfrac{3}{i^2-3i+2}=\dfrac{3}{(i-1)(i-2)}=3\left(\dfrac{1}{i-2}-\dfrac{1}{i-1}\right)$, so $f(50)=3\sum_{i=3}^{50}\left(\dfrac{1}{i-2}-\dfrac{1}{i-1}\right)=3\left(1-\dfrac{1}{49}\right)$ $=3\left(\tfrac{48}{49}\right)=\tfrac{144}{49}$