Answers:

1. B
2. C
3. C
4. C
5. A
6. E
7. D
8. A
9. D
10. A
11. C
12. A
13. A
14. D
15. B
16. D
17. D
18. E
19. C
20. B
21. D
22. E
23. C
24. C
25. B
26. A
27. C
28. C
29. C
30. A
Solutions:

1. \[ \int_{-3}^{3} (9 - x^2) \, dx = 2 \int_{0}^{3} (9 - x^2) \, dx = 2 \left( \left. 9x - \frac{1}{3}x^3 \right|_{0}^{3} \right) = 2 \left( \left. 9 \cdot 3 - \frac{1}{3} \cdot 3^3 \right) = 2 \cdot 18 = 36 \]

2. \[ \frac{1}{3} \int_{0}^{3} (x^3 - 5x) \, dx = \frac{1}{3} \left( \left. \frac{1}{4}x^4 - \frac{5}{2}x^2 \right|_{0}^{3} \right) = \frac{1}{3} \left( \left. \frac{81}{4} - \frac{45}{2} \right) \right) = \frac{1}{3} \left( \left. -\frac{9}{4} \right) \right) = -\frac{3}{4} \]

3. Substituting \( u = 3x - 1 \), \( du = 3 \, dx \) makes the integral \( \frac{1}{9} \int (u + 1)^{\frac{1}{2}} \, du \)
\[ = \frac{1}{9} \int \left( u^{\frac{3}{2}} + u^{\frac{1}{2}} \right) \, du = \frac{1}{9} \left( \frac{2}{5}u^{\frac{5}{2}} + \frac{2}{3}u^{\frac{3}{2}} \right) + C = \frac{2}{9}u^{\frac{3}{2}} \left( \frac{1}{5} u + \frac{1}{3} \right) + C = \frac{2}{9} (3x - 1)^{\frac{3}{2}} \left( \frac{9x + 2}{15} \right) + C \]
\[ = \frac{2(9x + 2)(3x - 1)^{\frac{3}{2}}}{135} + C \]

4. Substituting \( u = x + 7 \), \( du = dx \) gives \[ \int_{-1}^{3} (f(x + 7) + 1) \, dx = \int_{0}^{10} (f(u) + 1) \, du = A + (10 - 6) = A + 4. \]

5. This is the area above the \( y \)-axis inside the unit circle with radius 10, so the value is \( \frac{1}{2} \left( \pi (10)^2 \right) = 50\pi \).

6. \[ \int \tan^2 x \, dx = \int (\sec^2 x - 1) \, dx = \tan x - x + C \]

7. Since \( x \geq 2011 \), by the Fund. Thm of Calc., \[ F'(x) = \frac{\left( x^2 \right)^2 \sin \left( x^2 \right)}{1 + \sqrt{x^2}} \cdot 2x = \frac{2x^5 \sin \left( x^2 \right)}{1 + x}. \]

8. Since \( v(t) > 0 \) when \( 2 \leq t \leq 4 \), the total distance traveled is the displacement, which is \[ \int_{2}^{4} (2t^3 + 15) \, dt = \left. \left( \frac{1}{2} t^4 + 15t \right) \right|_{2}^{4} = (128 + 60) - (8 + 30) = 150. \]

9. \[ \pi \int_{0}^{1} \left( (2 + x)^2 - (2 + x^2)^2 \right) \, dx = \pi \int_{0}^{1} (4x - 3x^2 - x^4) \, dx = \pi \int_{0}^{1} \left( (x^2 + x + 4)(x - x^2) \right) \, dx \]

10. Graphically, we can see that \[ \int_{1}^{2} g(x) \, dx = 2 - \int_{0}^{1} (x^3 + 1) \, dx = 2 - \left. \left( \frac{1}{4} x^4 + x \right) \right|_{0}^{1} \]
\[
= 2 - \left( \frac{1}{4} + 1 \right) = \frac{3}{4}
\]

11. 
\[
\int_{0}^{4} \frac{2x + 3}{\sqrt{4 - x^2}} \, dx = \int_{0}^{4} \frac{2x}{\sqrt{4 - x^2}} \, dx + \int_{0}^{4} \frac{3}{\sqrt{4 - x^2}} \, dx = \left( -2\sqrt{4 - x^2} + 3 \arcsin \frac{x}{2} \right)_{0}^{4}
\]
\[
= (-2 + \pi) - (-4) = \pi + 2
\]

12. 
\[
\pi \int_{-1}^{0} (x\sqrt{x + 1})^2 \, dx = \pi \left[ \left( x^3 + x^2 \right) \right]_{-1}^{0} = -\pi \left( \frac{1}{4} - \frac{1}{3} \right) = \frac{\pi}{12}
\]

13. 
\[
\frac{1}{2} \left( \frac{1}{2^2} + \frac{2}{3^2} + \frac{1}{4^2} \right) = \frac{1}{2} \left( \frac{1}{4} + \frac{9}{16} \right) = \frac{77}{288}
\]

14. 
\[
10 + \int_{4}^{11} 3\sqrt{t + 5} \, dt = 10 + \left( 2(t + 5)^{3/2} \right)_{4}^{11} = 10 + 128 - 54 = 84
\]

15. Since \( \int_{2}^{2} f(t) \, dt = -6 \), \( \int_{-2}^{2} f(t) \, dt = 6 \), making 6 the last number in the first row. Similarly, the other number in the upper row would be -5. Now, since \( \int_{0}^{2} g(t) \, dt = 4 \) and \( g \) is odd, \( \int_{0}^{2} g(t) \, dt = -\int_{0}^{0} g(t) \, dt = \int_{-2}^{2} g(t) \, dt = 4 \), making 4 the last number in the second row. Similarly, the next to last number in the lower row would be 2. Finally, the middle number in the second row is 0 since that would be a definite integral with 0 as both the upper and lower limits. Thus, \( 6 - 5 + 4 + 2 + 0 = 7 \).

16. 
\[
\int_{1}^{2} 2^{3x} \, dx = \left( \frac{2^{3x}}{3\ln 2} \right)_{1}^{2} = \frac{64 - 8}{3\ln 2} = \frac{56}{\ln 8}
\]

17. 
\[
2\pi \int_{0}^{1} (2 - x)(2x - 5x^{2/3} + 3) \, dx = 2\pi \int_{0}^{1} \left( -2x^2 + 5x^{5/3} + x - 10x^{2/3} + 6 \right) \, dx
\]
\[
= 2\pi \left( \frac{2}{3}x^3 + \frac{15}{8}x^{8/3} + \frac{1}{2}x^2 - 6x^{5/3} + 6x \right)_{0}^{1} = 2\pi \left( \frac{2}{3} + \frac{15}{8} + \frac{1}{2} - 6 + 6 \right) = \frac{41}{12} \pi
\]

18. Letting \( a = 0 \), \( b = 2 \), \( c = 0.1 \), \( d = 0.2 \), \( f(x) = (x - 1)^2 \), and \( g(x) = x(2 - x) \), we can see that A, B, C, and D are not necessarily true.

19. Since \( a \) and \( b \) are distinct, there are \( 2P_2 = 42 \) different choices of limits. Since \( f \) is
strictly increasing and odd, the only possibility of the integral equaling 0 is if the limits are negatives of each other, and there are 6 choices for those. Therefore, the sought probability is \(\frac{42 - 6}{42} = \frac{36}{42} = \frac{6}{7}\).

20. \[\int_0^5 (5 - y)^2 \, dy = \int_0^5 (25 - 10y + y^2) \, dy = \left[25y - 5y^2 + \frac{1}{3}y^3\right]_0^5 = 125 - 125 + \frac{125}{3} = \frac{125}{3}\]

21. \[\int_0^{\pi/2} x \cos x \, dx = \left(x \sin x + \cos x\right)_{\pi/2}^0 = \left(\frac{\pi}{2} + 0\right) - (0 + 1) = \frac{\pi}{2} - 1\]

22. \[
\frac{dy}{dx} = 3x^2 (y^2 + 4) \Rightarrow \int \frac{dy}{y^2 + 4} = \int 3x^2 \, dx \Rightarrow \frac{1}{2} \tan^{-1} \frac{y}{2} = x^3 + C, \text{ which is not equivalent to any of the answer choices.}
\]

23. Since the graphs intersect when \(x = 0\) and \(x = 3\), \[x = \int_0^3 x (3x - x^2) \, dx = \int_0^3 (3x^2 - x^3) \, dx \]
\[= \left[\frac{x^3}{3} - \frac{x^4}{4}\right]_0^1 = \frac{27}{4} - \frac{81}{4} = \frac{27}{4} = \frac{3}{2}.
\]

24. Since the graph of \(f\) is concave down, the exact value will be larger than the Trapezoidal approximation, so \(T < A\). Additionally, since Simpson’s Rule gives exactly values for polynomials with degree less than 3, we must have \(A = S\). Therefore, \(T < S = A\).

25. For this limit, \(\Delta x = 1\) and \(x_0 = 0\). Therefore, \[I = \lim_{n \to \infty} \frac{1}{n} \sum_{i=1}^{n} \frac{i/n}{1 + (i/n)} = \int_0^1 \frac{x}{1 + x} \, dx
\]
\[= \left[x - \ln|1 + x|\right]_0^1 = 1 - \ln 2, \text{ so } e' = e^{1-\ln 2} = \frac{e}{2}.
\]

26. The shaded regions constitute areas where \(\frac{x}{y}\) is closest to an odd integer (the regions moving clockwise continue to intersect the right side of the square at numbers of the form \(\frac{2}{\text{odd integer}}\)). Moving clockwise, the first triangle contains an
area of $\frac{1}{2} \cdot \frac{1}{2} \cdot 1 = \frac{1}{4}$, and for the triangle to the right of the line $y = x$, each has an
altitude of 1, so the total enclosed area is $\frac{1}{2} \cdot 1 \cdot \left(\frac{1}{3} + 2 \left(\frac{1}{5} + \frac{1}{7} + \frac{1}{9} + \frac{1}{11} + \ldots\right)\right)$. Taking
that $\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \ldots$ for $-1 \leq x \leq 1$, the innermost set of parentheses
evaluates to $\tan^{-1} 1 - 1 + \frac{1}{3} = \frac{\pi}{4} - \frac{2}{3}$. Therefore, the sum of all shaded areas is
\[
\frac{1}{4} + \frac{1}{2} \left(\frac{1}{3} + 2 \left(\frac{\pi}{4} - \frac{2}{3}\right)\right) = \frac{1}{4} + \frac{1}{2} \left(\frac{\pi}{2} - 1\right) = \frac{\pi - 1}{4}.
\]

27. $\int_{-2}^{1} \left( f'(x)g(x) + f(x)g'(x) \right) dx = \int_{-2}^{1} \left( f(x)g(x) \right)' dx = f(x)g(x) \bigg|_{-2}^{1} = 1 \cdot 3 - (-5) \cdot 9 = 48$

28. $\int_{-1}^{2} \left( \frac{f'(x)g(x) - f(x)g'(x)}{(f(x) + g(x))^2} \right) dx = \left. \frac{f(x)}{f(x) + g(x)} \right|_{-1}^{2} = \frac{7}{7 + 9} - \frac{1}{1 + 3} = \frac{7}{16} - \frac{1}{4} = \frac{3}{16}$

29. Interpreting the integral as area, $\int_{-2}^{1} |x + 1| dx = \frac{1}{2} \cdot 1 \cdot 1 + \frac{1}{2} \cdot 2 \cdot 2 = \frac{5}{2}$

30. $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cot^3 x \csc x dx = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cos^3 x \sin^2 x dx = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{\cos x \left( 1 - \sin^2 x \right)}{\sin x} dx = \left( - \frac{1 + \sin^2 x}{\sin x} \right) \bigg|_{\frac{\pi}{4}}^{\frac{\pi}{2}} = \left( - \frac{1 + 1}{1} \right)$

$= -2 + \sqrt{2} + \frac{\sqrt{2}}{2} = \frac{3\sqrt{2} - 4}{2}$