Answers:

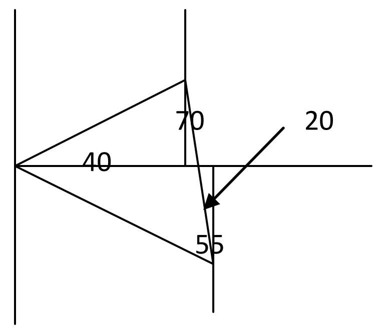
1. A
2. B
3. C
4. D
5. C
6. B
7. A
8. C
9. B
10. D
11. E
12. A
13. C
14. C
15. B
16. B
17. D
18. D
19. C
20. B
21. D
22. A
23. C
24. B
25. A
26. D
27. A
28. B
29. C
30. C

Solutions:

1. The decay is governed by $m = m_0 e^{kt}$. Therefore, $\frac{1}{2} = e^{k(60)} \Rightarrow 60k = \ln\frac{1}{2} = -\ln 2$ $\Rightarrow k = -\frac{\ln 2}{60}$.

2. Let s and c be the number of students and chaperones on the field trips, respectively. Then $s + c = 26$ and $6s + 10c = 168$, and solving this system gives $s = 23$ and $c = 3$. Therefore, the trip to the movie theater cost $\$7(23) + \$12(3)$ $= \$197$.

3. The route the SS Anne took is an isosceles triangle. $\text{N } 50^\circ \text{ E}$ means she was bearing 50° east of due north, and then when she went 20° east of due south, this means the vertex angle was 70°, making each of the base angles 55°. Since the 40° in the right angle from the initial bearing was less than 55°, point B is lower in the plane than Vermillion City. The bearing for this point, as seen in the diagram, is the 55° from the base angle plus the 20° the second leg of the trip was off of vertical. Therefore, the bearing was $\text{N } 75^\circ \text{ W}$.



4. $\sin x + \sin 5x = \sin(3x - 2x) + \sin(3x + 2x) = \sin 3x \cos 2x - \cos 3x \sin 2x + \sin 3x \cos 2x$ $+ \cos 3x \sin 2x = 2\sin 3x \cos 2x$

5. The volume is the $1/6$ of the absolute value of the scalar triple product, and using the origin as the point of radiation, the other points are the vectors for the tetrahedron. Therefore, $A = \frac{1}{6}\left\| \begin{matrix} 1 & 1 & 2 \\ -3 & 1 & 5 \\ 7 & 2 & 4 \end{matrix} \right\| = \frac{1}{6}\left|4 + 35 - 12 - 14 - 10 + 12\right| = \frac{5}{2} = 2.5$.

6. $3x^2 - 6x + 3y^2 - 18y + 3 \le 0 \Rightarrow 3(x-1)^2 + 3(y-3)^2 \le 27 \Rightarrow (x-1)^2 + (y-3)^2 \le 9$, so the region is a disc with radius 3 centered at the point $(1,3)$. Since this region is rotated about $x = 1$, the object is a solid sphere with radius 3. For a sphere, $\frac{A}{V} = \frac{4\pi r^2}{4/3\,\pi r^3} = \frac{3}{r}$, and since $r = 3$, this ratio equals 1.

7. $x=(3x)^2=9x^2 \Rightarrow x=0$ or $x=\frac{1}{9}$. However, 0 is an extraneous solution because $\log_0 0$ is not defined.

8. $\frac{(.25)(.96)}{(.3)(.99)+(.25)(.96)+(.45)(.97)}=\frac{160}{649}$

9. $x^{1/x+1}=x^x \Rightarrow \frac{1}{x}+1=x$ for positive x-values. Therefore, $x^2-x-1=0 \Rightarrow x=\frac{1+\sqrt{5}}{2}$ (the other root doesn't work because it is negative with a 2 in the denominator, meaning a square root would have to be taken of a negative number, contradicting the assumption that the values were real). Checking 1 and -1 also shows that 1 works and -1 does not, but since the product of solutions is being sought, the 1 doesn't make a difference. Therefore, the product is $\frac{1+\sqrt{5}}{2}$.

10. The expected value of the take for the casino is $\frac{1}{6}(P-1)+\frac{5}{36}(P-2)+\frac{25}{216}(P-3)$ $+\frac{125}{216}(P-1)=P-\frac{296}{216}$. This value is wanted to be positive, so $P>\frac{296}{216}\approx 1.37$, so, according to the stipulations, the sought value is \$1.50.

11. This series is $\frac{1}{5}-\frac{2}{25}+\frac{1}{125}-\frac{2}{625}+\ldots=\frac{3}{25}+\frac{3}{625}+\ldots=\frac{3/25}{1-1/25}=\frac{1}{8}$.

12. The lockers that are left open are the ones that have an odd number of divisors, or the perfect squares. The largest perfect square less than 2011 is $44^2=1936$, so the sum of the open lockers' numbers is $\frac{44\cdot 45\cdot 89}{6}=29370$.

13. In 1 second, the pulley does 5 revolutions, and the belt moves 40 feet. Therefore, $5(\pi d)=40 \Rightarrow d=\frac{8}{\pi}$.

14. The units' digit of 9^{412} is 1 since 412 is an even number. The units' digit of 16 to any power is 6, and the units' digit of 5 to any power is 5. Therefore, the units' digit of the sum is the units' digit of $1+6+5=12$, so it is 2.

15. $\log_{30}(\log_{1991} x)>0 \Rightarrow \log_{1991} x>1 \Rightarrow x>1991$

16. The two lines are in the directions of the vectors $\langle 1,-2,0\rangle$ and $\langle -1,5,-2\rangle$, and the angle θ between these vectors satisfies $\cos\theta = \dfrac{\vec{u}\cdot\vec{v}}{\|\vec{u}\|\|\vec{v}\|}$

$$= \frac{-1-10+0}{\sqrt{1^2+(-2)^2+0^2}\sqrt{(-1)^2+5^2+(-2)^2}} = -\frac{11}{\sqrt{150}} = -\frac{11\sqrt{6}}{30}.$$

17. $1 = \log_{\sin\theta}\cos\theta + \log_{\cos\theta}\tan\theta = \dfrac{\ln\cos\theta}{\ln\sin\theta} + \dfrac{\ln\sin\theta - \ln\cos\theta}{\ln\cos\theta} = X + \dfrac{1}{X} - 1$, where $X = \dfrac{\ln\cos\theta}{\ln\sin\theta}$. Therefore, $X + \dfrac{1}{X} = 2 \Rightarrow 0 = X^2 - 2X + 1 = (X-1)^2 \Rightarrow X = 1$. So $\ln\cos\theta = \ln\sin\theta \Rightarrow \cos\theta = \sin\theta \Rightarrow \theta = \dfrac{\pi}{4}$ or $\theta = \dfrac{5\pi}{4}$. However, this second angle is extraneous since its sine and cosine values are negative.

18. $\log x(x+2) = \log 8 \Rightarrow 0 = x^2 + 2x - 8 = (x+4)(x-2) \Rightarrow x = -4$ or $x = 2$. However, the negative answer is extraneous since the logarithms aren't defined at that value.

19. Using Cramer's Rule,

$$z = \frac{\begin{vmatrix} 19 & -12 & 90 \\ 27 & 13 & 72 \\ 3 & -51 & 61 \end{vmatrix}}{\begin{vmatrix} 19 & -12 & 15 \\ 27 & 13 & 75 \\ 3 & -51 & 32 \end{vmatrix}} = \frac{15067 - 2592 - 123930 - 3510 + 69768 + 19764}{7904 - 2700 - 20655 - 585 + 72675 + 10368} = -\frac{25433}{67007}.$$

20. $12 = x^2 - y^2 = (x-y)(x+y) = 2(x+y) \Rightarrow x + y = 6$, which when combined with $x - y = 2$ gives the solutions $x = 4$ and $y = 2$. $xy = 4\cdot 2 = 8$

21. $1.0575(0.8)(15+0.9(30)) = 1.0575(0.8)(42) = 1.0575(33.60) = 35.532$, so rounded to the nearest cent is \$35.53.

22. $\dfrac{4}{5}\cdot\dfrac{1}{5}\cdot\dfrac{9}{16} + \dfrac{4}{5}\cdot\dfrac{4}{5}\cdot\dfrac{7}{16} + \dfrac{1}{5}\cdot\dfrac{1}{5}\cdot\dfrac{7}{16} = \dfrac{36+112+7}{400} = \dfrac{155}{400} = \dfrac{31}{80}$

23. $15 = 18 - 9x + 5x^2 - 2x^2 - 27 + 15x \Rightarrow 0 = 3x^2 + 6x - 24 = 3(x+4)(x-2) \Rightarrow x = -4$ or $x = 2$

24. Mason's July reading total is $2^0+2^1+2^2+...+2^{30}$ since July has 31 days, which is equal to $\frac{1-2^{31}}{1-2}=2^{31}-1$.

25. The sought coefficient is $\binom{7}{3}(2)^3\left(-\frac{1}{3}\right)^4=35(8)\left(\frac{1}{81}\right)=\frac{280}{81}$.

26. ${}_8P_5=\frac{8!}{(8-5)!}=8\cdot7\cdot6\cdot5\cdot4=6720$

27. On January 6, Melissa's favorite time is 12:45 am and Anthony's favorite time is 6:15 pm (Anthony's sequence of favorite times up through January 6 is 12 pm, 10 pm, 2:30 pm, 7:30 pm, 3:45 pm, 6:15 pm). The time halfway between them on January 6 is 9:30 am, and the smaller angle between the clock hands would be $\frac{1}{2}\left|60(9)-11(30)\right|=\frac{1}{2}\left(210^\circ\right)=105^\circ$.

28. Let J be the number of hours it would take Jimmy to prune the garden working alone. Then $\frac{5}{8}+\frac{5}{J}=1\Rightarrow\frac{5}{J}=\frac{3}{8}\Rightarrow J=\frac{40}{3}$ hours.

29. The remainder is the value when -1 is plugged into the polynomial. Therefore, the remainder is $(-1)^{2011}-5(-1)^{1991}+2(-1)^{999}-(-1)^2+3(-1)-1=-1+5-2-1-3-1$ $=-3$.

30. $0!+\lim_{x\to0}\frac{\tan x}{x}=1+\lim_{x\to0}\left(\frac{\sin x}{x}\right)\left(\frac{1}{\cos x}\right)=1+1\left(\frac{1}{\cos 0}\right)=1+1(1)=1+1=2$