Answers:

1. A
2. B
3. C
4. D
5. C
6. B
7. A
8. C
9. B
10. D
11. E
12. A
13. C
14. C
15. B
16. B
17. D
18. D
19. C
20. B
21. D
22. A
23. C
24. B
25. A
26. D
27. A
28. B
29. C
30. C
Solutions:

1. The decay is governed by \( m = m_0e^{kt} \). Therefore, \( \frac{1}{2} = e^{k(60)} \Rightarrow 60k = \ln \frac{1}{2} = -\ln 2 \)
   \[ \Rightarrow k = -\frac{\ln 2}{60}. \]

2. Let \( s \) and \( c \) be the number of students and chaperones on the field trips, respectively. Then \( s + c = 26 \) and \( 6s + 10c = 168 \), and solving this system gives \( s = 23 \) and \( c = 3 \). Therefore, the trip to the movie theater cost \( $7(23) + $12(3) = $197 \).

3. The route the SS Anne took is an isosceles triangle. 
   N 50° E means she was bearing 50° east of due north, and then when she went 20° east of due south, this means the vertex angle was 70°, making each of the base angles 55°. Since the 40° in the right angle from the initial bearing was less than 55°, point B is lower in the plane than Vermillion City. The bearing for this point, as seen in the diagram, is the 55° from the base angle plus the 20° the second leg of the trip was off of vertical. Therefore, the bearing was N 75° W.

4. \[ \sin x + \sin 5x = \sin(3x - 2x) + \sin(3x + 2x) = \sin 3x \cos 2x - \cos 3x \sin 2x + \sin 3x \cos 2x + \cos 3x \sin 2x = 2 \sin 3x \cos 2x \]

5. The volume is the \( \frac{1}{6} \) of the absolute value of the scalar triple product, and using the origin as the point of radiation, the other points are the vectors for the tetrahedron. Therefore, \( A = \frac{1}{6} \begin{vmatrix} 1 & 1 & 2 \\ -3 & 1 & 5 \\ 7 & 2 & 4 \end{vmatrix} = \frac{1}{6} |4 + 35 - 12 - 14 - 10 + 12| = \frac{5}{2} = 2.5. \)

6. \[ 3x^2 - 6x + 3y^2 - 18y + 3 \leq 0 \Rightarrow 3(x - 1)^2 + 3(y - 3)^2 \leq 27 \Rightarrow (x - 1)^2 + (y - 3)^2 \leq 9, \]
   so the region is a disc with radius 3 centered at the point (1,3). Since this region is rotated about \( x = 1 \), the object is a solid sphere with radius 3. For a sphere, \( \frac{A}{V} = \frac{4\pi r^2}{\frac{4}{3}\pi r^3} = \frac{3}{r} \), and since \( r = 3 \), this ratio equals 1.
7. \( x = (3x)^2 = 9x^2 \Rightarrow x = 0 \) or \( x = \frac{1}{9} \). However, 0 is an extraneous solution because \( \log_0 0 \) is not defined.

8. \[
\frac{(0.25)(0.96)}{(0.3)(0.99)+(0.25)(0.96)+(0.45)(0.97)} = \frac{160}{649}
\]

9. \( \frac{1}{x} + 1 = x \) for positive \( x \)-values. Therefore, \( x^2 - x - 1 = 0 \Rightarrow x = \frac{1 + \sqrt{5}}{2} \) (the other root doesn’t work because it is negative with a 2 in the denominator, meaning a square root would have to be taken of a negative number, contradicting the assumption that the values were real). Checking 1 and \(-1\) also shows that 1 works and \(-1\) does not, but since the product of solutions is being sought, the 1 doesn’t make a difference. Therefore, the product is \( \frac{1 + \sqrt{5}}{2} \).

10. The expected value of the take for the casino is \[
\frac{1}{6}(P - 1) + \frac{5}{36}(P - 2) + \frac{25}{216}(P - 3) + \frac{125}{216}(P - 1) = P - \frac{296}{216}. \] This value is wanted to be positive, so \( P > \frac{296}{216} \approx 1.37 \), so, according to the stipulations, the sought value is \$1.50\.

11. This series is \[
\frac{1}{5} - \frac{2}{25} + \frac{1}{125} - \frac{2}{625} + \ldots = \frac{3}{25} + \frac{3}{625} + \ldots = \frac{3/25}{1 - 1/25} = \frac{1}{8}.
\]

12. The lockers that are left open are the ones that have an odd number of divisors, or the perfect squares. The largest perfect square less than 2011 is \( 44^2 = 1936 \), so the sum of the open lockers’ numbers is \( 44 \cdot 45 \cdot 89 = 29370 \).

13. In 1 second, the pulley does 5 revolutions, and the belt moves 40 feet. Therefore, \( 5(\pi d) = 40 \Rightarrow d = \frac{8}{\pi} \).

14. The units’ digit of \( 9^{412} \) is 1 since 412 is an even number. The units’ digit of 16 to any power is 6, and the units’ digit of 5 to any power is 5. Therefore, the units’ digit of the sum is the units’ digit of \( 1 + 6 + 5 = 12 \), so it is 2.

15. \( \log_{30}(\log_{1991} x) > 0 \Rightarrow \log_{1991} x > 1 \Rightarrow x > 1991 \)
16. The two lines are in the directions of the vectors \( \langle 1, -2, 0 \rangle \) and \( \langle -1, 5, -2 \rangle \), and the angle \( \theta \) between these vectors satisfies 
\[
\cos \theta = \frac{\mathbf{u} \cdot \mathbf{v}}{\| \mathbf{u} \| \| \mathbf{v} \|} = \frac{-1 - 10 + 0}{\sqrt{1^2 + (-2)^2 + 0^2} \sqrt{(-1)^2 + 5^2 + (-2)^2}} = -\frac{11}{\sqrt{150}} = -\frac{11\sqrt{6}}{30}.
\]

17. 
\[
\log_{\sin \theta} \cos \theta + \log_{\cos \theta} \tan \theta = \frac{\ln \cos \theta}{\ln \sin \theta} + \frac{\ln \sin \theta - \ln \cos \theta}{\ln \cos \theta} = X + \frac{1}{X} - 1,
\]
where 
\[
X = \frac{\ln \cos \theta}{\ln \sin \theta}.
\]
Therefore, 
\[
X + \frac{1}{X} = 2 \Rightarrow 0 = X^2 - 2X + 1 = (X - 1)^2 \Rightarrow X = 1.
\]
So \( \ln \cos \theta = \ln \sin \theta \Rightarrow \cos \theta = \sin \theta \Rightarrow \theta = \frac{\pi}{4} \) or \( \theta = \frac{5\pi}{4} \). However, this second angle is extraneous since its sine and cosine values are negative.

18. 
\[
\log x(x + 2) = \log 8 \Rightarrow 0 = x^2 + 2x - 8 = (x + 4)(x - 2) \Rightarrow x = -4 \text{ or } x = 2.
\]
However, the negative answer is extraneous since the logarithms aren’t defined at that value.

19. Using Cramer’s Rule,
\[
z = \begin{vmatrix}
19 & -12 & 90 \\
27 & 13 & 72 \\
3 & -51 & 61 \\
\end{vmatrix} = \frac{15067 - 2592 - 123930 - 3510 + 69768 + 19764}{-25433} = \frac{7904 - 2700 - 20655 - 585 + 72675 + 10368}{6707} = \frac{25433}{-6707}.
\]

20. 
\[
12 = x^2 - y^2 = (x - y)(x + y) = 2(x + y) \Rightarrow x + y = 6, \text{ which when combined with } x - y = 2 \text{ gives the solutions } x = 4 \text{ and } y = 2. \quad xy = 4 \cdot 2 = 8
\]

21. 
\[
1.0575(0.8)(15 + 0.9(30)) = 1.0575(0.8)(42) = 1.0575(33.60) = 35.532, \text{ so rounded to the nearest cent is $35.53.}
\]

22. 
\[
\frac{4}{5} + \frac{1}{16} + \frac{9}{16} + \frac{4}{5} + \frac{7}{5} = \frac{36 + 112 + 7}{400} = \frac{155}{400} = \frac{31}{80}
\]

23. 
\[
15 = 18 - 9x + 5x^2 - 2x^2 - 27 + 15x \Rightarrow 0 = 3x^2 + 6x - 24 = 3(x + 4)(x - 2) \Rightarrow x = -4 \text{ or } x = 2
\]
24. Mason’s July reading total is $2^0 + 2^1 + 2^2 + \ldots + 2^{30}$ since July has 31 days, which is equal to $\frac{1 - 2^{31}}{1 - 2} = 2^{31} - 1$.

25. The sought coefficient is \[ \binom{7}{3} \left(\frac{1}{3}\right)^4 = 35 \left(\frac{1}{81}\right) = \frac{280}{81}. \]

26. \[ sP_5 = \frac{8!}{(8-5)!} = 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 = 6720 \]

27. On January 6, Melissa’s favorite time is 12:45 am and Anthony’s favorite time is 6:15 pm (Anthony’s sequence of favorite times up through January 6 is 12 pm, 10 pm, 2:30 pm, 7:30 pm, 3:45 pm, 6:15 pm). The time halfway between them on January 6 is 9:30 am, and the smaller angle between the clock hands would be \[ \frac{1}{2} [60(9) - 11(30)] = \frac{1}{2} (210^\circ) = 105^\circ. \]

28. Let $J$ be the number of hours it would take Jimmy to prune the garden working alone. Then \[ \frac{5}{8} + \frac{5}{J} = 1 \Rightarrow \frac{5}{J} = \frac{3}{8} \Rightarrow J = \frac{40}{3} \text{ hours}. \]

29. The remainder is the value when $-1$ is plugged into the polynomial. Therefore, the remainder is \[ (-1)^{2011} - 5(-1)^{1991} + 2(-1)^{999} - (-1)^2 + 3(-1) - 1 = -1 + 5 - 2 - 1 - 3 - 1 = -3. \]

30. \[ \lim_{x \to 0} \frac{\tan x}{x} = 1 + \lim_{x \to 0} \left(\frac{\sin x}{x}\right) \left(\frac{1}{\cos x}\right) = 1 + 1 \left(\frac{1}{\cos 0}\right) = 1 + 1(1) = 1 + 1 = 2 \]