

For all questions, answer choice "E) NOTA" means none of the above answers is correct.

1. Evaluate:  $\begin{bmatrix} 6 & 7 \\ 2 & 1 \end{bmatrix} - \begin{bmatrix} 5 & -2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & 4 \\ 2 & -1 \end{bmatrix}$

A)  $\begin{bmatrix} 2 & 0 \\ 0 & 5 \end{bmatrix}$       B)  $\begin{bmatrix} 5 & -15 \\ -9 & -7 \end{bmatrix}$       C)  $\begin{bmatrix} 9 & 1 \\ -3 & 14 \end{bmatrix}$       D)  $\begin{bmatrix} -5 & 4 \\ -2 & -7 \end{bmatrix}$       E) NOTA

2. Given  $\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = \begin{bmatrix} 4 & 11 & 6 \\ 7 & 4 & 2 \\ 8 & -3 & 9 \end{bmatrix} \cdot \begin{bmatrix} 3 & 1 & 1 \\ -2 & 7 & 8 \\ 4 & 6 & 5 \end{bmatrix}$ , find the value of  $b - a - e - i$ .

A) 13      B) 17      C) 24      D) 27      E) NOTA

3. Determine the area enclosed by the parallelogram spanned by the vectors  $\vec{u} = \langle 3, 2, 2 \rangle$  and  $\vec{v} = \langle 1, -4, 5 \rangle$ .

A)  $\sqrt{593}$       B)  $\sqrt{689}$       C)  $4\sqrt{46}$       D)  $2\sqrt{218}$       E) NOTA

4. Find the sum of the real values of  $x$  such that  $\vec{u} = \langle e^{3x/2}, e^x, 2e^{x/2}, 1 \rangle$  and

$\vec{v} = \langle e^{3x/2}, -e^x, -e^{x/2}, 2 \rangle$  are perpendicular.

A)  $\ln\sqrt{2}$       B)  $\ln 3$       C)  $\ln\sqrt{5}$       D)  $\ln 6$       E) NOTA

5. Given that  $A$  and  $B$  are invertible  $3 \times 3$  matrices, which of the following is equivalent to

$((AB)^T)^{-1}$ ?

A)  $((A^{-1})(B^{-1}))^T$       B)  $((BA)^T)^{-1}$       C)  $((A^T)(B^T))^{-1}$       D)  $(A^T)^{-1}(B^T)^{-1}$       E) NOTA

6. Find the unit vector in the direction of the vector  $\langle 8, -6 \rangle$ .

A)  $\left\langle -\frac{3}{\sqrt{10}}, \frac{1}{\sqrt{10}} \right\rangle$       B)  $\left\langle \frac{4}{5}, -\frac{3}{5} \right\rangle$       C)  $\left\langle -\frac{1}{\sqrt{10}}, \frac{3}{\sqrt{10}} \right\rangle$       D)  $\left\langle -\frac{4}{\sqrt{53}}, \frac{7}{\sqrt{53}} \right\rangle$       E) NOTA

7. Find the solution to the system of equations 
$$\begin{cases} 2x + 5y - 3z = 8 \\ 5x - y + 2z = 3 \\ x + 2y - z = 6 \end{cases}$$

A)  $\left(-\frac{13}{4}, \frac{53}{4}, \frac{69}{4}\right)$  B)  $\left(-\frac{11}{4}, \frac{57}{4}, \frac{79}{4}\right)$  C)  $\left(-\frac{17}{4}, \frac{51}{4}, \frac{75}{4}\right)$  D)  $\left(-\frac{15}{4}, \frac{55}{4}, \frac{71}{4}\right)$  E) NOTA

8. Which of the following matrices is singular?

A)  $\begin{bmatrix} -3 & 1 & 2 \\ 1 & -2 & -4 \\ 7 & 4 & 8 \end{bmatrix}$  B)  $\begin{bmatrix} 4 & 5 & 2 \\ -4 & 6 & 9 \\ -3 & 1 & 1 \end{bmatrix}$  C)  $\begin{bmatrix} 3 & 4 & 5 \\ 9 & 2 & 0 \\ -1 & 8 & -3 \end{bmatrix}$  D)  $\begin{bmatrix} -3 & 1 & 2 \\ 7 & 0 & -2 \\ 8 & 4 & 5 \end{bmatrix}$  E) NOTA

9. Solve for matrix  $A$  in the equation  $A \begin{bmatrix} -2 & 3 \\ 5 & 1 \end{bmatrix} = 17 \begin{bmatrix} 4 & -1 \\ 6 & 5 \end{bmatrix}$ .

A)  $\begin{bmatrix} -6 & 14 \\ 5 & 10 \end{bmatrix}$  B)  $\begin{bmatrix} -9 & 10 \\ 19 & 28 \end{bmatrix}$  C)  $\begin{bmatrix} -11 & 13 \\ 7 & 26 \end{bmatrix}$  D)  $\begin{bmatrix} -12 & 32 \\ 16 & 7 \end{bmatrix}$  E) NOTA

10. Let  $\theta$  be the smaller angle formed by the vectors  $\langle 3, 1, -4 \rangle$  and  $\langle 2, -1, 5 \rangle$ . What is the value of  $\tan^2 \theta - 1$ ?

A)  $\frac{13}{15}$  B)  $\frac{22}{15}$  C)  $\frac{28}{15}$  D)  $\frac{32}{15}$  E) NOTA

11. Which of the following planes is perpendicular to the plane  $4x - 3y + 2z = 11$ ?

A)  $-x + 2y + z = 9$  B)  $7x - y - z = 4$  C)  $x + 2y + z = 2$  D)  $3x + y - 5z = 2$  E) NOTA

12. Find the trace of the matrix  $\begin{bmatrix} \ln 5 & 23 & \ln 6 \\ 2 & -\ln 3 & 8 \\ 7 & \ln 2 & e^2 \end{bmatrix}$ .

A)  $\ln\left(\frac{5e}{3}\right)$  B)  $\ln\left(\frac{5e^e}{3}\right)$  C)  $\ln\left(\frac{5e^{e^2}}{3}\right)$  D)  $\ln\left(\frac{5e^{e^{e^2}}}{3}\right)$  E) NOTA

13. Evaluate: 
$$\begin{vmatrix} 2 & -3 & 0 & -2 \\ 3 & -1 & 1 & 1 \\ -2 & 1 & -1 & 0 \\ 0 & 3 & 1 & 2 \end{vmatrix}$$

- A) 0                      B) 1                      C) 2                      D) 3                      E) NOTA

14. Which of the following describes the matrix  $\begin{bmatrix} 0 & 3 & -5 \\ -3 & 0 & -1 \\ 5 & 1 & 0 \end{bmatrix}$ ?

- A) invertible      B) symmetric      C) skew-symmetric      D) radial      E) NOTA

15. Find the product of the real values of  $x$  that make the matrix  $\begin{bmatrix} x^2 - 4 & x + 2 \\ 2 & x + 3 \end{bmatrix}$  singular?

- A) 4                      B) 8                      C) 12                      D) 16                      E) NOTA

16. A solid rectangular prism lies in the 3D coordinate system such that one of its vertices is at the origin and two connecting edges are formed by the vectors  $\langle 0, 8, 0 \rangle$  and  $\langle 4, 0, 0 \rangle$ . If the volume of the prism is 64 cubic units, find the length of the longest diagonal of the prism.

- A)  $2\sqrt{21}$                       B)  $4\sqrt{21}$                       C)  $8\sqrt{21}$                       D)  $16\sqrt{21}$                       E) NOTA

17. Given that  $\vec{a} = \langle 3, -5 \rangle$  and  $\vec{b} = \langle -6, 2 \rangle$ , find the vector that is the projection of  $\vec{a}$  onto  $\vec{b}$ .

- A)  $\left\langle -\frac{34}{5}, -\frac{9}{5} \right\rangle$       B)  $\left\langle \frac{21}{5}, -\frac{7}{5} \right\rangle$       C)  $\left\langle -\frac{14}{5}, -\frac{4}{5} \right\rangle$       D)  $\left\langle -\frac{9}{5}, -\frac{1}{5} \right\rangle$       E) NOTA

18. Find the length of the vector  $\langle \sin 23^\circ, \sin 42^\circ, \sin 48^\circ, \sin 67^\circ \rangle$

- A)  $\frac{\sqrt{2}}{2}$                       B)  $\sqrt{2}$                       C)  $\pi$                       D)  $e$                       E) NOTA

19. If  $\vec{a} = \langle 0, 0, -3 \rangle$  and  $\vec{b} = \langle 0, 2, 0 \rangle$ , in what direction is the vector  $\vec{a} \times \vec{b}$ ?

- A) positive  $x$ -axis      B) negative  $y$ -axis      C)  $z$ -axis      D) it is the zero vector      E) NOTA

20. Compute and simplify:  $\begin{vmatrix} \cos 2x & -1 \\ i \sin 2x & 1 \end{vmatrix}$

- A)  $e^{ix}$       B)  $e^{2ix}$       C)  $e^{3ix}$       D)  $e^{4ix}$       E) NOTA

21. Which of the following is equivalent to  $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}^{2011}$ , where  $I$  is the  $3 \times 3$  identity matrix?

- A)  $2^{2010}I$       B)  $2^{2011}I$       C)  $2^{2012}I$       D)  $2^{2013}I$       E) NOTA

22. Given that  $\vec{a} = \langle 2, -1, 7 \rangle$ ,  $\vec{b} = \langle 0, 2, -1 \rangle$ , and  $\vec{c} = \langle 4, -3, 5 \rangle$ , find the scalar triple product  $\vec{a} \cdot (\vec{b} \times \vec{c})$ .

- A) 17      B) -38      C) -26      D) 48      E) NOTA

23. Evaluate:  $\begin{bmatrix} 10.8 & 4.1 \\ 62.4 & 39.3 \end{bmatrix} + \begin{bmatrix} 1072.3 & 2.4 \\ 21.8 & 96.1 \end{bmatrix}$

- A)  $\begin{bmatrix} 1093.1 & 7.5 \\ 24.2 & 35.4 \end{bmatrix}$       B)  $\begin{bmatrix} 1083.1 & 6.5 \\ 84.2 & 135.4 \end{bmatrix}$       C)  $\begin{bmatrix} 1083.1 & 9.5 \\ 64.2 & 1305.4 \end{bmatrix}$       D)  $\begin{bmatrix} 183.1 & 6.5 \\ 4.2 & 105.4 \end{bmatrix}$   
 E) NOTA

24. Given that  $A$  is a square matrix such that  $|A| = 36$ , find  $|A^T|$ .

- A) -36      B)  $\frac{1}{36}$       C) 36      D) 1296      E) NOTA

25. Given that  $A = \begin{bmatrix} -3 & 2 \\ 7 & 5 \end{bmatrix}$  and  $B = \begin{bmatrix} 6 & 4 \\ -2 & 3 \end{bmatrix}$ , find the product  $A^{-1}B^{-1}$ .

- A)  $-\frac{1}{764} \begin{bmatrix} 11 & -32 \\ -27 & 10 \end{bmatrix}$       B)  $\frac{1}{764} \begin{bmatrix} 11 & -32 \\ -27 & 10 \end{bmatrix}$       C)  $-\frac{1}{754} \begin{bmatrix} 11 & -32 \\ -27 & 10 \end{bmatrix}$       D)  $\frac{1}{754} \begin{bmatrix} 11 & -32 \\ -27 & 10 \end{bmatrix}$   
 E) NOTA

26. Which of the following matrices can be used to rotate a point or vector through an angle of  $60^\circ$  counterclockwise?

A)  $\begin{bmatrix} -1/2 & -\sqrt{3}/2 \\ -\sqrt{3}/2 & 1/2 \end{bmatrix}$     B)  $\begin{bmatrix} 1/2 & -\sqrt{3}/2 \\ \sqrt{3}/2 & -1/2 \end{bmatrix}$     C)  $\begin{bmatrix} -1/2 & \sqrt{3}/2 \\ \sqrt{3}/2 & 1/2 \end{bmatrix}$     D)  $\begin{bmatrix} 1/2 & -\sqrt{3}/2 \\ \sqrt{3}/2 & 1/2 \end{bmatrix}$

E) NOTA

27. What is the magnitude of the vector  $6\vec{i} - 4\vec{j} + 3\vec{k}$ ?

A)  $\sqrt{61}$     B) 6    C) 72    D)  $\langle 6, -4, 3 \rangle$     E) NOTA

28. What is the minimum value of the determinant  $|AB|$  if  $A = \begin{bmatrix} x & -3 \\ 2 & 1 \end{bmatrix}$  and  $B = \begin{bmatrix} 4 & -x \\ 1 & 2 \end{bmatrix}$ ?

A) -1    B) -6    C) -8    D) 8    E) NOTA

29. How many of the following products are defined for matrices  $A$ ,  $B$ ,  $C$ , and  $D$  with respective dimensions  $2 \times 2$ ,  $3 \times 2$ ,  $2 \times 3$ , and  $4 \times 3$ ?

I)  $AB$     II)  $CB$     III)  $DB$     IV)  $BC$     V)  $DC$

A) 1    B) 2    C) 3    D) 4    E) NOTA

30. Find the value of  $x$  such that the matrix  $\begin{bmatrix} 3 & 4 \\ x & -3 \end{bmatrix}$  is its own inverse.

A) -3    B) -1    C) 1    D) 3    E) NOTA