Answers:

1. D
2. B
3. E
4. A
5. D
6. A
7. C
8. B
9. C
10. A
11. D
12. C
13. E
14. C
15. B
16. B
17. D
18. B
19. B
20. B
21. C
22. A
23. E
24. D
25. D
26. A
27. D
28. C
29. A
30. D

Solutions:

1. The angle satisfies $\cos\theta = \dfrac{\vec{u}\cdot\vec{v}}{\|\vec{u}\|\|\vec{v}\|} = \dfrac{-3+8+10}{\sqrt{3^2+4^2+5^2}\sqrt{(-1)^2+2^2+2^2}} = \dfrac{15}{5\sqrt{2}\cdot 3} = \dfrac{\sqrt{2}}{2}$, so the acute angle would be 45°.

2. Solving for y, $x^2-1=y^2-2xy \Rightarrow 2x^2-1=(y-x)^2 \Rightarrow y-x=\pm\sqrt{2x^2-1}$ $\Rightarrow y=x\pm\sqrt{2x^2-1}$. Since if (x,y) is on the graph, so is $(-x,-y)$, the graph is odd and thus the center of this conic section is the origin; therefore, the asymptotes go through the origin also. Therefore, the slopes of the asymptotes satisfy $\lim\limits_{x\to\infty}\dfrac{y}{x}=\lim\limits_{x\to\infty}\dfrac{x\pm\sqrt{2x^2-1}}{x}=\lim\limits_{x\to\infty}\left(1\pm\sqrt{2-\dfrac{1}{x^2}}\right)=1\pm\sqrt{2}$, so the smaller slope is $1-\sqrt{2}$.

3. The first graph is the upper half of the circle centered at $(-2,0)$ with radius 2. The distance from the center to the other graph is $\dfrac{|2(-2)+0-11|}{\sqrt{2^2+1^2}}=\dfrac{15}{\sqrt{5}}=3\sqrt{5}$. Therefore, the distance from the half-circle to the line is $3\sqrt{5}-2$ (subtract the radius length).

4. By Shoelace method,
$$\begin{array}{r|rr|r} & 1 & 8 & \\ 24 & 3 & 4 & 4 \\ -4 & -1 & -2 & -6 \\ 0 & 0 & 4 & -4 \\ -28 & -7 & 3 & 0 \\ 3 & 1 & 8 & -56 \\ -5 & & & -62 \end{array} \Rightarrow A=\frac{1}{2}\left|-5-(-62)\right|=\frac{57}{2}=28.5$$

5. Using the first point as the tail for two vectors, the plane contains the vectors $\langle -3,1,-2\rangle$ and $\langle 2,-1,2\rangle$, and the cross-product of those vectors is $\vec{a}\times\vec{b}$ $=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ -3 & 1 & -2\\ 2 & -1 & 2\end{vmatrix}=2\vec{j}+\vec{k}$, so the plane has equation $2y+z=D$. Plugging in any point on the plane, namely $(4,0,5)$, will give $D=2\cdot 0+5=5$, so the equation of the plane is $2y+z=5$. Checking the answer choices, the point in choice D works.

6. Solving the second equation for t and plugging into the first equation gives

$x=8\left(\frac{y+2}{4}\right)^2+6 \Rightarrow x=\frac{1}{2}(y+2)^2+6$, so the vertex of the parabola is at the point $(6,-2)$. $\frac{1}{4p}=\frac{1}{2} \Rightarrow p=\frac{1}{2}$ and $a>0$, so the focus is $\frac{1}{2}$ a unit to the right of the vertex, making it at the point $(6.5,-2)$.

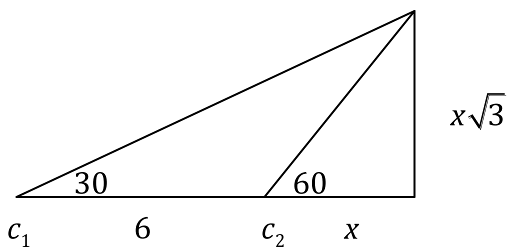
7. This is the upper half of the circle with equation $(y-2)^2+(x-1)^2=4$. Since the reflection followed by rotation of 90° clockwise are the translations to perform, the left-most point of the original graph will have the largest y-coordinate. That point is $(-1,2)$, which becomes $(-1,-2)$ after the reflection, which becomes the point $(-1,-6)$ after the rotation. Therefore, -6 is the largest y-value.

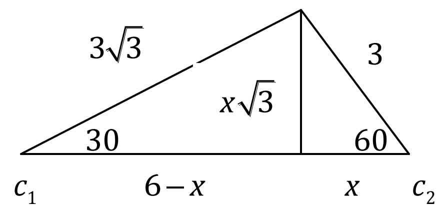
8. Since the area enclosed by the square is 16, the edge length of the square is 4. Also, since opposite corners are the foci of one of the ellipses, the length of the major axis is the sum of two of these edges, which is 8. The distance between foci is the length of a diagonal of the square, which is $4\sqrt{2}$. The length of the minor axis is also the length of the diagonal of the square. Therefore, the area enclosed by an ellipse is $\pi ab=\pi\left(\frac{8}{2}\right)\left(\frac{4\sqrt{2}}{2}\right)=8\pi\sqrt{2}$.

9. The slope of the line is $-1/2$, so the closest point would be the point where the tangent also have slope $-1/2$. $\cos a=-1/2 \Rightarrow a=2\pi/3$.

10. The 90° counterclockwise rotation matrix is $\begin{bmatrix}\cos 90^\circ & -\sin 90^\circ \\ \sin 90^\circ & \cos 90^\circ\end{bmatrix}=\begin{bmatrix}0 & -1 \\ 1 & 0\end{bmatrix}$. A matrix that would project a point onto the x-axis must satisfy $A\begin{bmatrix}x \\ y\end{bmatrix}=\begin{bmatrix}x \\ 0\end{bmatrix}$, so we must have $A=\begin{bmatrix}1 & 0 \\ 0 & 0\end{bmatrix}$. Therefore, the matrix combining the two operation is $\begin{bmatrix}1 & 0 \\ 0 & 0\end{bmatrix}\begin{bmatrix}0 & -1 \\ 1 & 0\end{bmatrix}=\begin{bmatrix}0+0 & -1+0 \\ 0+0 & 0+0\end{bmatrix}=\begin{bmatrix}0 & -1 \\ 0 & 0\end{bmatrix}$.

11. To find the shortest path, reflect $(2,8)$ over the x- and y-axes to yield $(-2,-8)$, then find the distance between that point and $(6,3)$, which is $\sqrt{8^2+11^2}=\sqrt{64+121}$ $=\sqrt{185}$.

12. 



These are the only two diagrams where the product of the coordinates of the third vertex would be positive (other diagrams are reflections of these where P would be in the 2nd or 4th quadrants). For the first diagram, $x\sqrt{3}\cdot\sqrt{3}=6+x \Rightarrow 3x=x+6$ $\Rightarrow x=3$, which makes $P=\left(2+3,3\sqrt{3}\right)=\left(5,3\sqrt{3}\right)$. For the second diagram, $x={}^{3}\!/_{2}$, which makes $P=\left(2-{}^{3}\!/_{2},{}^{3\sqrt{3}}\!/_{2}\right)=\left({}^{1}\!/_{2},{}^{3\sqrt{3}}\!/_{2}\right)$. $(5)\left(3\sqrt{3}\right)+\left({}^{1}\!/_{2}\right)\left({}^{3\sqrt{3}}\!/_{2}\right)=\dfrac{63\sqrt{3}}{4}$.

13. If s is the boat's speed, then $s\tan\theta$ is always constant. $\sin\theta={}^{1}\!/_{2} \Rightarrow \tan\theta={}^{\sqrt{3}}\!/_{3}$, so the constant of proportionality is $5\sqrt{3}$. Therefore, $20\tan\theta=5\sqrt{3} \Rightarrow \tan\theta=\dfrac{\sqrt{3}}{4}$ $\Rightarrow \cos\theta=\dfrac{4}{\sqrt{19}}$.

14. $$\frac{\langle 6,4\rangle\cdot\langle 4,7\rangle}{\left\|\langle 4,7\rangle\right\|^2}\langle 4,7\rangle=\frac{24+28}{16+49}\langle 4,7\rangle=\frac{52}{65}\langle 4,7\rangle=\frac{4}{5}\langle 4,7\rangle=\left\langle\frac{16}{5},\frac{28}{5}\right\rangle$$

15. Since the coefficient of θ is odd, that number is the number of petals, which is 9.

16. The slopes of the two lines are $-\dfrac{3k}{4}$ and $\dfrac{2}{5\sqrt{k}}$, so $-1=-\dfrac{3k}{4}\cdot\dfrac{2}{5\sqrt{k}}=-\dfrac{3\sqrt{k}}{10}$ $\Rightarrow \sqrt{k}=\dfrac{10}{3} \Rightarrow k=\dfrac{100}{9}$.

17. Using Shoelace method, the area enclosed by the triangle is $\begin{array}{r|rr|l} & 1 & 3 & \\ 12 & 4 & 5 & 5 \\ 35 & 7 & 1 & 4 \\ 1 & 1 & 3 & 21 \\ 48 & & & 30 \end{array} \Rightarrow A=\dfrac{1}{2}(48$ $-30)=9$. Therefore, the longest altitude, which is to the shortest side, is $h=\dfrac{2A}{c}$

$$=\frac{2\cdot 9}{\sqrt{(4-1)^2+(5-3)^2}}=\frac{18}{\sqrt{9+4}}=\frac{18}{\sqrt{13}}=\frac{18\sqrt{13}}{13}.$$

18. The vector between the two points is $\langle 2+4,3-1,2-4\rangle=\langle 6,2,-2\rangle$, and this vector is perpendicular to the plane, so the equation of the plane is $6x+2y-2z=A$ for some number A. The midpoint between the two points is $\left(\frac{2-4}{2},\frac{3+1}{2},\frac{2+4}{2}\right)=(-1,2,3)$, and this point is on the plane, so $A=6(-1)+2(2)-2(3)=-8$, making the equation of the plane $6x+2y-2z=-8\Rightarrow x+\frac{1}{3}y-\frac{1}{3}z=-\frac{4}{3}$. $\frac{1}{3}-\frac{1}{3}-\frac{4}{3}=-\frac{4}{3}$

19. $r=6\sin\theta-2\cos\theta\Rightarrow r^2=6r\sin\theta-2r\cos\theta\Rightarrow x^2+y^2=6y-2x\Rightarrow(x+1)^2+(y-3)^2$ $=10$, so the area enclosed is 10π.

20. The midpoint of the segment is $\left(\frac{2-4}{2},\frac{3+1}{2},\frac{2+4}{2}\right)=(-1,2,3)$, and the vector between this point and $(-3,6,1)$ is $\langle -1-(-3),2-6,3-1\rangle=\langle 2,-4,2\rangle$. Therefore the line has parametric equations $x=-3+2t$, $y=6-4t$, and $z=1+2t$. Solving each of these equations for t and setting the expressions equal makes the equation $\frac{x+3}{2}=\frac{y-6}{-4}=\frac{z-1}{2}\Rightarrow\frac{x+3}{1}=\frac{y-6}{-2}=\frac{z-1}{1}$, so $1\cdot-2\cdot1-(1-2+1)=-2$.

21. $r=\frac{2}{3-3\sin\theta}\Rightarrow 3r-3y=2\Rightarrow 3r=3y+2\Rightarrow 9x^2+9y^2=9y^2+12y+4\Rightarrow y=\frac{3}{4}x^2-\frac{1}{3}$. Therefore, the latus rectum length is $4/3$ (the reciprocal of the coefficient of x^2).

22. The distance between the foci is 6, so $c=3$. The sum of the distances from $(4,2)$ to each of the foci is the major axis length, so $2a=\sqrt{(4-2)^2+(2-4)^2}$ $+\sqrt{(4-2)^2+(2+2)^2}=\sqrt{4+4}+\sqrt{4+16}=\sqrt{8}+\sqrt{20}=2\sqrt{2}+2\sqrt{5}\Rightarrow a=\sqrt{2}+\sqrt{5}$. Also, for an ellipse, $b^2=a^2-c^2=\left(\sqrt{2}+\sqrt{5}\right)^2-3^2=7+2\sqrt{10}-9=-2+2\sqrt{10}$. Thus we have $(ab)^2=\left(7+2\sqrt{10}\right)\left(-2+2\sqrt{10}\right)=-14+14\sqrt{10}-4\sqrt{10}+40=26+10\sqrt{10}$.

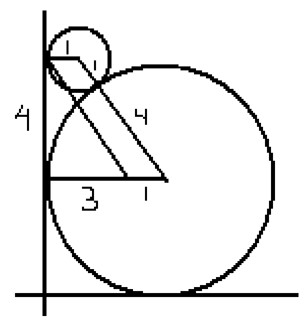
23. $4x^2-24x-2y^2+8y+44=0\Rightarrow 4(x-3)^2-2(y-2)^2=-16\Rightarrow\frac{(y-2)^2}{8}-\frac{(x-3)^2}{4}=1$.

Therefore, $c^2 = 8+4 = 12 \Rightarrow c = 2\sqrt{3}$, and with $a = \sqrt{8} = 2\sqrt{2}$, we have $\varepsilon = \frac{c}{a} = \frac{2\sqrt{3}}{2\sqrt{2}}$

$= \frac{\sqrt{6}}{2}$, meaning $\sin a = \frac{\varepsilon}{2} = \frac{\sqrt{6}}{4}$.

24. The radius of the circle lies on the angle bisector of the two lines whose slopes are 2 and $1/2$, so the angle bisector has slope 1 and passes through the point $(1,3)$, which is the intersection of the two lines. Therefore, the angle bisector has equation $y = x+2$. Now we want a point on this line that is a distance 2 away from the two given lines. Let $(a, a+2)$ be the center of the circle. Then $2 = \frac{|2a-(a+2)+1|}{\sqrt{2^2+(-1)^2}}$ $= \frac{|a-1|}{\sqrt{5}} \Rightarrow |a-1| = 2\sqrt{5}$. Since $a > 0$, $a = 1+2\sqrt{5}$, and therefore $ab = a(a+2)$ $= (1+2\sqrt{5})(3+2\sqrt{5}) = 3+2\sqrt{5}+6\sqrt{5}+20 = 23+8\sqrt{5}$.

25. The line connecting the centers of the two circles in the first quadrant has slope $-4/3$, and it passes through the point $(4,4)$, which is the center of circle A. Therefore, the line has equation $y-4 = -\frac{4}{3}(x-4)$. This line must intersect the line $y = -x$ since that's where the center of circle C must be. The intersection of these two lines satisfies $4-\frac{4}{3}(x-4) = -x$ $0 = 12-4x+16+3x = 28-x \Rightarrow x = -28$. Therefore, the radius of circle C is 28.



26. I is false by looking at an example. If $c = d = 2$, then the equation of the circle becomes $(x+1)^2 + (y+1)^2 = 2$, making the center 1 unit away from both axes but having a radius with length $\sqrt{2}$. II is false also by argument. Since c and d cannot be equal, one of them is smaller; without loss of generality, suppose c is smaller. The equation of the circle could also be expressed as $\left(x+c/2\right)^2 + \left(y+d/2\right)^2 = \frac{c^2}{4}+\frac{d^2}{4}$. In order to be a Chelsea circle, the radius would need to have length $c/2$, or squared radius of $\frac{c^2}{4}$. Clearly the radius of the circle will actually be longer than that, so the circle is not a Chelsea circle.

27. Since f is continuous, we must have $3^2 - a = b\sqrt{3-2} + a \Rightarrow 2a + b = 9$ and $b\sqrt{6-2} + a$ $= 2\cdot 6 + b \Rightarrow b + a = 12$. Solving this system, we get $a = -3$ and $b = 15$. Therefore, $f(1) + f(3) + f(7) = (1^2 + 3) + (15\sqrt{3-2} - 3) + (2\cdot 7 + 15) = 4 + 12 + 29 = 45$.

28. $\dfrac{|18-9|}{\sqrt{4^2 + (-3)^2}} = \dfrac{9}{\sqrt{16+9}} = \dfrac{9}{\sqrt{25}} = \dfrac{9}{5} = 1.8$

29. $0 = 9x^2 - 18x - 4y^2 + 16y - 43 = 9(x-1)^2 - 4(y-2)^2 - 36 \Rightarrow \dfrac{(y-2)^2}{9} - \dfrac{(x-1)^2}{4} = 1$, so the slopes of the asymptotes are $\pm\dfrac{3}{2}$, making the product of those slopes $-\dfrac{9}{4}$.

30. The distance from P to the center of the square will always be $\sqrt{1^2 + 1^2} = \sqrt{1+1}$ $= \sqrt{2}$, so P acts as the center of a circle with that radius length. Therefore, the parametric equations must be sinusoidal. Verifying the points at $t = 0$, $t - 0.5$, and $t = 1$ shows that choice D is correct.