Answers:

1. C
2. C
3. A
4. A
5. E
6. D
7. D
8. C
9. C
10. C
11. C
12. D
13. A
14. B
15. D
16. D
17. A
18. E
19. B
20. B
21. C
22. D
23. D
24. D
25. D
26. D
27. B
28. A
29. D
30. C

Solutions:

1. $\dfrac{dp}{dt}=3000e^{2t/5}\Rightarrow p=7500e^{2t/5}+C$. Since $p(0)=7500$, $C=0\Rightarrow p=7500e^{2t/5}$.
Therefore, at time $t=5$, $p=7500e^{2}$.

2. $SA=2\pi\int_0^2 x\sqrt{1+\left(\dfrac{1}{2\sqrt{y}}\right)^2}dy=2\pi\int_0^2\sqrt{y}\sqrt{1+\dfrac{1}{4y}}dy=\pi\int_0^2\sqrt{4y+1}dy=\dfrac{\pi}{6}(4y+1)^{3/2}\Big|_0^2$
$=\dfrac{\pi}{6}(27-1)=\dfrac{13\pi}{3}$

3. Let ρ be the linear density, then $\rho=x^4$. The center of mass is $\bar{x}=\dfrac{\int_0^L x\rho dx}{\int_0^L \rho dx}$

$$=\frac{\int_0^L x^5dx}{\int_0^L x^4dx}=\frac{\left(\frac{x^6}{6}\right)\Big|_0^L}{\left(\frac{x^5}{5}\right)\Big|_0^L}=\frac{L^6/6}{L^5/5}=\frac{5L}{6}$$

4. $y'=x^{1/2}$, so $L=\int_0^3\sqrt{1+\left(x^{1/2}\right)^2}dx=\int_0^3\sqrt{1+x}dx=\dfrac{2}{3}(1+x)^{3/2}\Big|_0^3=\dfrac{2}{3}(8-1)=\dfrac{14}{3}$

5. When $100\pi=SA=4\pi r^2$, $r=5$. $V=\dfrac{4}{3}\pi r^3\Rightarrow\dfrac{dV}{dt}=4\pi r^2\dfrac{dr}{dt}\Rightarrow\dfrac{dV}{dt}\Big|_{r=5,\ \frac{dr}{dt}=0.3}$
$=4\pi(5)^2(0.3)=30\pi$.

6. I converges because it is a p-series with $p>1$. II diverges because it is the harmonic series. III converges because it is an alternating series with $\dfrac{1}{\sqrt{n}}$ decreasing and $\dfrac{1}{\sqrt{n}}\to 0$ as $n\to\infty$.

7. A is true by Mean Value Theorem. B is trivial by definition of a function. C is true by Intermediate Value Theorem. D is not necessarily true because we don't know if g goes above $y=1$.

8. The x-coordinate of the point is $x = x(0) + \int_0^1 (t+1)\,dt = 1 + \left(\frac{1}{2}t^2 + t\right)\Big|_0^1 = 1 + \frac{1}{2} + 1$ $= 2.5$, and therefore the y-coordinate is $\ln 2.5$.

9. $2 \cdot \frac{1}{2}\int_0^\pi (1-\cos\theta)^2\,d\theta = \int_0^\pi \left(1 - 2\cos\theta + \cos^2\theta\right)d\theta = \int_0^\pi \left(1 - 2\cos\theta + \frac{1}{2} + \frac{1}{2}\cos 2\theta\right)d\theta$

$= \int_0^\pi \left(\frac{3}{2} - 2\cos\theta + \frac{1}{2}\cos 2\theta\right)d\theta = \left(\frac{3\theta}{2} - 2\sin\theta + \frac{1}{4}\sin 2\theta\right)\Big|_0^\pi = \frac{3\pi}{2}$

10. Since $g'(x) < 0$ whenever $x \ge 0$, $xg'(x) \le 0$. Therefore, F takes on negative values. Since g is twice differentiable, g' is continuous, making the integrand continuous, and therefore, F is continuous. $F'(x) = xg'(x)$ by Fundamental Theorem of Calculus, and therefore, $F'(x) < 0$, making F a decreasing function.

11. $\frac{f'''(0)}{3!} = \frac{3}{4} \Rightarrow f'''(0) = 6 \cdot \frac{3}{4} = \frac{9}{2}$

12. $g(x) \approx g(3) + g'(3)(x-3) = 5 - 2(x-3) \Rightarrow g(2.98) \approx 5 - 2(-0.02) = 5.04$

13. Let $s = \text{speed}$. Then $s = \sqrt{100 - 10t + 4t^2} \Rightarrow s' = \frac{8t - 10}{2\sqrt{100 - 10t + 4t^2}} \Rightarrow s'\big|_{t=1} = -\frac{1}{\sqrt{94}}$, so the speed is decreasing, meaning the particle is slowing down.

14. $x'(t) = 2t - 4$ and $y'(t) = \frac{1}{t} \Rightarrow x'(1) = -2$ and $y'(1) = 1$, meaning the particle is moving to the left and up.

15. The step size is 0.5, and Euler's Method would take two iterations. Therefore, $y_1 = 2 + 0.5(-3 + 2\cdot 2) = 2.5;\ y_2 = 2.5 + 0.5(-2.5 + 2\cdot 2.5) = 3.75$.

16. $\frac{dy}{y-2} = k\,dx \Rightarrow \ln|y-2| = kx + c \Rightarrow y - 2 = Ce^{kx} \Rightarrow y = Ce^{kx} + 2$

17. $\int_0^1 e^{x^2}dx \approx \int_0^1 \left(1 + x^2 + \frac{x^4}{2} + \frac{x^6}{6}\right)dx = \left(x + \frac{x^3}{3} + \frac{x^5}{10} + \frac{x^7}{42}\right)\Big|_0^1 = 1 + \frac{1}{3} + \frac{1}{10} + \frac{1}{42}$

18. Using Limit Comparison Test with the series whose terms are of the form $\frac{1}{k^{2a-5}}$.

$\lim_{k\to\infty}\frac{1/k^{2a-5}}{k^2/\left(k^{2a-3}+4\right)}=\lim_{k\to\infty}\frac{k^{2a-3}+4}{k^{2a-3}}=1$, which is finite and positive, so both series would converge as long as $2a-5>1\Rightarrow 2a>6\Rightarrow a>3$.

19. Using L'hopital's Rule, $\lim_{x\to0}xf(x)=\lim_{x\to0}\frac{f(x)}{1/x}=\lim_{x\to0}\frac{f'(x)}{-1/x^2}=\lim_{x\to0}\left(-x^2f'(x)\right)=0\cdot4=0$, which means that $\lim_{x\to0}\left(e^x\right)^{f(x)}=e^0=1$.

20. $\int_4^{10}f(x)dx\approx\frac{2}{2}(24+2\cdot37+2\cdot47+58)=250$

21. The values of θ closest to 0 where $r=0$ occur when $\sin\theta=1/2$, which are $\theta=\pi/6$ and $\theta=5\pi/6$. Since the graph is symmetric to the y-axis, we can find the area between $\theta=\pi/6$ and $\theta=\pi/2$. Thus, the area is $2\cdot\frac{1}{2}\int_{\pi/6}^{\pi/2}r^2d\theta=2\cdot\frac{1}{2}\int_{\pi/6}^{\pi/2}(1-2\sin\theta)^2d\theta$ $=\int_{\pi/6}^{\pi/2}(1-2\sin\theta)^2d\theta$.

22. One arch of the cycloid is spanned through the angles in the interval $[0,2\pi]$. Therefore, the length is $L=\int_0^{2\pi}\sqrt{\left(x'(t)\right)^2+\left(y'(t)\right)^2}dt=\int_0^{2\pi}\sqrt{(1-\cos t)^2+(\sin t)^2}dt$

$=\int_0^{2\pi}\sqrt{1-2\cos t+\cos^2t+\sin^2t}dt=\int_0^{2\pi}\sqrt{2-2\cos t}dt=2\int_0^{2\pi}\sqrt{\frac{1-\cos t}{2}}dt$

$=2\int_0^{2\pi}\sin\frac{t}{2}dt=-4\cos\frac{t}{2}\Big|_0^{2\pi}=4+4=8$.

23. This integral would have to be split into $\int_2^3\frac{dx}{(x-3)^2}$ and $\int_3^4\frac{dx}{(x-3)^2}$, and both integrals must converge for the whole integral to converge. $\int_2^3\frac{dx}{(x-3)^2}$

$=\lim_{t\to3^-}\int_2^t\frac{dx}{(x-3)^2}=\lim_{t\to3^-}\left(-\frac{1}{x-3}\right)\Big|_2^t=\lim_{t\to3^-}\left(-\frac{1}{t-3}-1\right)=\infty$, so the integral diverges.

24. The volume of a slice of water is $\Delta V = \pi x^2 \Delta y$, where $x = 2$. A slice at height y must be lifted $(6-y)$ ft. $\Delta W = 4w\pi\Delta y(6-y) \Rightarrow W = 4\pi w\int_0^3 (6-y)dy = 4\pi w\left(6y - \frac{1}{2}y^2\right)\Big|_0^3$ $= 4\pi w\left(18 - \frac{9}{2}\right) = 54w\pi$ (3 is the upper limit because that is the height of the water).

25. Since the area is symmetric to the y-axis, the area is $2\int_0^1 \frac{4}{\sqrt{1-x^2}}dx$ $= 2\lim_{t\to 1^-}\int_0^t \frac{4}{\sqrt{1-x^2}}dx = 2\lim_{t\to 1^-}\left(4\sin^{-1}x\right)\Big|_0^t = 2\lim_{t\to 1^-}\left(4\sin^{-1}t - 4\sin^{-1}0\right) = 2\cdot 4\cdot\frac{\pi}{2} = 4\pi$.

26. For this spring, $F = kx = 5x$ since 20 lbs are needed to compress the spring 4 in. Therefore, the work needed is $\int_4^8 5x dx = \left(\frac{5}{2}x^2\right)\Big|_4^8 = 160 - 40 = 120$.

27. Since the area enclosed by an equilateral triangle with side length s is $A = \frac{s^2\sqrt{3}}{4}$, the area is $\frac{\sqrt{3}}{4}\int_0^4 (2x)^2 dy = \sqrt{3}\int_0^4 x^2 dy = 8\sqrt{3}\int_0^4 y dy = 8\sqrt{3}\left(\frac{1}{2}y^2\right)\Big|_0^4 = 8\sqrt{3}(8) = 64\sqrt{3}$.

28. Center the sphere at the origin, and thus the great circle has equation $x^2 + y^2 = r^2$. Therefore, the volume is $\pi\int_h^r \left(r^2 - x^2\right)dx = \pi\left(r^2x - \frac{1}{3}x^3\right)\Big|_h^r = \pi\left(r^3 - \frac{1}{3}r^3 - r^2h + \frac{1}{3}h^3\right)$ $= \frac{\pi}{3}\left(2r^3 + h^3 - 3r^2h\right)$.

29. This graph is symmetric to both axes, so quadrupling the first quadrant area would suffice to find the area. Because the graph sketches parametrically from the positive x-axis to the positive y-axis, the integral must be negative. Therefore, the first quadrant area is $-\int_0^{\pi/2} y dx = -\int_0^{\pi/2} \sin^3 t\cdot 3\cos^2 t\cdot -\sin t dt = 3\int_0^{\pi/2}\sin^4 t\cos^2 t dt$, making the total area $12\int_0^{\pi/2}\sin^4 t\cos^2 t dt$.

30. $\int_1^5 f(x)dx \approx \frac{1}{3}(1.62 + 4\cdot 4.15 + 2\cdot 7.5 + 4\cdot 9 + 12.13) = 27.11\overline{6}$, so to 2 decimal places, the area is approximately 27.12.