For all questions, answer choice "E) NOTA" means none of the above answers is correct.

1. The number of bacteria in a culture is growing at a rate of $3000e^{2t/5}$ bacteria per time unit. At $t=0$, there were 7500 bacteria in the culture. Find the number of bacteria in the culture at $t=5$.

A) $1200e^2$ B) $3000e^2$ C) $7500e^2$ D) $7500e^5$ E) NOTA

2. The arc of the curve $y=x^2$ between $(0,0)$ and $\left(\sqrt{2},2\right)$ is revolved about the y-axis. What is the area of the surface generated?

A) $9\pi/2$ B) $13\pi/6$ C) $13\pi/3$ D) $3\pi/2$ E) NOTA

3. A thin rod of length L lies along that part of the x-axis with $0\le x\le L$. Its density at the point $(x,0)$ is equal to $f(x)=x^4$ grams/unit of length. What is the x-coordinate of the center of mass of the rod?

A) $5L/6$ B) $4L/5$ C) $3L/4$ D) $2L/3$ E) NOTA

4. What is the length of the curve $y=\frac{2}{3}x^{3/2}$ between the points $(0,0)$ and $\left(3,2\sqrt{3}\right)$?

A) $14/3$ B) $16/3$ C) 7 D) 8 E) NOTA

5. The radius of a sphere is increasing at the uniform rate of 0.3 in/sec. At the instant when the sphere's surface area becomes 100π in^2, what is the rate of increase of the volume of the sphere, in in^3/sec?

A) 10π B) 12π C) 22.5π D) 25π E) NOTA

6. Which of the following series converge?

I) $\sum_{n=1}^{\infty}\frac{1}{n^2}$ II) $\sum_{n=1}^{\infty}\frac{1}{n}$ III) $\sum_{n=1}^{\infty}\frac{(-1)^n}{\sqrt{n}}$

A) I only B) III only C) I & II only D) I & III only E) NOTA

7. Let g be a continuous function on the interval $[0,1]$, with $g(0)=1$ and $g(1)=0$. Which of the following is NOT necessarily true?

A) There exists a number h in $[0,1]$ such that $g(h)\geq g(x)$ for all x in $[0,1]$.
B) For all a and b in $[0,1]$, if $a=b$, then $g(a)=g(b)$.
C) There exists a number h in $[0,1]$ such that $g(h)=0.5$.
D) There exists a number h in $[0,1]$ such that $g(h)=1.5$.
E) NOTA

8. A particle moves on the curve $y=\ln x$ such that the x-coordinate has velocity $x'(t)=t+1$ for $t\geq 0$. At time $t=0$, the particle is at the point $(1,0)$. At time $t=1$, the particle is at what point?

A) $(2,\ln 2)$ B) $(e^2,2)$ C) $(2.5,\ln 2.5)$ D) $(3,\ln 3)$ E) NOTA

9. What is the area of the region enclosed by the polar curve with equation $r=1-\cos\theta$?

A) $3\pi/4$ B) π C) $3\pi/2$ D) 2π E) NOTA

10. Suppose g is twice differentiable with $g'(x)<0$ for all $x\geq 0$, and let $F(x)=\int_0^x tg'(t)dt$ for all $x\geq 0$. Which of the following statements is false?

A) F takes on negative values B) F is continuous for all $x>0$
C) F is an increasing function D) $F'(x)$ exists for all $x>0$ E) NOTA

11. If the first five terms of the Maclaurin expansion for $f(x)$ are $3-7x+\frac{5}{2}x^2+\frac{3}{4}x^3-6x^4$, then $f'''(0)=?$

A) $1/8$ B) $3/4$ C) $9/2$ D) 6 E) NOTA

12. The tangent line to the graph of $y=g(x)$ at the point $(3,5)$ has a slope of -2. Use the equation of the tangent line to estimate $g(2.98)$.

A) 2.5 B) 4.98 C) 5.02 D) 5.04 E) NOTA

13. A particle moves in the xy-plane so that its velocity vector at time t, $0 \le t \le 10$, is $\left\langle \sqrt{100-10t}, 2t \right\rangle$. Which of the following statements is true about the particle at $t = 1$?

A) The particle is slowing down. B) The particle is speeding up.
C) The particle is at rest. D) The speed of the particle is $2+3\sqrt{10}$. E) NOTA

14. A particle moves in the xy-plane for $t > 0$ such that $x(t) = t^2 - 4t$ and $y(t) = \ln t$. At time $t = 1$, the particle is moving in which directions?

A) up and to the right B) up and to the left C) down and to the right
D) down and to the left E) NOTA

15. Let $y(x)$ be the solution to the differential equation $\frac{dy}{dx} = x + 2y$. Using Euler's Method, starting with the point $(-3,2)$ and step size $\Delta x = 0.5$, what is the approximate value of $y(-2)$?

A) 1 B) 2.5 C) 3 D) 3.75 E) NOTA

16. If $\frac{dy}{dx} = k(y-2)$, then $y =$?

A) Ce^{x-2} B) $e^{kx} + C$ C) $\frac{k}{2}(x-2)^2 + C$ D) $Ce^{kx} + 2$ E) NOTA

17. Which expression represents the first four terms of the Maclaurin approximation of the area bounded by $f(x) = e^{x^2}$ and $f(x) = 0$ between $x = 0$ and $x = 1$?

A) $1 + \frac{1}{3} + \frac{1}{10} + \frac{1}{42}$ B) $1 + 1 + \frac{1}{4} + \frac{1}{36}$ C) $1 + 1 + \frac{1}{2} + \frac{1}{6}$ D) $1 + \frac{1}{2} + \frac{1}{6} + \frac{1}{24}$ E) NOTA

18. What are all values of a for which the series $\sum_{k=1}^{\infty} \frac{k^2}{k^{2a-3} + 4}$ converges?

A) $a > 2$ B) $a \ge 3$ C) $a < 3$ D) $a > 1$ E) NOTA

19. Let f be a continuous function with the properties that $\lim_{x\to 0} f(x) = \infty$ and $\lim_{x\to 0} f'(x) = 4$. What is the value of $\lim_{x\to 0} \left(e^x\right)^{f(x)}$?

A) 0 B) 1 C) 4 D) ∞ E) NOTA

20. Let f be a continuous function defined on the interval $[4,10]$. A table of selected values of f is shown. What is the estimate of $\int_4^{10} f(x)dx$ produced by a trapezoidal approximation with $n=3$ subdivisions of equal width?

x	4	6	8	10
$f(x)$	24	37	47	58

A) 216 B) 250 C) 262 D) 270 E) NOTA

21. The area bounded by the small loop of the limaçon $r = 1-2\sin\theta$ is given by which integral?

A) $\frac{1}{4}\int_{\pi/3}^{5\pi/3} (1-2\sin\theta)^2\, d\theta$ B) $\int_{7\pi/6}^{3\pi/2} (1-2\sin\theta)^2\, d\theta$ C) $\int_{\pi/6}^{\pi/2} (1-2\sin\theta)^2\, d\theta$

D) $\int_0^{\pi/3} (1-2\sin\theta)^2\, d\theta$ E) NOTA

22. What is the length of one arch of the cycloid with equations $x = t-\sin t$ and $y = 1-\cos t$?

A) 3π B) 4 C) 16 D) 8 E) NOTA

23. Evaluate: $\int_2^4 \frac{dx}{(x-3)^2}$

A) 2 B) -2 C) 0 D) divergent E) NOTA

24. A cylindrical reservoir of diameter 4 ft and height 6 ft is half full of water weighing w lb/ft^3. How much work is needed, in ft-lbs, to empty the water over the top?

A) $216w\pi$ B) $18w$ C) $72w\pi$ D) $54w\pi$ E) NOTA

25. Determine the area enclosed between the curve $y=\dfrac{4}{\sqrt{1-x^2}}$, its asymptotes, and $y=0$.

A) $\pi/2$ B) π C) 2π D) 4π E) NOTA

26. A force of 20 lb compresses a spring from its natural length of 30 in to 26 in. How much work is needed, in inch-lbs, to compress the spring from 26 in to 22 in?

A) 20 B) 40 C) 80 D) 120 E) NOTA

27. The base of a solid is the region bounded by the parabola $x^2=8y$ and the line $y=4$. Each plane section perpendicular to the *y*-axis is an equilateral triangle. Find the volume of this solid.

A) $64\sqrt{3}/3$ B) $64\sqrt{3}$ C) $32\sqrt{3}$ D) 32 E) NOTA

28. A sphere with radius of length r is divided into 2 parts by a plane at a distance h, where $0<h<r$, from the center. What is the volume of the smaller part?

A) $\dfrac{\pi}{3}\left(2r^3+h^3-3r^2h\right)$ B) $\dfrac{\pi}{3}\left(2r^3-h^3+3r^2h\right)$ C) $\dfrac{4}{3}\left(2r^3+h^3-3r^2h\right)$ D) $\dfrac{\pi h}{3}\left(3r^2-h^2\right)$
E) NOTA

29. The area enclosed by the hypocycloid with parametric equations $x=\cos^3 t$ and $y=\sin^3 t$ is given by which of the following integrals?

A) $3\int_{\pi/3}^{0}\sin^4 t\cos^2 t\,dt$ B) $4\int_0^1\sin^3 t\,dt$ C) $-4\int_{\pi/2}^{0}\sin^6 t\,dt$ D) $12\int_0^{\pi/2}\sin^4 t\cos^2 t\,dt$
E) NOTA

30. Let f be a continuous function defined on the interval $[1,5]$. A table of selected values of f is shown. What is the estimate of $\int_1^5 f(x)\,dx$ produced by a Simpson's Rule approximation with $n=4$ subdivisions of equal width, to 2 decimal places?

x	1	2	3	4	5
$f(x)$	1.62	4.15	7.5	9	12.13

A) 16.20 B) 20.30 C) 27.12 D) 40.68 E) NOTA