For all questions, answer choice "E) NOTA" means none of the above answers is correct.

1. What is the area enclosed by a regular hexagon with side length 4?

A) $12\sqrt{3}$ B) $24\sqrt{3}$ C) $36\sqrt{3}$ D) $48\sqrt{3}$ E) NOTA

2. Find the volume enclosed by a sphere with radius of length 6.

A) 288π B) 36π C) 144π D) 72π E) NOTA

3. Find the area bounded by the graphs of $y=3e^{-3x}$, $y=0$, $x=0$, and $x=2$.

A) $2e^{-3}$ B) $1-e^{-9}$ C) $1-e^{-6}$ D) $27-9e$ E) NOTA

4. What is the area bounded by the graphs of $y=x^2-x-3$ and $y=2x+7$?

A) $^{497}/_{6}$ B) $^{863}/_{12}$ C) $^{153}/_{10}$ D) $^{343}/_{6}$ E) NOTA

5. Find the volume of the solid obtained when the region bounded by the graphs of $y=\dfrac{1}{\sqrt[4]{1-x^2}}$, $y=0$, $x=0$, and $x=0.5$ is revolved about the x-axis.

A) $^{\pi^4}/_{90}$ B) $^{\pi}/_{3}$ C) $^{2}/_{3}$ D) $^{\pi^2}/_{6}$ E) NOTA

6. Calculate the volume of the solid obtained when the region bounded by the graphs of $y=4\sqrt{x}$ and $y=2x$ is revolved about the x-axis.

A) $^{128\pi}/_{3}$ B) $^{256\pi}/_{5}$ C) $^{128\pi}/_{15}$ D) $^{273\pi}/_{5}$ E) NOTA

7. A region is determined by the inequalities $0 \le y \le 5x^{-1}$ and $1 \le x \le 5$. What is the volume when this region is revolved about the y-axis?

A) 45π B) 40π C) 35π D) 30π E) NOTA

8. If $\int_0^3 |3x-2|\,dx = {}^{m}/_{n}$, where m and n are relatively prime positive integers, find $m+n$.

A) 61 B) 59 C) 57 D) 55 E) NOTA

9. If the units of x are feet$\cdot$seconds and the units of $f(x)$ are $\text{feet}^3/\text{second}^2$, what are the units of $\int_a^b f(x)dx$?

A) $\text{feet}^4/\text{second}$ B) feet$\cdot$seconds C) seconds^2 D) $\text{feet}^3/\text{second}^2$ E) NOTA

10. Let $f(x)$ be a continuous, real-valued function on the interval $[a,b]$. Which of the following is necessarily an appropriate interpretation of $\int_a^b f(x)dx$?

A) The area of the region bounded by the graphs of $y=f(x)$, the x-axis, and the lines $x=a$ and $x=b$

B) The volume when the region bounded by the graphs of $y=\sqrt{\frac{f(x)}{\pi}}$, the x-axis, and the lines $x=a$ and $x=b$ is revolved about the x-axis

C) The volume when the region bounded by the graphs of $y=\frac{f(x)}{2\pi x}$, the x-axis, and the lines $x=a$ and $x=b$ is revolved about the y-axis

D) All of A, B, and C are valid interpretations

E) NOTA

11. A parabolic sector is a "wedge" created by the bounded enclosure of a parabola and a line perpendicular to its axis of symmetry. For a parabola with vertical axis of symmetry and negative leading term, a parabolic sector with base of length b and altitude of length h can be modeled by the region bounded by the graphs of $y=h-\frac{4hx^2}{b^2}$ and the x-axis. Find the area of a parabolic sector with base of length 6 and altitude of length 4.

A) 16 B) 36 C) 18 D) 32 E) NOTA

12. Let R be the region bounded by the graphs of $y=\sin x$ and the x-axis on the interval $0\le x\le\pi$. The lines $x=a$ and $x=b$, where $0<a<b<\pi$, divide R into three regions of equal area. Find the value of $\sin(b-a)$.

A) 0 B) $4\sqrt{2}/9$ C) $2\sqrt{5}/3$ D) $\frac{\sqrt{6}-\sqrt{3}}{4}$ E) NOTA

13. The base of a solid is given by the region bounded by the graphs of $y=5-x$, $x=0$, and $y=0$. Cross-sections of this solid perpendicular to the x-axis are equilateral triangles. Find the volume of the solid.

A) $25/2$ B) $125\sqrt{3}/12$ C) $125/6$ D) $25\sqrt{15}/6$ E) NOTA

14. Approximate the value of $\int_0^4 \left(x^3-4x+2\right)dx$ using Simpson's Rule with 1000 equal partitions of the interval $[0,4]$.

A) 60 B) 50 C) 30 D) 40 E) NOTA

15. Two real numbers x and y are chosen at random from the interval $(-4,4]$. Find the probability that $x^2+y^2\le 4$.

A) $\pi/16$ B) $\pi/8$ C) $\pi/4$ D) 0 E) NOTA

16. Let $f(n)=\int_0^{2\pi n}|\sin x|dx$, where n is a whole number. Evaluate the sum $\sum_{n=1}^{100} f(n)$.

A) 10100 B) 15150 C) 20200 D) 25250 E) NOTA

17. Find the area of the triangle formed by the graphs of the x- and y-axes and the line tangent to the graph of $y=x^5$ at the point where $x=5$.

A) 30000 B) 35000 C) 20000 D) 25000 E) NOTA

18. Let $g(x)=\lfloor x\rfloor$ represent the greatest integer function. Evaluate $\int_0^5 g(x)dx$.

A) 5 B) 10 C) 15 D) 20 E) NOTA

19. Let R be the region given by $(x-6)-|y-6|\ge 0$ for $6\le x\le 9$ and $(x-9)^2+(y-6)^2\le 9$ for $9\le x\le 12$. Find the volume of the solid formed when R is revolved about the x-axis.

A) $54\pi(\pi+2)$ B) $63\pi^2-27$ C) $72\pi(\pi+2)$ D) $64\pi(\pi-1)$ E) NOTA

20. Find the area of the region in the first quadrant bounded by the graphs of $x=4$ and $y=|x-1|+|x-2|+|x-3|$.

A) 15 B) 13 C) 16 D) 14 E) NOTA

21. Let $h(x)=f^{-1}(x)$, where $f(x)=x^3+3x+2$. Evaluate $\int_2^{78} h(x)dx$.

A) 188 B) 202 C) 216 D) 230 E) NOTA

22. Find the maximum value of $f(a,b)=4(b^2-a^2)-\frac{1}{3}(b^3-a^3)-15(b-a)$, where a and b are real numbers.

A) $4/3$ B) 7 C) 8 D) $5/6$ E) NOTA

23. Evaluate: $2\pi\int_6^{14}(x-3)\left(4-\frac{(x-10)^2}{4}\right)^{1/2}dx$

A) $40\pi^2$ B) $48\pi^2$ C) $56\pi^2$ D) $64\pi^2$ E) NOTA

24. A cup is in the shape of a right conical frustum with top and bottom radii of lengths 8 cm and 5 cm, respectively, and the height of the cup is 12 cm. The cup is being filled with water at the rate of π cm^3/sec such that the water level at any moment is parallel to the bottom of the cup. At the moment when the water level is 6 cm from the bottom of the cup, the water level is changing at a rate of m/n cm/sec, where m and n are relatively prime positive integers. Find the value of $\sqrt{m}+\sqrt{n}$.

A) 9 B) 12 C) 15 D) 18 E) NOTA

25. Find the value of a^2+b^2 if a and b are positive numbers where $\pi\int_0^a\left(b-\frac{bx}{a}\right)^2dx=800\pi$ and $\pi\int_0^b\left(a-\frac{ay}{b}\right)^2dy=1920\pi$.

A) 225 B) 1024 C) 64 D) 676 E) NOTA

26. Let R be the region bounded by the graphs of $y=x^3-x$ and the x-axis in the fourth quadrant. A line with negative slope m passes through the origin and intersects the graph of $y=x^3-x$ in exactly three distinct points. The area of the portion of R above the line is equal to half the area of R. Find the value of m.

A) $\dfrac{\sqrt{2}-2}{2}$ B) $\dfrac{\sqrt{2}-\sqrt{6}}{\sqrt{2}+\sqrt{6}}$ C) $\dfrac{1-\sqrt{5}}{4}$ D) -1 E) NOTA

27. Two lines, one with positive slope and one with negative slope, are each tangent to the graph of $y=x^2$. The points of tangency of these lines, along with the intersection point of the two lines, are the vertices of an equilateral triangle. Find the area enclosed by the triangle.

A) $3/2$ B) $3\sqrt{3}/4$ C) $\sqrt{3}/12$ D) $4\sqrt{3}/9$ E) NOTA

28. Let R be the region described by the inequalities $x\ge 1$, $xy\ge 1$, and $y\le \left(g(x)\right)^{-1}$, where $g(x)=\lfloor x\rfloor$, the greatest integer function. Given that $\sum_{n=1}^{\infty}\dfrac{1}{n^2}=\dfrac{\pi^2}{6}$, find the volume obtained when R is revolved about the x-axis.

A) ∞ B) $\dfrac{\pi^5}{36}-\pi$ C) $\pi^2-\dfrac{6}{\pi^2}$ D) $\dfrac{\pi^3}{6}-\pi$ E) NOTA

29. Find the maximum area of a rectangle inscribed in a parabolic sector of base 2 and height 1. The rectangle is inscribed such that one side lies along the base of the parabolic sector, and the axis of symmetry of the curved portion of the sector passes through the center of the rectangle.

A) $\sqrt{2}/2$ B) $3/4$ C) $1/3$ D) $4\sqrt{3}/9$ E) NOTA

30. Suppose a and b are nonnegative real numbers such that $a^2+b\le 4$. Find the probability that the graphs of $y=x^2+2bx+1$ and $y=2a(x+b)$ do not intersect.

A) $3\pi/16$ B) $3\pi/8$ C) $3\pi/64$ D) $3\pi/32$ E) NOTA