Answers:

1. D
2. C
3. C
4. C
5. A
6. E
7. B
8. B
9. A
10. B
11. D
12. D
13. B
14. C
15. B
16. B
17. A
18. C
19. B
20. D
21. C
22. A
23. D
24. A
25. B
26. B
27. A
28. C
29. B
30. D

Solutions:

1. The circle has diameter 36, so the square has side length $18\sqrt{2}$ and thus area $\left(18\sqrt{2}\right)^2 = 648$.

2. The triangle must be a right triangle being inscribed the way it is, so the hypotenuse, which is the diameter, is 10; thus the area is $\frac{1}{2}(5)\left(5\sqrt{3}\right) = 12.5\sqrt{3}$.

3. The parallelogram doesn't have to be a rhombus unless the sides form right angles. However, the parallelogram doesn't have to be a square either as the sides don't have to be equal. A kite isn't necessarily a parallelogram. So the parallelogram must be a rectangle.

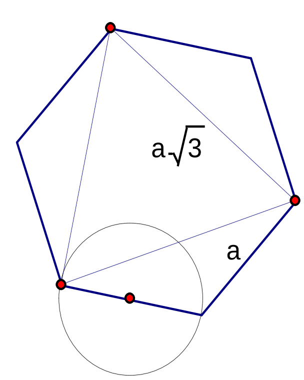
4. The arcs's degree measures are each the lengths divided by the radius length, and those are each double the angles of the triangle, so the interior angles of the triangle are in that same ratio. Therefore, the angles are all different and the largest angle is $180 \cdot \frac{65}{16+63+65} = 81.25^\circ$, making the triangle scalene.

5. $5^2 + 5^2 = 25 + 25 = 50 \neq 25$, so $(5,5)$ is not on the circle.

6. Both angles $\angle A$ and $\angle B$ must be 45°, and the arcs are both double the inscribed angles that intersect them. Therefore, arc AC has measure 90°.

7. Since the radius of the circle is 5 and the chord is 4 units away from the center, the length of the chord is $2 \cdot 3 = 6$. Since the base angles of the triangle are both 45°, the legs of the triangle are both $6/\sqrt{2} = 3\sqrt{2}$. The perimeter is $2\left(3\sqrt{2}\right) + 6 = 6 + 6\sqrt{2}$.

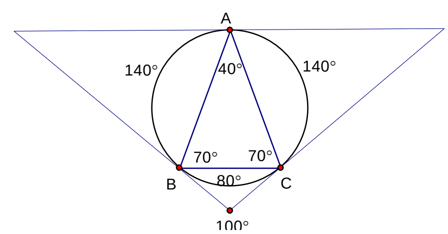
8. $A = 8 \cdot \frac{1}{2} \cdot 5^2 \sin 45^\circ = 50\sqrt{2}$



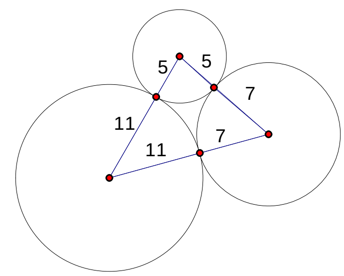
9. The angle made by the tangents is $x = \frac{1}{2}\left(270^\circ - 90^\circ\right) = 90^\circ$, and the two angles made at the points of tangency are both 90°, so the quadrilateral must be a rectangle. Additionally, two consecutive sides (two radii, two tangents) are equal, so the rectangle must be a square.

10. $3a\sqrt{3}=9 \Rightarrow a=\sqrt{3}$, and this is the length of the diameter, so the area enclosed by the circle is $\pi\left(\sqrt{3}\right)^2 \big/ 4 = 3\pi/4$.

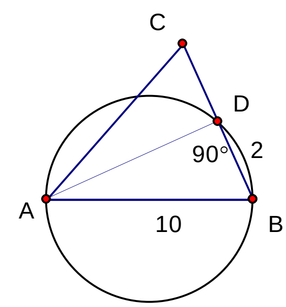
11. The measure of arc BC is 80° since it is double the inscribed angle $\angle A$. Therefore, the other two arcs have measure 140°. This makes the two base angles of the larger triangle made with the tangents equal to $\frac{1}{2}\left(220^\circ - 140^\circ\right) = 40^\circ$, and thus the other angle would have measure 100°.



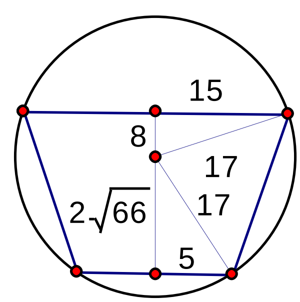
12. The circles have radii in the ratio 5:7:11, and since the circumference of the smallest circle is 10π, the radii have measure 5, 7, and 11. The triangles have side lengths equal to the sum of two of these radii, so the lengths of the triangle are 12, 16, and 18. Using Hero's formula, the area enclosed by the triangle is $\sqrt{23\cdot 5\cdot 7\cdot 11}=\sqrt{8855}$.



13. Since the radius has length 5, the leg of the triangle has length 10. Because the triangle is isosceles and the length of side $\overline{BC}$ is 4, the length of side $\overline{BD}$ is 2 and $\overline{AD}$ is an altitude. Therefore, $\left|\overline{AD}\right|=\sqrt{10^2-2^2}=\sqrt{96}$ $=4\sqrt{6}$.



14. Half the shorter base has length 5, so by the diagram, the distance the shorter base is from the center of the circle is $\sqrt{17^2-5^2}=\sqrt{264}=2\sqrt{66}$, making the altitude of the trapezoid $8+2\sqrt{66}$. Doing a similar calculation for the longer base, half the longer base has length $\sqrt{17^2-8^2}=15$, so the longer base has length 30. Therefore, the area enclosed by the trapezoid is $\frac{1}{2}(10+30)\left(8+2\sqrt{66}\right)=160+40\sqrt{66}$.



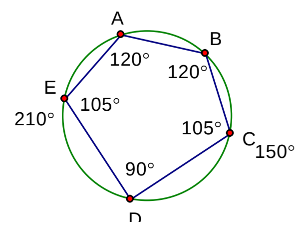
15. $2\pi r = 45 \Rightarrow r = \dfrac{45}{2\pi} \Rightarrow A = \pi r^2 = \pi\left(\dfrac{45}{2\pi}\right)^2 = \dfrac{2025}{4\pi}$, which is the area enclosed by the circle. Using the Shoelace method, the area enclosed by the triangle is

$$\begin{array}{r|rr|r|} & 2 & 0 & \\ 0 & 4 & 6 & 12 \\ 30 & 5 & -2 & -8 \\ -4 & 2 & 0 & 0 \\ 26 & & & 4 \end{array} \Rightarrow A = \frac{1}{2}\left|26-4\right| = 11$$

Therefore the ratio of these two areas is

$$\frac{2025/4\pi}{11} = 2025:44\pi.$$

16. Since the ratio of angles are 8:8:7:7:6 and the interior angles of a pentagon sum to 540°, the angles are 120°, 120°, 105°, 105°, and 90°. Arc $BAED$ has measure 210° because it is intercepted by inscribed angle $\angle C$, so minor arc BD must have measure 150°.



17. The length of the common external tangent is $\sqrt{(8+5+5)^2-(8-5)^2} = \sqrt{324-9} = \sqrt{315} = 3\sqrt{35}$. The length of the common internal tangent is $\sqrt{(8+5+5)^2-(8+5)^2} = \sqrt{324-169} = \sqrt{155}$. The sum of these lengths is $3\sqrt{35}+\sqrt{155}$.

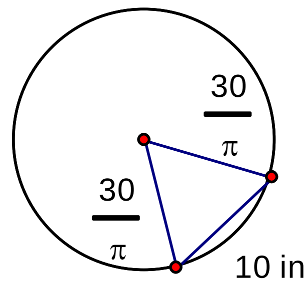
18. Completing the square gives $4x^2-8x+4+4y^2-12y+9 = 1+4+9$ $\Rightarrow 4(x-1)^2+4\left(y-3/2\right)^2 = 14 \Rightarrow (x-1)^2+\left(y-3/2\right)^2 = 7/2$. Therefore, the circle has radius $\dfrac{\sqrt{14}}{2}$, so the square that circumscribes the circle has side length $\sqrt{14}$, so the area of the square is 14.

19. Let b be the other leg length and c the hypotenuse length. Therefore, $b+c=32$ and $64 = c^2-b^2 = (c+b)(c-b) \Rightarrow c-b=2$. Solving this system, $c=17$ and $b=15$. Therefore, the radius of the inscribed circle is $r = \dfrac{A}{s} = \dfrac{1/2(8)(15)}{20} = 3$ and the area enclosed by the circle is $\pi(3)^2 = 9\pi$.

20. If a is the leg length of the triangle, $\frac{1}{2}a^2 = 8 \Rightarrow a = 4$, so the hypotenuse has length $4\sqrt{2}$, but since the triangle is inscribed in a circle, the hypotenuse is the diameter. Therefore, the distance traveled is $7\left(\pi \cdot 4\sqrt{2}\right) = 28\pi\sqrt{2}$.

21. The centroid of the triangle is the point $\left(\frac{2+0+7}{3}, \frac{0+3+6}{3}\right) = (3,3)$, and since the circle passes through the origin, it has radius $3\sqrt{2}$, making the circumference $2\pi\left(3\sqrt{2}\right) = 6\pi\sqrt{2}$.

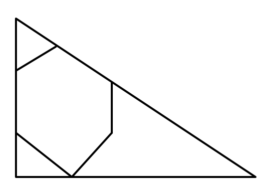
22. Using $s = r\theta$, $10 = r\left(\frac{\pi}{3}\right) \Rightarrow r = \frac{30}{\pi}$, and because the central angle is 60°, the third length of the triangle has the same length. Therefore, the perimeter of the portion of the sector remaining has perimeter $10 + \frac{30}{\pi}$.



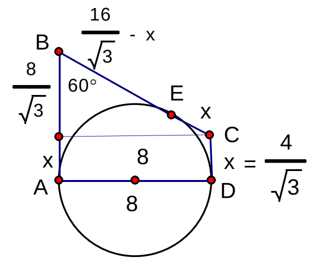
23. The side length of the hexagon is $\frac{44\sqrt{3}}{6} = \frac{22\sqrt{3}}{3}$, so the side length of the image of the dilation is $\frac{22\sqrt{3}}{3} \cdot 5 = \frac{110\sqrt{3}}{3}$. The radius of the inscribed circle of the dilation, therefore, is $\frac{110\sqrt{3}/3}{2} \cdot \sqrt{3} = 55$, making the area enclosed by the dilation 3025π.

24. Since the triangle is right, the circumcenter of the triangle is the midpoint of the hypotenuse. Therefore, the area of the intersection of the enclosures of the triangle and the circle is just half the enclosure of the circle. $A = \frac{1}{2}\left(\pi \cdot 3^2\right) = \frac{9\pi}{2}$

25. In order to inscribe the hexagon so that one side is on the hypotenuse and one is on the short leg, the triangle must be a 30-60-90 triangle. Let x be the length of a side of the hexagon. This creates a small equilateral triangle in the corner with side length x and a right triangle in another corner which is also a 30-60-90 and which has a hypotenuse of length x. Therefore, the leg of the smaller right triangle on the smaller leg of the big right triangle has length $x/2$.

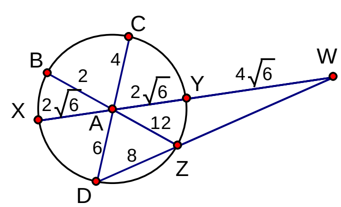


Therefore, summing the lengths along the short leg of the big right triangle, $18 = x + x + x/2 = 5x/2 \Rightarrow x = 36/5$, and thus, the perimeter of the hexagon must be $6 \cdot \frac{36}{5} = \frac{216}{5}$.

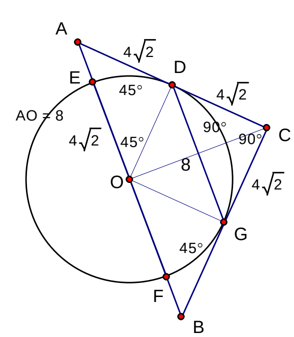


26. Let x be the length of the shorter tangent. As in the diagram, the longer of the first two tangents must contain the 60° angle. Therefore, the longer of the first two tangents must have length $\frac{8}{\sqrt{3}} + x$, and thus we have that $\frac{8}{\sqrt{3}} + x = \frac{16}{\sqrt{3}} - x \Rightarrow x = \frac{4}{\sqrt{3}}$. Therefore, the area of the quadrilateral, which is a trapezoid, is $\frac{1}{2}\left(\frac{12}{\sqrt{3}} + \frac{4}{\sqrt{3}}\right)(8) = \frac{64\sqrt{3}}{3}$.

27. $A = \frac{4^2\sqrt{3}}{4} = 4\sqrt{3}$; $B = \left(4 + 4\sqrt{2}\right)^2 - 4\left(\frac{1}{2}\left(2\sqrt{2}\right)^2\right) = 32 + 32\sqrt{2}$; $C = \frac{4\sqrt{3}}{6} = \frac{2\sqrt{3}}{3}$; and $D = 2\sqrt{3}$. Therefore, $\frac{AB}{CD} = \frac{\left(4\sqrt{3}\right)\left(32 + 32\sqrt{2}\right)}{\left(\frac{2\sqrt{3}}{3}\right)\left(2\sqrt{3}\right)} = 32\sqrt{3} + 32\sqrt{6}$.



28. We have $\left|\overline{AZ}\right| = \frac{\left|\overline{AC}\right|\left|\overline{AD}\right|}{\left|\overline{AB}\right|} = \frac{4 \cdot 6}{2} = 12$, and, since $\left|\overline{AX}\right| = \left|\overline{AY}\right|$, $\left|\overline{AX}\right| = \sqrt{\left|\overline{AC}\right|\left|\overline{AD}\right|} = 2\sqrt{6}$. Therefore, $\left|\overline{WY}\right| = \left|\overline{XY}\right| = 2\left|\overline{AX}\right| = 4\sqrt{6}$, and thus we have $\left(\left|\overline{WZ}\right|\right)\left(\left|\overline{WZ}\right| + 8\right) = \left(8\sqrt{6}\right)\left(4\sqrt{6}\right) = 192 \Rightarrow \left(\left|\overline{WZ}\right|\right)^2 + 8\left|\overline{WZ}\right| - 192 = 0 \Rightarrow \left|\overline{WZ}\right| = -4 + 4\sqrt{13}$. Therefore, the sought perimeter is $12 + \left(-4 + 4\sqrt{13}\right) + 6\sqrt{6} = 8 + 6\sqrt{6} + 4\sqrt{13}$.



29. Since $\left|\overline{AD}\right| = \left|\overline{BG}\right|$, arc ED also has measure 45°. Since DG is a 90° arc, if O is the center of the circle, then quadrilateral $ODCG$ is a square, and thus has side length $4\sqrt{2}$, which is also the radius length of the circle. $\angle EAD$ is also 45°, so $\left|\overline{AD}\right| = 4\sqrt{2}$. Therefore, by similarity of

triangles, $\left|\overline{AB}\right| = 16$, making $\left|\overline{AE}\right| = 8 - 4\sqrt{2}$. Now, since ED is a 45° arc, its length is $s = \left(4\sqrt{2}\right)\left(\dfrac{\pi}{4}\right) = \pi\sqrt{2}$. Therefore, the sought perimeter is $4\sqrt{3} + \left(8 - 4\sqrt{3}\right) + \pi\sqrt{2}$ $= 8 + \pi\sqrt{2}$.

30. The diameter of the sphere is 20, so the length of one side of the cube has length $\dfrac{20}{\sqrt{3}}$, making the length of a face diagonal $\dfrac{20\sqrt{2}}{\sqrt{3}}$. Since the sphere contains two face diagonals and two edges of the cube, the plane passes through the center of the sphere. Therefore, the area enclosed by the circle outside the cube is $\pi\left(10\right)^2 - \left(\dfrac{20\sqrt{2}}{\sqrt{3}}\right)\left(\dfrac{20}{\sqrt{3}}\right) = 100\pi - \dfrac{400\sqrt{2}}{3}$.