For all questions, answer choice "E) NOTA" means none of the above answers is correct.

1. Which of the following is not an equivalent way of writing the polar point $\left(-4,-\pi/3\right)$ in the polar coordinate plane?

I) $\left(-4,\pi/3\right)$ II) $\left(4,2\pi/3\right)$ III) $\left(-4,11\pi/3\right)$ IV) $\left(4,\pi/3\right)$ V) $\left(-4,8\pi/3\right)$

A) II & III only B) II & IV only C) I, IV, & V only D) none are equivalent E) NOTA

2. Which point with polar coordinates is equivalent to the rectangular point $(-2,-2)$?

A) $\left(-2\sqrt{2},3\pi/4\right)$ B) $\left(-2\sqrt{2},5\pi/4\right)$ C) $\left(2\sqrt{2},-3\pi/4\right)$ D) $\left(2\sqrt{2},\pi/4\right)$ E) NOTA

3. What is the angle argument, in degrees and on the interval $\left[0^\circ,360^\circ\right)$, of the product $\left(\left(2cis47^\circ\right)\left(3cis18^\circ\right)\right)^3$?

A) 6° B) 305° C) 216° D) 195° E) NOTA

4. The equation $x^4+2x^2y^2+y^4-4x^2+4y^2=0$ describes the graph of which of the following?

A) cardioid B) inner loop limaçon C) lemniscate D) dimpled limaçon E) NOTA

5. I have a small cage where I keep my snail collection. Oddly enough, my cage is in the shape of a limaçon (Get it? "Limaçon" is French for "snail"). My limaçon cage has the equation $z=1+3e^{i\theta}+e^{2i\theta}$ on the imaginary polar coordinate plane. Given that the area enclosed by a limaçon with polar equation $r=b+a\cos\theta$ is $\left(b^2+0.5a^2\right)\pi$, each snail has a base area of $\pi/2$, and the snails cannot be placed on top of one another, what is the maximum whole number of snails I can put in my cage? Assume the snails can be placed in such a way as to cover the entire floor of the cage, if necessary.

A) 9 B) 11 C) 17 D) 22 E) NOTA

Use the polar equation $r=\dfrac{4}{2+\cos\theta}$ to answer questions 6-8.

6. What is the shape of the conic section defined in the given equation?

A) ellipse B) circle C) parabola D) hyperbola E) NOTA

7. What is the length of the latus rectum of the conic section defined in the given equation?

A) 1 B) 2 C) 4 D) 8 E) NOTA

8. What is the eccentricity of the conic section defined in the given equation?

A) $1/4$ B) $1/2$ C) 2 D) 4 E) NOTA

9. The famous polar curve defined by $r=\theta$ was discussed by this mathematician in a book called *On Spirals*, and the curve is even named after said mathematician; who is he?

A) Euclid B) Pythagoras C) Apollonius D) Archimedes E) NOTA

10. What is the distance between the points whose polar coordinates are $\left(4,5\pi/3\right)$ and $\left(-\sqrt{2},5\pi/4\right)$?

A) $2\sqrt{3}-1$ B) $2\sqrt{7+\sqrt{3}}$ C) $2\sqrt{7-\sqrt{3}}$ D) $2\sqrt{7+\sqrt{2}}$ E) NOTA

11. For the rose curve with equation $r=\left(\cos 4\theta\right)\left(\sin 4\theta\right)$, how many petals does the graph have?

A) 2 B) 4 C) 8 D) 16 E) NOTA

12. Define a vector on the polar coordinate plane by the notation $\langle r,\theta\rangle$, where (r,θ) is the polar terminal point of the vector and $(0,0)$ is the polar initial point of the vector. Given polar vectors $\vec{u}=\left\langle -3,\pi/3\right\rangle$ and $\vec{v}=\left\langle -6,3\pi/4\right\rangle$, find the value of $\overrightarrow{u_c}\cdot\overrightarrow{v_c}$, where $\overrightarrow{u_c}$ and $\overrightarrow{v_c}$ are the rectangular vectors corresponding to $\vec{u}$ and $\vec{v}$, respectively.

A) $18+\frac{\pi^2}{4}$ B) $\frac{9\left(\sqrt{6}-\sqrt{2}\right)}{2}$ C) $\frac{9\left(\sqrt{2}-\sqrt{6}\right)}{2}$ D) $\frac{9\left(\sqrt{6}+\sqrt{2}\right)}{2}$ E) NOTA

13. On the polar coordinate plane, a convex quadrilateral is defined by the vertices $\left(2,{}^{13\pi}\!/_{6}\right)$, $\left(-3,{}^{5\pi}\!/_{3}\right)$, $\left(4,{}^{5\pi}\!/_{4}\right)$, and $\left(5,-{}^{\pi}\!/_{6}\right)$. Find this quadrilateral's enclosed area.

A) $\dfrac{8\sqrt{6}-8\sqrt{2}+3}{2}$ B) $\dfrac{8\sqrt{6}-5\sqrt{3}-8\sqrt{2}+6}{2}$ C) $\dfrac{8\sqrt{6}+8\sqrt{2}+5\sqrt{3}+6}{2}$

D) $4\sqrt{6}+4\sqrt{2}-6$ E) NOTA

14. For the cardioid with equation $r=1+\sin\theta$, a cusp occurs at the origin. If the cusp is viewed as pointing toward the origin, in which direction does the cusp point?

A) upward B) downward C) to the left D) to the right E) NOTA

15. Evaluate as the principal value of the quantity: i^{-i}

A) $e^{-\pi/2}$ B) $e^{\pi/2}$ C) -1 D) 1 E) NOTA

16. A limaçon with equation $r=2+4\cos\theta$ would be classified as which of the following?

A) having an inner loop B) dimpled C) convex D) cardioid E) NOTA

17. On the polar coordinate system, the curves with equations $r=\sin\theta$ and $r=\cos\theta$ are drawn. A point is chosen randomly from the region enclosed by at least one of the graphs of these curves. Assuming each point is equally likely to be chosen, what is the probability that the point chosen is from the region enclosed by both curves' graphs?

A) $\dfrac{\pi-2}{8}$ B) $\dfrac{\pi-2}{4\pi}$ C) $\dfrac{\pi-2}{5\pi-2}$ D) $\dfrac{\pi-2}{3\pi+2}$ E) NOTA

18. What are the rectangular coordinates of the point with polar coordinates $\left(25,\tan^{-1}\left(-\dfrac{7}{24}\right)+\cos^{-1}\left(\dfrac{4}{5}\right)\right)$?

A) $\left({}^{117}\!/_{5},{}^{44}\!/_{5}\right)$ B) $\left({}^{44}\!/_{5},{}^{117}\!/_{5}\right)$ C) $(44,8)$ D) $(8,44)$ E) NOTA

19. Which of the following is not an equation of a line in the polar coordinate plane?

A) $\theta={}^{\pi}\!/_{6}$ B) $r=3\sec\theta$ C) $r=3$ D) $r=r\tan\theta$ E) NOTA

20. What is the degree measure equivalent to the angle $\left(\frac{3\pi}{4}-\frac{4\pi}{5}+\frac{5\pi}{6}\right)$ radians?

A) 141° B) 159° C) 282° D) 318° E) NOTA

21. Evaluate: $\frac{\left(3cis72^\circ\right)^2}{\left(2cis18^\circ\right)^3}$

A) $\frac{9}{8}$ B) $\frac{9}{8}i$ C) $\frac{9}{8}cis36^\circ$ D) $\frac{9}{8}cis72^\circ$ E) NOTA

22. 2011° is coterminal with which of the following angles?

A) $14{,}077^\circ$ B) 31° C) -1229° D) 3271° E) NOTA

23. Simplify: $\left(cisx\right)^2\left(cisy\right)^3\left(cisz^4\right)$

A) $\left(cis2x\right)\left(cis3y\right)\left(cis4z\right)$ B) $cis\left(x^2+y^3+z^4\right)$ C) $cis\left(2x+3y+4z\right)$
D) $cis\left(2x+3y+z^4\right)$ E) NOTA

24. Which of the following pairs of parametric equations is not a parameterization of the polar curve with equation $r=9$?

A) $\begin{cases}x=9\cos t\\y=9\sin t\end{cases}$ B) $\begin{cases}x=9\sin t\\y=9\cos t\end{cases}$ C) $\begin{cases}x=18\sin t\cos t\\y=9-18\sin^2 t\end{cases}$ D) $\begin{cases}x=9\sqrt{\frac{1-\cos t}{2}}\\y=9\sqrt{\frac{1+\cos t}{2}}\end{cases}$ E) NOTA

25. What point, in rectangular coordinates, is the center of the graph given by the polar equation $r=3\sin\theta-4\cos\theta$?

A) $\left(-3/2,2\right)$ B) $\left(3/2,-2\right)$ C) $\left(2,-3/2\right)$ D) $\left(-2,3/2\right)$ E) NOTA

26. What is the length of the radius of the graph given by the polar equation $r=3\sin\theta-4\cos\theta$?

A) 1 B) 2.5 C) 5 D) 6.25 E) NOTA

27. What are the cylindrical coordinates of the point with rectangular coordinates $(-2,2,5)$?

A) $\left(-2\sqrt{2},\frac{3\pi}{4},5\right)$ B) $\left(2\sqrt{2},\frac{3\pi}{4},5\right)$ C) $\left(2\sqrt{2},\frac{3\pi}{4},\frac{5\sqrt{2}}{2}\right)$ D) $\left(-2\sqrt{2},\frac{3\pi}{4},\frac{5\sqrt{2}}{2}\right)$ E) NOTA

28. What are the rectangular coordinates of the point with cylindrical coordinates $\left(4,\frac{5\pi}{3},7\right)$?

A) $\left(2,-2\sqrt{3},7\right)$ B) $\left(2,2\sqrt{3},7\right)$ C) $\left(-2,2\sqrt{3},7\right)$ D) $\left(-2,-2\sqrt{3},7\right)$ E) NOTA

29. Finally, we couldn't end the Polar Coordinate System test without talking about the North Pole, where Santa Claus lives! In July Santa Claus is about to embark on his summer vacation. He can't get too far away from the cold, so he plots his location on a model Earth with radius 4, where the *xy*-plane passes through the equator and the *z*-plane is perpendicular to the *xy*-plane through the center of the model Earth. His location on the surface of the model Earth lies at an angle of $\frac{2\pi}{3}$ radians in the *xy*-plane in standard position, and he stays on the 60° north latitude. What are the rectangular coordinates of Santa Claus' location?

A) $\left(1,-\sqrt{3},2\sqrt{3}\right)$ B) $\left(1,\sqrt{3},2\sqrt{3}\right)$ C) $\left(-1,\sqrt{3},2\sqrt{3}\right)$ D) $\left(-1,-\sqrt{3},2\sqrt{3}\right)$ E) NOTA

30. Santa Claus decides to circumnavigate the planet, but he can't go too close to the equator. To be safe, but still warm enough to travel in July, he maintains a travel trajectory along the 75° north latitude. Given a radius of 4 from the model Earth as in the previous problem, how far will Santa Claus travel around the globe on that trajectory, as depicted by the model? Set aside your differences and assume Earth is a perfect sphere.

A) 4π B) $4\pi\sqrt{4-2\sqrt{3}}$ C) $4\pi\sqrt{2-\sqrt{3}}$ D) $8\pi\sqrt{2-\sqrt{3}}$ E) NOTA