Answers:

1. C
2. A
3. B
4. A
5. D
6. D
7. A
8. B
9. C
10. C
11. A
12. B
13. A
14. A
15. B
16. C
17. A
18. A
19. D
20. A
21. E
22. C
23. D
24. C
25. D
26. B
27. E
28. B
29. B
30. A

Solutions:

1. The largest term in the rows whose numbers are answer choices are $\binom{6}{3}=20$, $\binom{7}{3}=35$, $\binom{8}{4}=70$, and $\binom{9}{4}=126$, so the lowest such row is row 8.

2. $(x+2y)^6 = x^6+6x^5\cdot 2y+15x^4(2y)^2+20x^3(2y)^3+15x^2(2y)^4+6x(2y)^5+(2y)^6$
$=x^6+12x^5y+60x^4y^2+160x^3y^3+240x^2y^4+192xy^5+64y^6$

3. $\sum_{j=0}^{8}\binom{8}{j}=2^8=256$

4. The difference in consecutive terms in A is $2/5$, thus creating an arithmetic sequence. B is geometric and D is harmonic .

5. $\sqrt{2}=\sqrt{2}r^3 \Rightarrow r^3=1 \Rightarrow r=1$ or $r=-\frac{1}{2}\pm\frac{\sqrt{3}}{2}i$, so the second term could be $\sqrt{2}\cdot 1=\sqrt{2}$ or $\sqrt{2}\left(-\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)=\frac{1}{2}\left(-\sqrt{2}+\sqrt{6}i\right)$ or $\sqrt{2}\left(-\frac{1}{2}-\frac{\sqrt{3}}{2}i\right)=-\frac{1}{2}\left(\sqrt{2}+\sqrt{6}i\right)$. The third number here is D.

6. Since the sum of the roots is 15, 5 must be a root. Therefore, $x^3-15x^2+71x-105$ $=(x-5)(x^2-10x+21)=(x-5)(x-3)(x-7)$, so the roots are 3, 5, and 7, making their common difference 2.

7. Let L be the limit. Then $L=\lim_{n\to\infty}a_n=\lim_{n\to\infty}a_{n+1}=\lim_{n\to\infty}\sqrt{12-a_n}=\sqrt{12-L}\Rightarrow L^2=12-L$ $\Rightarrow 0=L^2+L-12=(L+4)(L-3)\Rightarrow L=-4$ or $L=3$, but since the terms in the sequence are all positive, $L=3$.

8. $\sum_{n=1}^{\infty}a_n=\lim_{n\to\infty}b_n=2$, so the series $\sum_{n=1}^{\infty}a_n$ converges. Since $\lim_{n\to\infty}b_n=2$, $\sum_{n=1}^{\infty}b_n$ diverges and it must have been the case that $\lim_{n\to\infty}a_n=0\neq 2$, ruling out A and D. C is wrong also because $\{b_n\}$ converges to 2, not diverges.

9. $x=1+\cfrac{2}{1+\cfrac{2}{1+\cfrac{2}{1+\ldots}}}=1+\frac{2}{x}\Rightarrow x^2=x+2\Rightarrow 0=x^2-x-2=(x-2)(x+1)\Rightarrow x=2$ or $x=-1$, but the quantity must be positive since it only adds and divides positive numbers, so $x=2$.

10. The series is the harmonic series, which diverges, but $\frac{1}{n}\to 0$ as $n\to\infty$, so the sequence converges.

11. The series is a p-series with $p=2$, so the series converges. Also, $\frac{1}{n^2}\to 0$ as $n\to\infty$, so the sequence converges.

12. This is the exact statement of the Direct Comparison Test, so because $\sum_{n=1}^{\infty} b_n$, the "larger" series, converges, $\sum_{n=1}^{\infty} a_n$, the "smaller" series also converges. Additionally, because $\sum_{n=1}^{\infty} c_n$, the "smaller" series, diverges, $\sum_{n=1}^{\infty} a_n$, the "larger" series, diverges.

13. The series consists only of positive terms, so the Limit Comparison Test can be used, and $\lim\limits_{n\to\infty}\dfrac{\dfrac{4n^3+3n^2-7n+19}{6n^5+15n^4+126n^3+11n^2+12}}{\dfrac{1}{n^2}}=\lim\limits_{n\to\infty}\dfrac{4n^5+3n^4-7n^3+19n^2}{6n^5+15n^4+126n^3+11n^2+12}=\dfrac{2}{3}$, so both series converge or diverge. Since $\sum_{n=1}^{\infty}\frac{1}{n^2}$ converges (see problem 11), the series in question also converges. Since all the terms are positive, it also converges absolutely (since the absolute values of the terms are just the terms themselves).

14. The first 58 terms of the series are negative, but after that, the terms become positive (it isn't necessary to be able to pinpoint the exact number, just that eventually the terms will become positive since exponentials grow faster than any polynomial). Therefore, we will examine the series $\sum_{n=59}^{\infty}\frac{1}{2^n-n^{10}}$. Using the Limit Comparison Test on this series, $\lim\limits_{n\to\infty}\dfrac{\dfrac{1}{2^n}}{\dfrac{1}{2^n-n^{10}}}=\lim\limits_{n\to\infty}\dfrac{2^n-n^{10}}{2^n}=\lim\limits_{n\to\infty}\left(1-\dfrac{n^{10}}{2^n}\right)=1-0=1$

(again, because exponentials grow faster than polynomials). Therefore, either both series converge or both series diverge. Because $\sum_{n=1}^{\infty}\frac{1}{2^n}$ is geometric with ratio $\frac{1}{2}$, the series converges, meaning the series $\sum_{n=59}^{\infty}\frac{1}{2^n-n^{10}}$ converges (and absolutely, just as in problem 13). Therefore, adding 58 terms (or any finite number of terms) doesn't change the convergence, so the series in question converges absolutely.

15. $\sum_{n=2}^{\infty}\frac{(-1)^n}{n\ln n}$ is an alternating series, and since $\frac{1}{n\ln n}$ decreases and $\frac{1}{n\ln n}\to 0$ as $n\to\infty$, by the Alternating Series Test, this series converges. To determine if it converges absolutely, consider $\sum_{n=2}^{\infty}\frac{1}{n\ln n}$. $\int_2^{\infty}\frac{1}{x\ln x}dx=\lim_{t\to\infty}\ln\left|\ln x\right|\Big|_2^t=\lim_{t\to\infty}\left(\ln\ln t-\ln\ln 2\right)=\infty$, so this new series diverges, meaning the original series does not converge absolutely. It therefore converges conditionally.

16. $\lim_{n\to\infty}(-1)^n\sin n$ does not exist (the function oscillates), so by the Test for Divergence, the series diverges.

17. $\lim_{n\to\infty}\left|\dfrac{\frac{(-10)^{n+1}}{(n+2)4^{2n+3}}}{\frac{(-10)^n}{(n+1)4^{2n+1}}}\right|=\lim_{n\to\infty}\frac{10(n+1)}{16(n+2)}=\frac{5}{8}<1$, so by the Ratio Test, the series converges absolutely.

18. $\lim_{n\to\infty}\sqrt[n]{\left|\left(\frac{1}{n}\right)^n\right|}=\lim_{n\to\infty}\frac{1}{n}=0<1$, so by the Root Test, the series converges absolutely.

19. The first four terms in the Maclaurin expansion for $\sin x$ are $x-\frac{x^3}{3!}+\frac{x^5}{5!}-\frac{x^7}{7!}$, so for $\sin\left(x^2\right)$, the first four terms are $x^2-\frac{\left(x^2\right)^3}{3!}+\frac{\left(x^2\right)^5}{5!}-\frac{\left(x^2\right)^7}{7!}=x^2-\frac{x^6}{3!}+\frac{x^{10}}{5!}-\frac{x^{14}}{7!}$.

20. For $f(x)=\sin(\cos x)$, $f'(x)=\cos(\cos x)\cdot-\sin x$ and $f''(x)=\cos(\cos x)\cdot-\cos x$ $+(-\sin x)(-\sin(\cos x))(-\sin x)\Rightarrow f(0)=\sin 1,\ f'(0)=0,$ and $f''(0)=-\cos 1$. This

makes the second-degree Taylor polynomial $f(0)+f'(0)x+\frac{f''(0)}{2!}x^2$

$=\sin 1-\frac{x^2}{2}\cos 1$

21. Since the first-degree Taylor polynomial for e^x is $1+x$, the third-degree Taylor polynomial for e^{x^3} is $1+x^3$. Therefore, $\int_0^2 e^{x^3}dx \approx \int_0^2 (1+x^3)dx = \left(x+\frac{1}{4}x^4\right)\Big|_0^2$

$=2+4=6$.

22. $y'=\cos x-\sin x+\sec^2 x-\csc x\cot x-\csc^2 x$, so $y\left(\frac{\pi}{4}\right)=\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}+1+\sqrt{2}+1$

$=2+2\sqrt{2}$ and $y'\left(\frac{\pi}{4}\right)=\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}+2-\sqrt{2}\cdot 1-2=-\sqrt{2}$, making the linearization

$L(x)=2+2\sqrt{2}-\sqrt{2}\left(x-\frac{\pi}{4}\right)$. Therefore, $y\left(\sqrt{2}\right)\approx L\left(\sqrt{2}\right)=2+2\sqrt{2}-2+\frac{\sqrt{2}\pi}{4}$

$=\frac{\sqrt{2}}{4}(8+\pi)$.

23. A well-known result of the Fibonacci sequence is that $\sum_{k=1}^{n}F_k=F_{n+2}-1$. Therefore,

$\sum_{k=1}^{30}F_k=F_{32}-1=2{,}178{,}309-1=2{,}178{,}308$.

24. $\sum_{k=1}^{n}\sum_{i=1}^{k}i=\sum_{k=1}^{n}\frac{k(k+1)}{2}=\frac{1}{2}\sum_{k=1}^{n}\left(k^2+k\right)=\frac{1}{2}\left(\frac{n(n+1)(2n+1)}{6}+\frac{n(n+1)}{2}\right)$

$=\frac{1}{2}\cdot\frac{n(n+1)(2n+4)}{6}=\frac{(n+2)(n+1)n}{3\cdot 2\cdot 1}=\binom{n+2}{3}$

25. Using the Ratio test, $\lim_{n\to\infty}\left|\frac{\frac{2^{n+1}4^{n+1}}{n+1}(x-2)^{n+1}}{\frac{2^n4^n}{n}(x-2)^n}\right|=\lim_{n\to\infty}\frac{8n}{n+1}|x-2|=8|x-2|<1\Rightarrow|x-2|<\frac{1}{8}$,

so the radius of convergence is $1/8=0.125$.

26. Using the Ratio test, $\lim_{n\to\infty}\left|\dfrac{\frac{(-x)^{n+1}}{n+1}}{\frac{(-x)^n}{n}}\right| = \lim_{n\to\infty}\dfrac{n|-x|}{n+1} = |-x| < 1 \Rightarrow -1 < x < 1$, but the endpoints must be tested as well. If $x=-1$, the series becomes $\sum_{n=1}^{\infty}\frac{(1)^n}{n} = \sum_{n=1}^{\infty}\frac{1}{n}$, which is the harmonic series, which diverges. If $x=1$, the series becomes $\sum_{n=1}^{\infty}\frac{(-1)^n}{n}$, which converges by the Alternating Series test because $\frac{1}{n}$ is decreasing and $\frac{1}{n}\to 0$ as $n\to\infty$. Therefore, the interval of convergence is $(-1,1]$.

27. $\sum_{n=0}^{\infty}\frac{1}{2^n n!} = \sum_{n=0}^{\infty}\frac{\left(\frac{1}{2}\right)^n}{n!} = e^{\frac{1}{2}}$ since the power series for e^x is $\sum_{n=0}^{\infty}\frac{x^n}{n!}$, which converges to e^x for all x

28. $\sum_{n=0}^{\infty}9^{-n}$ is a geometric series, so $\sum_{n=0}^{\infty}9^{-n} = \dfrac{1}{1-\frac{1}{9}} = \dfrac{1}{\frac{8}{9}} = \dfrac{9}{8}$

29. Let S be the sum of the series; then $S = \frac{1}{4}+\frac{2}{16}+\frac{3}{64}+\frac{4}{256}+\dots$. Multiplying this by $\frac{1}{4}$ yields $\frac{1}{4}S = \frac{1}{16}+\frac{2}{64}+\frac{3}{256}+\dots$. Subtracting this second equation from the first yields $\frac{3}{4}S = \frac{1}{4}+\frac{1}{16}+\frac{1}{64}+\frac{1}{256}+\dots = \dfrac{\frac{1}{4}}{1-\frac{1}{4}} = \dfrac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{3} \Rightarrow S = \frac{4}{9}$.

30. $\dfrac{1}{1+x^2} = \dfrac{1}{1-\left(-x^2\right)} = \sum_{n=0}^{\infty}\left(-x^2\right)^n = \sum_{n=0}^{\infty}\left((-1)^n x^{2n}\right)$