Answers:

1. A
2. C
3. B
4. B
5. A
6. A
7. D
8. D
9. B
10. B
11. E
12. B
13. C
14. B
15. A
16. E
17. D
18. C
19. C
20. A
21. B
22. D
23. A
24. C
25. C
26. C
27. D
28. B
29. C
30. A

Solutions:

1. $21 \le 3x + 11 \Rightarrow 3x \ge 10 \Rightarrow x \ge {}^{10}\!/_{3}$

2. $4A + 6Bi = (3+2i)(1-3i) = 3 - 9i + 2i + 6 = 9 - 7i \Rightarrow A = \frac{9}{4}$ and $B = -\frac{7}{6} \Rightarrow \frac{A}{B} = \frac{{}^{9}\!/_{4}}{-{}^{7}\!/_{6}}$

$= -{}^{27}\!/_{14}$

3. $0 = \begin{vmatrix} 1 & 3 & 2 \\ 4 & x & 6 \\ 5 & 2 & 1 \end{vmatrix} = x + 90 + 16 - 10x - 12 - 12 = -9x + 82 \Rightarrow 9x = 82 \Rightarrow x = \frac{82}{9}$

4. $18 = 2\big(2(x+3) - (x+1)\cdot 3\big) = 2(2x + 6 - 3x - 3) = 2(-x+3) = -2x + 6 \Rightarrow -2x = 12$
$x = -6$

5. $3 = 11 - 8 = B - A < C < B + A = 11 + 8 = 19$. Each of B, C, and D are incorrect because, although they are in the acceptable interval, the value of C does not necessarily lie in the intervals given as answers B, C, or D.

6. Considered as a function, the solutions are the roots of $f(x) = x^3 - 27$, so the sum of the roots for this polynomial is the negative x^2 coefficient divided by the x^3 coefficient, or $-\frac{0}{1} = 0$.

7. This is the sum of all products of the roots taking two at a time, so for this polynomial, this value is the x^2 coefficient divided by the x^4 coefficient, or $\frac{-53}{1} = -53$.

8. Written as exponentials with base 2, the equation becomes $\left(2^2\right)^{3x+4} = \left(2^5\right)^{x-1}$
$\Rightarrow 2^{6x+8} = 2^{5x-5} \Rightarrow 6x + 8 = 5x - 5 \Rightarrow x = -13$.

9. $-14 \le 6x - 4 \le 14 \Rightarrow -10 \le 6x \le 18 \Rightarrow -{}^{5}\!/_{3} \le x \le 3$. The integers in this interval are -1, 0, 1, 2, and 3, so there are 5 integer solutions total.

10. Multiplying this inequality by 144 yields $481-160x+16x^2-54y+9y^2\le 144$, and completing the square yields
$16\left(x^2-10x+25\right)+9\left(y^2-6y+9\right)\le 144-481+400+81=144$
$\Rightarrow \dfrac{(x-5)^2}{9}+\dfrac{(y-3)^2}{16}\le 1$, so this is the area enclosed by an ellipse who semi-major and semi-minor axes have lengths 3 and 4. Therefore, the area is $\pi\cdot 4\cdot 3=12\pi$.

11. $f(3-x)=\dfrac{x^2-6x}{x+2}=\dfrac{\left(-(3-x)\right)^2-9}{-(3-x)+5}\Rightarrow f(x+1)=\dfrac{\left(-(x+1)\right)^2-9}{-(x+1)+5}=\dfrac{x^2+2x-8}{-x+4}$

12. The domain of $f^{-1}(x)$ is the range of $f(x)$, which is all numbers greater than 11, so $x>11$.

13. $x=\sqrt{90+x}\Rightarrow x^2=90+x\Rightarrow 0=x^2-x-90=(x-10)(x+9)\Rightarrow x=10$ or $x=-9$, but x must be positive, so $x=10$.

14. $g(f(2))=g\left(6\cdot 2+\dfrac{11}{2}-3\right)=g\left(\dfrac{29}{2}\right)=\left(\dfrac{29}{2}\right)^2=\dfrac{841}{4}$

15. $a+b\sqrt{c}=\dfrac{\sqrt{24-8\sqrt{5}}}{\sqrt{6+2\sqrt{5}}}=\dfrac{2\sqrt{5}-2}{\sqrt{5}+1}=3-\sqrt{5}$, so $abc=3\cdot -1\cdot 5=-15$

16. $x^2+5x-8\ge 2x^2+2\Rightarrow 0\ge x^2-5x+10=\left(x-\dfrac{5}{2}\right)^2+\dfrac{15}{4}$, which can never be less than $\dfrac{15}{4}$, so there is no solution.

17. $f(z)=15z^2+16z+4=(3z+2)(5z+2)$, so the zeros are $-\dfrac{2}{3}$ and $-\dfrac{2}{5}$

18. If getting a common denominator on the right-hand side of the equation, A, B, and C would be the x^2 coefficients of each fraction, so their sum would be the x^2 coefficient in the numerator on the left-hand side, which is 21.

19. Subtracting the second equation from the first yields $2x-6z=11$, and doubling the third equation yields $2x+6z=8$. Adding these two equations together yields $4x=19\Rightarrow x=4.75$.

20. $x^2-x-12=(x+3)(x-4)>(x-6)(x+2)=x^2-4x-12\Rightarrow 3x>0\Rightarrow x>0$

21. The graph is a rhombus connecting the vertices $\left(\pm\frac{1}{2},0\right)$ and $\left(0,\pm\frac{1}{3}\right)$. Therefore, the enclosed area is $\frac{1}{2}\cdot 1\cdot\frac{2}{3}=\frac{1}{3}$.

22. Adding all three equations together yields $3x+3y+3z=9\Rightarrow x+y+z=3$.

23. $2=8+\frac{5}{x}+\frac{1}{x^2}\Rightarrow 0=6+\frac{5}{x}+\frac{1}{x^2}=\frac{6x^2+5x+1}{x^2}=\frac{(3x+1)(2x+1)}{x^2}\Rightarrow x=-\frac{1}{3}$ or $x=-\frac{1}{2}$. Therefore, $\frac{1}{A}+\frac{1}{B}=(-3)+(-2)=-5$.

24. The coefficients in the expansion are $3^4=81$, $4(3)^3(5)=540$, $6(3)^2(5)^2=1350$, $4(3)(5)^3=1500$, and $5^4=625$. Therefore, the largest coefficient is 1500.

25. Adding the second and third equations together yields $4x+4y+4z=8$ $\Rightarrow x+y+z=2$.

26. The sum of the coefficients would be the same as if $x=1$, so that sum is $(1+1)^7$ $=2^7=128$.

27. The only ways to get the left-hand side to equal 1 is if (1) the base is 1, (2) the exponent is 0 and the base is not, or (3) the base is -1 and the exponent is even. For case (1), $x^2-x-1=1\Rightarrow 0=x^2-x-2=(x-2)(x+1)\Rightarrow x=2$ or $x=-1$. For case (2), $0=x^2-x-6=(x-3)(x+2)\Rightarrow x=3$ or $x=-2$. For case (3), setting the base equal to -1, $-1=x^2-x-1\Rightarrow 0=x^2-x=x(x-1)\Rightarrow x=0$ or $x=1$, and both of these values make the exponent equal to -6, so both work. Therefore, the sum of the solutions to the equation is $2-1+3-2+0+1=3$.

28. Plugging $x=\frac{1}{2}$ into the equation yields $\frac{1}{4}+\frac{A}{4}+\frac{B}{2}-\frac{3}{2}=0\Rightarrow\frac{A}{4}+\frac{B}{2}=\frac{5}{4}$. Plugging $x=-\frac{1}{2}$ into the equation yields $-\frac{1}{4}+\frac{A}{4}-\frac{B}{2}-\frac{3}{2}=0\Rightarrow\frac{A}{4}-\frac{B}{2}=\frac{7}{4}$. Adding these two equations together yields $\frac{A}{2}=3\Rightarrow A=6\Rightarrow B=-\frac{1}{2}\Rightarrow A+B=6-\frac{1}{2}=\frac{11}{2}$.

29. $f\left(f\left(f\left(f\left(f\left(f(2)\right)\right)\right)\right)\right)=f\left(f\left(f\left(f\left(f(4)\right)\right)\right)\right)=f\left(f\left(f\left(f(6)\right)\right)\right)=f\left(f\left(f(-6)\right)\right)$
$=f\left(f(36)\right)=f(1)=2$

30. We must consider $4x-|3x+2|=6$ and $4x-|3x+2|=-6$. For the first equation, $4x-6=|3x+2|\Rightarrow 4x-6=3x+2$ or $4x-6=-3x-2\Rightarrow x=8$ or $7x=4\Rightarrow x=8$ or $x=4/7$. Checking these in the original equation, $|4\cdot 8-|3\cdot 8+2||=|32-|26||=6$ and $\left|4\cdot\frac{4}{7}-\left|3\cdot\frac{4}{7}+2\right|\right|=\left|\frac{16}{7}-\left|\frac{26}{7}\right|\right|=\frac{10}{7}\neq 6$, so 8 is our only solution from that equation. For the second equation, $4x+6=|3x+2|\Rightarrow 4x+6=3x+2$ or $4x+6=-3x-2\Rightarrow x=-4$ or $7x=-8\Rightarrow x=-4$ or $x=-8/7$. Checking these in the original equation, $|4\cdot -4-|3\cdot -4+2||=|-16-|-10||=26\neq 6$ and $\left|4\cdot -\frac{8}{7}-\left|3\cdot -\frac{8}{7}+2\right|\right|=\left|-\frac{32}{7}-\left|-\frac{10}{7}\right|\right|=6$, so $-\frac{8}{7}$ is our only solution from that equation. Therefore, the sum of the solutions is $8+\left(-\frac{8}{7}\right)=\frac{48}{7}$.