Select the best answer to each question. The choice "NOTA" denotes "None of the Above".
For this test, we assume that inverse trig functions have their restricted ranges, and $i = \sqrt{-1}$.

1) Which of the following most closely resembles the graph of the equation $x = |y|$ in the *xy*-plane?

A)  B)  C)  D)  E) NOTA

2) Which of the following points (x, y) satisfies the inequality $x^2 + 1 < y^3 - 3$?

A) $(2,2)$ B) $(8,4)$ C) $(11,5)$ D) $(14,6)$ E) NOTA

3) Find $A > 0$ and $\phi \in [0, 2\pi)$ such that $\sqrt{3}\sin(x) + \cos(x) = A\cos(x - \phi)$. What is $A\phi$?

A) $\pi/6$ B) $\pi/3$ C) $2\pi/3$ D) $10\pi/3$ E) NOTA

4) A value of $\theta \in [0, \pi]$ is chosen randomly. Find the probability that it satisfies $\sin(\theta) \le \cos(\theta)$.

A) $1/4$ B) $1/\pi$ C) $1/2$ D) $3/4$ E) NOTA

For questions 5-7, consider that $\{x\} = x - \lfloor x \rfloor$ where $\lfloor x \rfloor$ denotes the greatest integer function.

5) Find the value of $\{\pi\} + \{-\pi\}$.

A) $6 - 2\pi$ B) 0 C) $2\pi - 6$ D) 1 E) NOTA

6) Find an inequality that gives the range of $\{x\}$.

A) $-1 \le \{x\} \le 1$ B) $-1 < \{x\} < 1$ C) $0 \le \{x\} < 1$ D) $0 \le \{x\} \le 1$ E) NOTA

7) Find the sum of the solutions to the equation $2x + \{x\} = 2.1$. *Hint:* We must have $x > 0$.

A) $7/10$ B) $11/15$ C) $7/5$ D) $26/15$ E) NOTA

8) Solve for a: $\langle 5,a,3\rangle \bullet \langle -a,2a,6\rangle = 16$.

A) $\left\{-2,-\frac{1}{2}\right\}$ B) $\left\{-2,\frac{1}{2}\right\}$ C) $\left\{-\frac{1}{2},2\right\}$ D) $\left\{\frac{1}{2},2\right\}$ E) NOTA

9) Let $f(t) = A\cos(\omega t - \phi)$ have period P and phase lag ϕ. The time lag, t_0, of $f(t)$ is then given by $\frac{P\phi}{2\pi}$. Find the time lag of $f(t) = 2012\cos(\pi t + 2)$.

A) -2 B) $-2/\pi$ C) $2/\pi$ D) 2 E) NOTA

10) If $x^3 - y^3 = 98i$ and $x - y = 7i$, then $xy = a + bi$ where a and b are real numbers. Find the value of $a + b$.

A) 7 B) $35/3$ C) 21 D) 63 E) NOTA

11) Find the sum of the solutions to $(x+2)^{2012} + (x-1)^{2012} = 2012$.

A) -2012 B) -1006 C) 1006 D) 2012 E) NOTA

12) Consider the polar curves $r = \frac{1}{\cos(\theta)}$ and $r = \frac{1}{\cos\left(\theta - \frac{\pi}{4}\right)}$. How many distinct points of intersection are there between these graphs in the xy-plane?

A) 0 B) 1 C) 2 D) infinitely many E) NOTA

13) There is a famous inequality that relates the arithmetic mean, AM, the geometric mean, GM, and the harmonic mean, HM, of a set of positive real numbers. Which of the following correctly states that inequality?

A) $AM \geq GM \geq HM$ B) $AM \geq HM \geq GM$
C) $HM \geq AM \geq GM$ D) $GM \geq AM \geq HM$ E) NOTA

14) Find the number of solutions to the equation $\sin(x) = \tan(x)$.

A) 1 B) 2 C) 3 D) 4 E) NOTA

15) Given that $\log_a\left[\sin(x)\right]-2\log_a\left[\cos(x)\right]=2$ and $\sum_{k=1}^{\infty}\sin^k(x)=\frac{3}{2}$, find a. (Note: $a>0$.)

A) $\sqrt{5}/4$ B) $3/4$ C) $\sqrt{6}/3$ D) $\sqrt{15}/4$ E) NOTA

For questions 16-17, consider that one popular way to solve a system of linear equations is with Cramer's Rule. Consider the system of equations $\begin{cases} w-2x+4z=3 \\ 3w+x+y=4 \\ -w-5x-1y+8z=12 \\ 3w+8x+2y-12z=15 \end{cases}$.

16) Let the coefficient matrix be D. Find the determinant of D.

A) -56 B) 0 C) 7 D) 84 E) NOTA

17) Using Cramer's Rule, solve for the value of w in this system of equations.

A) 0 B) 1 C) 3 D) need more info E) NOTA

18) Find the smallest value of a such that $-x^2+x-1\le a$ for all real numbers x.

A) $-3/4$ B) $-1/2$ C) $1/4$ D) $1/2$ E) NOTA

19) If $a=\sin^2(2x)$ then which of the following is equivalent to $\sin^6(x)+\cos^6(x)$?

A) $1-\frac{3a}{4}$ B) $1-\frac{3a}{2}$ C) $1-3a$ D) $1-\frac{3a^2}{4}$ E) NOTA

20) If $f(2x+1)=x^5-3x^4+2x^3-1$, find the sum of the roots of $f(x)$.

A) 3 B) 4 C) 7 D) 11 E) NOTA

21) Find the subset of $[0,\pi]$ that satisfies $0<\sin(\theta)\cdot\tan(\theta)<1$.

A) $\left(0,\cos^{-1}\left(\frac{\sqrt{5}-1}{2}\right)\right)$ B) $\left(\cos^{-1}\left(\frac{\sqrt{5}-1}{2}\right),\frac{\pi}{2}\right)$

C) $\left(\cos^{-1}\left(\frac{\sqrt{5}-1}{2}\right),\pi\right)$ D) $\left(\cos^{-1}\left(\frac{\sqrt{5}-1}{2}\right),\frac{\pi}{2}\right)\cup\left(\frac{\pi}{2},\pi\right)$ E) NOTA

22) The subset of $[0,\pi]$ that satisfies $0\le 25\cos(\theta)-20\le 4$ is $[a,b]$. Find $\frac{\tan(b)}{\tan(a)}$.

A) $7/18$ B) $5/6$ C) $7/4$ D) $18/7$ E) NOTA

23) Let $a_0=2,\ a_1=1,$ and $a_n=\frac{a_{n-2}}{2}$ for $n\ge 2$. Find $\theta\in\left(0,\pi/2\right)$ such that $\sum_{n=0}^{\infty}\left[a_n\sin^n(\theta)\right]=4$.

A) $\pi/6$ B) $\sin^{-1}\left(\frac{\sqrt{15}-1}{4}\right)$ C) $\pi/3$ D) $\sin^{-1}\left(\frac{\sqrt{17}-1}{4}\right)$ E) NOTA

24) Using a certain inequality relating two types of means, find the minimum value of $x+\frac{1}{2x}$ for $x>0$.

A) $1/2$ B) $\sqrt{2}/2$ C) 1 D) $\sqrt{2}$ E) NOTA

25) Consider $P=\begin{bmatrix}0.9 & 0.1 & 0\\ 0.9 & 0 & 0.1\\ 0.9 & 0 & 0.1\end{bmatrix}$. There exists a vector $\mathbf{p}=\langle p_1,p_2,p_3\rangle$ such that $p_1+p_2+p_3=1$ and $\mathbf{p}P=\mathbf{p}$. Find p_2.

A) 0.009 B) 0.09 C) 0.1 D) 0.9 E) NOTA

26) Find the number of values of x such that $\left|1-\left|x^2+2x-1\right|\right|=-1$.

A) 0 B) 1 C) 2 D) 3 E) NOTA

27) Find the sum of all values of $\theta \in (0,2\pi)$ such that $4\sin^3(x)+2\sin^2(x)-2\sin(x)=1$.

A) 3π B) 4π C) 6π D) 7π E) NOTA

28) How many values of $\theta \in (0,2\pi)$ satisfy $\left|\sin(\theta)+\cos(\theta)\right|=\frac{3}{2}$?

A) 0 B) 1 C) 2 D) 4 E) NOTA

29) The two non-real solutions to the equation $(x+1)^3=2$ can be written in the form $a \pm bi$ where a and b are real numbers. Find $|a|$.

A) $\frac{\sqrt[3]{2}\cdot\sqrt{3}}{2}-1$ B) $1-\frac{\sqrt[3]{2}}{2}$ C) $1+\frac{\sqrt[3]{2}}{2}$ D) $1+\frac{\sqrt[3]{2}\cdot\sqrt{3}}{2}$ E) NOTA

30) Solve the equation $\sin\left(2^{2013}\cdot x\right)=\prod_{k=0}^{2012}\cos\left(2^k\cdot x\right)$ over the interval $x \in [0,\pi]$.

A) $\sin^{-1}\left(\frac{1}{2^{2013}}\right)$ B) $\frac{1}{2^{2012}}$ C) $\sin^{-1}\left(\frac{1}{2^{2012}}\right)$ D) $\frac{\pi}{2}$ E) NOTA