1. A  This is the graph of $y = |x|$ reflected about the line $y = x$.
2. D  We see that $14^2 + 1 = 197 < 213 = 6^3 - 3$. None of the other points satisfy the inequality.
3. C  Since the amplitude of $\sqrt{3} \sin(x) + \cos(x)$ is $\sqrt{\left(\sqrt{3}\right)^2 + 1^2} = 2$, we have that $A = 2$. Expand
   
   $2\cos(x - \phi) = 2\left[\sin(\phi)\sin(x) + \cos(\phi)\cos(x)\right]$, so we need $\sin(\phi) = \frac{\sqrt{3}}{2}$ and $\cos(\phi) = \frac{1}{2}$. Thus, $\phi = \frac{\pi}{3}$, so $A\phi = 2\frac{\pi}{3}$.
4. A  Over $[0, \pi]$, this inequality is satisfied on the interval $\left[0, \frac{\pi}{4}\right]$. Thus, the desired probability
   is $\frac{\pi}{4} = \frac{1}{4}$.
5. D  We have $\{\pi\} + \{-\pi\} = (\pi - 3) + (-\pi - (-4)) = 4 - 3 = 1$.
6. B  Naturally, for positive $x$, the fractional part can be any number in $[0,1)$. However, for
   negative $x$, $[x] \leq x$, so the fractional part becomes negative (it can be any number in $(-1,0]$). Thus, the desired range is $(-1,1)$.
7. D  We can write this as $2x + (x - \lfloor x \rfloor) = 3x - \lfloor x \rfloor = 2.1$. It is clear that $x$ cannot be greater than
   or equal to 2 (since for positive $x$, $\{x\}$ is non-negative). Thus, $\lfloor x \rfloor = 0$ or $\lfloor x \rfloor = 1$. Solving these
   separately gives $x = 2.1/5 = 0.4$ and $x = 3.1/3 = 1.0$. The sum is $21/15 + 31/30 = 52/30 = 26/15$.
8. D  We have $-5a + 2a^2 + 18 = 16$, so $2a^2 - 5a + 2 = 0$. We factor this as
   
   $2a^2 - 4a - a + 2 = 2(a - 2) - (a - 2) = (2a - 1)(a - 2) = 0$, so the solutions are $\left\{\frac{1}{2}, 2\right\}$.
9. B  The period is $\frac{2\pi}{\omega}$ so $\frac{P}{2\pi} = \frac{1}{\omega}$. The time lag is then $t_0 = \frac{1}{\pi} \cdot (-2) = -\frac{2}{\pi}$.
10. C  Note that $\frac{x^3 - y^3}{x - y} = x^2 + xy + y^2 = 14$. We have $(x - y)^2 = x^2 - 2xy + y^2 = -49$. Subtracting
    the two equations gives $3xy = 63$, so $xy = 21$. Thus, $a + b = 21 + 0 = 21$.
11. B  We need to find the coefficients on the $x^{2012}$ and $x^{2011}$ terms. We see that
    
    $(x + 2)^{2012} = x^{2012} + 2012 \cdot 2x^{2011} + \ldots$ and $(x - 1)^{2012} = x^{2012} - 2012x^{2011} + \ldots$, so
    
    $(x + 2)^{2012} + (x - 1)^{2012} = 2x^{2012} + 2012x^{2011} + \ldots$. The sum of the roots is $-2012/2 = -1006$.
12. B  Writing these in rectangular form gives $r\cos(\theta) = 1 \Rightarrow x = 1$ and
    
    $r\cos\left(\theta - \frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}(r\sin(\theta) + r\cos(\theta)) = 1 \Rightarrow x + y = \sqrt{2}$. These are non-parallel lines, so they
    intersect exactly once.
13. A  If you didn’t already know this, you could just substitute in some numbers.
14. E  This occurs where $\sin(x) = 0$ or $\cos(x) = 1$. Thus, $x = n\pi$ for all integers $n$, so there are
    infinitely many solutions.
15. D We have that \( \frac{\sin(x)}{1-\sin(x)} = \frac{3}{2} \), so \( \sin(x) = \frac{3}{2} - \frac{3}{2} \sin(x) \Rightarrow \frac{5}{2} \sin(x) = \frac{3}{2} \Rightarrow \sin(x) = \frac{3}{5} \).

By the domain of the logarithm, we know that \( \cos(x) > 0 \) so \( \cos(x) = \frac{4}{5} \). Then,

\[
\log_a\left( \frac{\sin(x)}{\cos^2(x)} \right) = \log_a\left( \frac{\frac{3}{5}}{\frac{16}{25}} \right) = \log_a\left( \frac{15}{16} \right) = 2, \text{ so } a = \sqrt[15]{16} = \sqrt[15]{4}.
\]

16. B The determinant is

\[
\begin{vmatrix}
1 & 1 & 0 & 3 & 1 & 0 & 3 & 1 & 1 \\
-5 & -1 & 8 & -1 & -1 & 8 & -4 & -1 & -5 \\
8 & 2 & -12 & 3 & 2 & -12 & 3 & 8 & 2 \\
\end{vmatrix}
= 1 \cdot 0 + 2 \cdot 0 - 4 \cdot 0 = 0. \text{ However, this is actually unnecessary – just do question 17, and when you see that row operations eliminate a row of the matrix, you know that the determinant is 0.}
\]

17. D Since the answer to the previous question is 0, the system is linearly dependent. Hence, \( w \) cannot be uniquely determined without values of at least some of the other variables.

18. A \( a \) is just the minimum value of \(-x^2 + x - 1\) which occurs at the vertex of \( x = \frac{1}{2(-1)} = \frac{1}{2} \).

Thus, \( a = -\frac{1}{4} + \frac{1}{2} - 1 = -\frac{3}{4} \).

19. A Using the sum of cubes factorization, write

\[
\sin^6(x) + \cos^6(x) = \left[ \sin^2(x) + \cos^2(x) \right] \left[ \sin^4(x) + \cos^4(x) - \sin^2(x) \cos^2(x) \right].
\]

Note then that

\[
\left[ \sin^2(x) + \cos^2(x) \right]^2 = 1 = \sin^4(x) + \cos^4(x) + 2 \sin^2(x) \cos^2(x).
\]

We can then write

\[
\sin^6(x) + \cos^6(x) = 1 - 3 \sin^2(x) \cos^2(x) = 1 - 3 \left( \frac{\sin(2x)}{2} \right)^2 = 1 - \frac{3a}{4}.
\]

20. D If we denote the 5 roots of \( f(2x+1) \) as \( r_i \) then the roots of \( f(x) \) are \( 2r_i + 1 \). Thus, the sum of the roots of \( f(x) \) is \( \sum_{k=1}^{5} (2r_i + 1) = 5 + 2 \sum_{k=1}^{5} r_i = 5 + 2 \cdot 3 = 11 \).

21. A We can write \( \sin(\theta) \tan(\theta) = \frac{\sin^2(\theta)}{\cos(\theta)} = \frac{1 - \cos^2(\theta)}{\cos(\theta)} \). Since the numerator is non-negative, we have a positive denominator, so \( \cos(\theta) > 0 \). Then, for \( \frac{1 - \cos^2(\theta)}{\cos(\theta)} < 1 \) we need \( \cos(\theta) > 1 - \cos^2(\theta) \) so \( \cos^2(\theta) + \cos(\theta) - 1 > 0 \). The roots of \( \cos^2(\theta) + \cos(\theta) - 1 = 0 \) are \( \cos(\theta) = \frac{-1 \pm \sqrt{5}}{2} \), so this is satisfied where \( \cos(\theta) < \frac{-1 - \sqrt{5}}{2} \) and \( \cos(\theta) > \frac{-1 + \sqrt{5}}{2} \). However, since \( \cos(\theta) > 0 \), the valid interval is where \( \cos(\theta) > \frac{-1 + \sqrt{5}}{2} \). Noting that cosine decreases on this interval, the correct answer is \( \left[ 0, \cos^{-1}\left( \frac{-1 + \sqrt{5}}{2} \right) \right] \).
22. D We need \(20 \leq 25\cos(\theta) \leq 24 \Rightarrow 4/5 \leq \cos(\theta) \leq 24/25\). This occurs over the interval 
\([\arccos(24/25), \arccos(4/5)]\), (note that the order is reversed because of arccosine), so
\[
\frac{\tan(b)}{\tan(a)} = \frac{\tan(\arccos(4/5))}{\tan(\arccos(24/25))} = \frac{3/4}{7/24} = \frac{3}{4} \cdot \frac{24}{7} = \frac{18}{7}.
\]

23. D Write this as the sum of two series. For even terms, we have \(a_{2n} = \frac{2}{2^n}\) and for odd terms we have \(a_{2n+1} = \frac{1}{2^n}\). Then, \(\sum_{n=0}^{\infty} \left[\frac{2}{2^n} \cdot \sin^2(\theta) \right] + \sum_{n=0}^{\infty} \left[\frac{1}{2^n} \cdot \sin^2(\theta) \right] = \frac{2}{1 - 0.5\sin^2(\theta)} + \frac{\sin(\theta)}{1 - 0.5\sin^2(\theta)} = 4\), so \(4 - 2\sin^2(\theta) = 2 + \sin(\theta) \Rightarrow 2\sin^2(\theta) + \sin(\theta) - 2 = 0\), so \(\sin(\theta) = \frac{-1 \pm \sqrt{17}}{4}\). Taking the value on the desired interval gives \(\theta = \sin^{-1}\left(\frac{-1 - \sqrt{17}}{4}\right)\).

24. D Apply AM-GM: \(\frac{x + \frac{1}{2x}}{2} \geq \sqrt{\frac{x}{2x}} = \sqrt{\frac{1}{2}}\), so \(x + \frac{1}{2x} \geq \sqrt{2}\). This minimum can indeed be obtained when \(x = \sqrt{\frac{1}{2}}\).

25. B Multiplying gives \(p_1 = 0.9(p_1 + p_2 + p_3) = 0.9, p_2 = 0.1p_1 = 0.09, \) and \(p_3 = 0.1p_2 = 0.01\).

26. A The absolute value of anything is non-negative; there cannot be any solutions.

27. D Let \(a = \sin(x)\). Then, \(4a^3 + 2a^2 - 2a - 1 = 2a^2(2a + 1) - 1(2a + 1) = (2a - 1)(2a + 1) = 0\).

This gives \(\sin(x) = \pm \frac{1}{\sqrt{2}}\) and \(\sin(x) = -\frac{1}{2}\). This corresponds to the solutions \(\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}\) and \(\frac{7\pi}{6}, \frac{11\pi}{6}\) for a sum of \(4\pi + 3\pi = 7\pi\).

28. A The maximum value of \(|\sin(\theta) + \cos(\theta)|\) is \(\sqrt{2} < 3/2\). We could also show this by squaring both sides of the given equation and obtaining \(\sin(2\theta) = \frac{-5}{4}\) which is obviously impossible.

29. C First we find the non-real cube roots of 2: we know that they are in the form \(\sqrt[3]{2}\cis(\theta)\), so
\[2\cis(3\theta) = 2 \Rightarrow 3\theta = 0, 2\pi, 4\pi, \] so the non-real cube roots of 2 are \(\sqrt[3]{2}\cis\left(\frac{2\pi}{3}\right) = \sqrt[3]{2}\cis\left(-\frac{1}{2} + \frac{i\sqrt{3}}{2}\right)\) and
\[\frac{3\sqrt[3]{2}}{2}\cis\left(\frac{4\pi}{3}\right) = \sqrt[3]{2}\cis\left(-\frac{1}{2} - \frac{i\sqrt{3}}{2}\right).\] Thus, \(x + 1 = \frac{\sqrt[3]{2}}{2} \pm \frac{\sqrt[3]{2}}{2}i\), so \(x = -\frac{\sqrt[3]{2}}{2} \pm \frac{\sqrt[3]{2}}{2}i\).

Thus, \(|a| = 1 + \frac{\sqrt[3]{2}}{2}i\).

30. A Note that
\[
\sin(x)\prod_{k=0}^{2012} \cos(2^k x) = \sin(x)\cos(x)\cos(2x)\cdots\cos(2^{2012} x) = \frac{\sin(2x)\cos(2x)\cos(4x)\cdots\cos(2^{2012} x)}{2}
\]
\[
\frac{\sin(4x) \cos(4x) \cos(8x) \cdots \cos(2^{2012}x)}{2^2} = \frac{\sin(8x) \cos(8x) \cos(16x) \cdots \cos(2^{2012}x)}{2^3} = \cdots = \frac{\sin(2^{2013}x)}{2^{2013}}.
\]

Thus, \( \sin(2^{2013}x) = \prod_{k=0}^{2012} \cos(2^k x) = \frac{\sin(2^{2013}x)}{2^{2013} \sin(x)} \Rightarrow \sin(x) = \frac{1}{2^{2013}} \Rightarrow x = \sin^{-1}\left(\frac{1}{2^{2013}}\right). \)