1. **A** This is the graph of $y = |x|$ reflected about the line $y = x$.
2. **D** We see that $14^2 + 1 = 197 < 213 = 6^3 - 3$. None of the other points satisfy the inequality.
3. **C** Since the amplitude of $\sqrt{3}\sin(x) + \cos(x)$ is $\sqrt{\left(\sqrt{3}\right)^2 + 1^2} = 2$, we have that $A = 2$. Expand $2\cos(x-\phi) = 2\left[\sin(\phi)\sin(x) + \cos(\phi)\cos(x)\right]$, so we need $\sin(\phi) = \frac{\sqrt{3}}{2}$ and $\cos(\phi) = \frac{1}{2}$. Thus, $\phi = \pi/3$, so $A\phi = 2\pi/3$.
4. **A** Over $[0,\pi]$, this inequality is satisfied on the interval $\left[0,\pi/4\right]$. Thus, the desired probability is $\frac{\pi/4}{\pi} = 1/4$.
5. **D** We have $\{\pi\} + \{-\pi\} = (\pi - 3) + (-\pi - (-4)) = 4 - 3 = 1$.
6. **B** Naturally, for positive *x*, the fractional part can be any number in $[0,1)$. However, for negative *x*, $\lfloor x \rfloor \le x$, so the fractional part becomes negative (it can be any number in $(-1,0]$). Thus, the desired range is $(-1,1)$.
7. **D** We can write this as $2x + (x - \lfloor x \rfloor) = 3x - \lfloor x \rfloor = 2.1$. It is clear that *x* cannot be greater than or equal to 2 (since for positive *x*, $\{x\}$ is non-negative). Thus, $\lfloor x \rfloor = 0$ or $\lfloor x \rfloor = 1$. Solving these separately gives $x = 2.1/3 = 7/10$ and $x = 3.1/3 = 31/30$. The sum is $21/30 + 31/30 = 52/30 = 26/15$.
8. **D** We have $-5a + 2a^2 + 18 = 16$, so $2a^2 - 5a + 2 = 0$. We factor this as $2a^2 - 4a - a + 2 = 2a(a-2) - (a-2) = (2a-1)(a-2) = 0$, so the solutions are $\left\{\frac{1}{2}, 2\right\}$.
9. **B** The period is $2\pi/\omega$ so $P/2\pi = 1/\omega$. The time lag is then $t_0 = 1/\pi \cdot (-2) = -2/\pi$.
10. **C** Note that $\frac{x^3 - y^3}{x - y} = x^2 + xy + y^2 = 14$. We have $(x-y)^2 = x^2 - 2xy + y^2 = -49$. Subtracting the two equations gives $3xy = 63$, so $xy = 21$. Thus, $a + b = 21 + 0 = 21$.
11. **B** We need to find the coefficients on the x^{2012} and x^{2011} terms. We see that $(x+2)^{2012} = x^{2012} + 2012 \cdot 2x^{2011} + \ldots$ and $(x-1)^{2012} = x^{2012} - 2012x^{2011} + \ldots$, so $(x+2)^{2012} + (x-1)^{2012} = 2x^{2012} + 2012x^{2011} + \ldots$ The sum of the roots is $-2012/2 = -1006$.
12. **B** Writing these in rectangular form gives $r\cos(\theta) = 1 \Rightarrow x = 1$ and $r\cos\left(\theta - \pi/4\right) = \frac{1}{\sqrt{2}}\left(r\sin(\theta) + r\cos(\theta)\right) = 1 \Rightarrow x + y = \sqrt{2}$. These are non-parallel lines, so they intersect exactly once.
13. **A** If you didn't already know this, you could just substitute in some numbers.
14. **E** This occurs where $\sin(x) = 0$ or $\cos(x) = 1$. Thus, $x = n\pi$ for all integers *n*, so there are infinitely many solutions.

15. **D** We have that $\frac{\sin(x)}{1-\sin(x)} = 3/2$, so $\sin(x) = 3/2 - 3/2\sin(x) \Rightarrow 5/2\sin(x) = 3/2 \Rightarrow \sin(x) = 3/5$. By the domain of the logarithm, we know that $\cos(x) > 0$ so $\cos(x) = 4/5$. Then, $\log_a\left(\frac{\sin(x)}{\cos^2(x)}\right) = \log_a\left(\frac{3/5}{16/25}\right) = \log_a\left(\frac{15}{16}\right) = 2$, so $a = \sqrt{15/16} = \sqrt{15}/4$.

16. **B** The determinant is $1\begin{vmatrix} 1 & 1 & 0 \\ -5 & -1 & 8 \\ 8 & 2 & -12 \end{vmatrix} + 2\begin{vmatrix} 3 & 1 & 0 \\ -1 & -1 & 8 \\ 3 & 2 & -12 \end{vmatrix} - 4\begin{vmatrix} 3 & 1 & 1 \\ -1 & -5 & -1 \\ 3 & 8 & 2 \end{vmatrix}$ $= 1\cdot 0 + 2\cdot 0 - 4\cdot 0 = 0$. However, this is actually unnecessary – just do question 17, and when you see that row operations eliminate a row of the matrix, you know that the determinant is 0.

17. **D** Since the answer to the previous question is 0, the system is linearly dependent. Hence, w cannot be uniquely determined without values of at least some of the other variables.

18. **A** a is just the minimum value of $-x^2 + x - 1$ which occurs at the vertex of $x = -\frac{1}{2(-1)} = 1/2$. Thus, $a = -1/4 + 1/2 - 1 = -3/4$.

19. **A** Using the sum of cubes factorization, write $\sin^6(x) + \cos^6(x) = \left[\sin^2(x) + \cos^2(x)\right]\left[\sin^4(x) + \cos^4(x) - \sin^2(x)\cos^2(x)\right]$. Note then that $\left[\sin^2(x) + \cos^2(x)\right]^2 = 1 = \sin^4(x) + \cos^4(x) + 2\sin^2(x)\cos^2(x)$. We can then write $\sin^6(x) + \cos^6(x) = 1 - 3\sin^2(x)\cos^2(x) = 1 - 3\left(\frac{\sin(2x)}{2}\right)^2 = 1 - \frac{3a}{4}$.

20. **D** If we denote the 5 roots of $f(2x+1)$ as r_k then the roots of $f(x)$ are $2x_k + 1$. Thus, the sum of the roots of $f(x)$ is $\sum_{k=1}^{5}(2x_k + 1) = 5 + 2\sum_{k=1}^{5} x_k = 5 + 2\cdot 3 = 11$.

21. **A** We can write $\sin(\theta)\tan(\theta) = \frac{\sin^2(\theta)}{\cos(\theta)} = \frac{1-\cos^2(\theta)}{\cos(\theta)}$. Since the numerator is non-negative, we have a positive denominator, so $\cos(\theta) > 0$. Then, for $\frac{1-\cos^2(\theta)}{\cos(\theta)} < 1$ we need $\cos(\theta) > 1 - \cos^2(\theta)$ so $\cos^2(\theta) + \cos(\theta) - 1 > 0$. The roots of $\cos^2(\theta) + \cos(\theta) - 1 = 0$ are $\cos(\theta) = \frac{-1 \pm \sqrt{5}}{2}$, so this is satisfied where $\cos(\theta) < \frac{-1-\sqrt{5}}{2}$ and $\cos(\theta) > \frac{-1+\sqrt{5}}{2}$. However, since $\cos(\theta) > 0$, the valid interval is where $\cos(\theta) > \frac{-1+\sqrt{5}}{2}$. Noting that cosine decreases on this interval, the correct answer is $\left(0, \cos^{-1}\left(\frac{\sqrt{5}-1}{2}\right)\right)$.

22. **D** We need $20 \le 25\cos(\theta) \le 24 \Rightarrow \frac{4}{5} \le \cos(\theta) \le \frac{24}{25}$. This occurs over the interval $\left[\arccos\left(\frac{24}{25}\right), \arccos\left(\frac{4}{5}\right)\right]$, (note that the order is reversed because of arccosine), so $\frac{\tan(b)}{\tan(a)} = \frac{\tan\left(\arccos\left(\frac{4}{5}\right)\right)}{\tan\left(\arccos\left(\frac{24}{25}\right)\right)} = \frac{\frac{3}{4}}{\frac{7}{24}} = \frac{3}{4}\cdot\frac{24}{7} = \frac{18}{7}$.

23. **D** Write this as the sum of two series. For even terms, we have $a_{2n} = \frac{2}{2^n}$ and for odd terms we have $a_{2n+1} = \frac{1}{2^n}$. Then, $\sum_{n=0}^{\infty}\left[\frac{2}{2^n}\cdot\sin^{2n}(\theta)\right] + \sum_{n=0}^{\infty}\left[\frac{1}{2^n}\cdot\sin^{2n+1}(\theta)\right] = \frac{2}{1-0.5\sin^2(\theta)} + \frac{\sin(\theta)}{1-0.5\sin^2(\theta)} = 4$, so $4 - 2\sin^2(\theta) = 2 + \sin(\theta) \Rightarrow 2\sin^2(\theta) + \sin(\theta) - 2 = 0$, so $\sin(\theta) = \frac{-1 \pm \sqrt{17}}{4}$. Taking the value on the desired interval gives $\theta = \sin^{-1}\left(\frac{\sqrt{17}-1}{4}\right)$.

24. **D** Apply AM-GM: $\frac{x + \frac{1}{2x}}{2} \ge \sqrt{x \cdot \frac{1}{2x}} = \frac{\sqrt{2}}{2}$, so $x + \frac{1}{2x} \ge \sqrt{2}$. This minimum can indeed be obtained when $x = \frac{\sqrt{2}}{2}$.

25. **B** Multiplying gives $p_1 = .9(p_1 + p_2 + p_3) = .9$, $p_2 = .1p_1 = 0.09$, and $p_3 = .1p_2 + .1p_3 \Rightarrow p_3 = .01$.

26. **A** The absolute value of anything is non-negative; there cannot be any solutions.

27. **D** Let $a = \sin(x)$. Then, $4a^3 + 2a^2 - 2a - 1 = 2a^2(2a+1) - 1(2a+1) = (2a^2 - 1)(2a+1) = 0$. This gives $\sin(x) = \pm\frac{1}{\sqrt{2}}$ and $\sin(x) = -\frac{1}{2}$. This corresponds to the solutions $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$ and $\frac{7\pi}{6}, \frac{11\pi}{6}$ for a sum of $4\pi + 3\pi = 7\pi$.

28. **A** The maximum value of $\left|\sin(\theta) + \cos(\theta)\right|$ is $\sqrt{2} < \frac{3}{2}$. We could also show this by squaring both sides of the given equation and obtaining $\sin(2\theta) = \frac{5}{4}$ which is obviously impossible.

29. **C** First we find the non-real cube roots of 2: we know that they are in the form $\sqrt[3]{2}\text{cis}(\theta)$, so $2\text{cis}(3\theta) = 2 \Rightarrow 3\theta = 0, 2\pi, 4\pi$, so the non-real cube roots of 2 are $\sqrt[3]{2}\text{cis}\left(\frac{2\pi}{3}\right) = \sqrt[3]{2}\left(-\frac{1}{2} + \frac{i\sqrt{3}}{2}\right)$ and $\sqrt[3]{2}\cdot\text{cis}\left(\frac{4\pi}{3}\right) = \sqrt[3]{2}\left(-\frac{1}{2} - \frac{i\sqrt{3}}{2}\right)$. Thus, $x + 1 = -\frac{\sqrt[3]{2}}{2} \pm \frac{\sqrt[3]{2}\cdot\sqrt{3}}{2}i$, so $x = \left(-1 - \frac{\sqrt[3]{2}}{2}\right) \pm \frac{\sqrt[3]{2}\cdot\sqrt{3}}{2}i$.
Thus, $|a| = 1 + \frac{\sqrt[3]{2}}{2}$.

30. **A** Note that

$$\sin(x)\prod_{k=0}^{2012}\cos\left(2^k x\right) = \sin(x)\cos(x)\cos(2x)\cdots\cos\left(2^{2012}x\right) = \frac{\sin(2x)\cos(2x)\cos(4x)\cdots\cos\left(2^{2012}x\right)}{2}$$

$$=\frac{\sin(4x)\cos(4x)\cos(8x)\cdots\cos\left(2^{2012}x\right)}{2^2}=\frac{\sin(8x)\cos(8x)\cos(16x)\cdots\cos\left(2^{2012}x\right)}{2^3}=\cdots=\frac{\sin\left(2^{2013}x\right)}{2^{2013}}.$$

Thus, $\sin\left(2^{2013}x\right)=\prod_{k=0}^{2012}\cos\left(2^k x\right)=\frac{\sin\left(2^{2013}x\right)}{2^{2013}\sin(x)}\Rightarrow\sin(x)=\frac{1}{2^{2013}}\Rightarrow x=\sin^{-1}\left(\frac{1}{2^{2013}}\right).$