The choice (E) NOTA denotes "None of the Above". All functions are assumed to have domain and range of the real numbers, or the appropriate subset of the real numbers, unless otherwise specified. Inverse trigonometric functions have their traditional restricted ranges. Good luck, and have fun!

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1. If $\cos(\theta)=\frac{1}{3}$, where $2011\pi<\theta<2012\pi$, find the value of $\cos\left(\theta+\frac{\pi}{2}\right)$.

(A) $-\frac{2\sqrt{2}}{3}$ (B) $-\frac{1}{3}$ (C) $\frac{1}{3}$ (D) $\frac{2\sqrt{2}}{3}$ (E) NOTA

2. Find the smallest integer $n>4$ such that the integer $\left(n^2+18n+2012\right)$ is divisible by 35.

(A) 34 (B) 35 (C) 39 (D) 48 (E) NOTA

3. A group of n students is sitting equally spaced around a circular table, and each student is labeled with a number from 1 through n (corresponding to the clockwise order in which they are sitting). If student 7 is sitting directly across from student 81, what is the value of n?

(A) 147 (B) 148 (C) 149 (D) 150 (E) NOTA

4. The three roots of $f(x)=3x^3-10x^2+kx-8$ are a, b, and c. If $a+b=ab$, and $c\in\mathbb{Z}$, find ab.

(A) $\frac{4}{3}$ (B) 2 (C) $\frac{8}{3}$ (D) need more info (E) NOTA

5. Find the sum of the value(s) of θ on the interval $[0,2\pi)$ such that M is not invertible, where

$$M=\begin{bmatrix}\sin\theta & 1 & \sec\theta\\ \cot\theta & \sec\theta & 1\\ 1 & \csc\theta & \tan\theta\end{bmatrix}$$

(A) π (B) 2π (C) 4π (D) no solutions (E) NOTA

6. Right triangle ABC has perimeter $18+12\sqrt{3}$ and area $36\sqrt{3}$. Find $\sin^2(A)+\sin^2(B)+\sin^2(C)$.

(A) 1 (B) 2 (C) 3 (D) need more info (E) NOTA

7. The graph of the solution set over the real numbers for the equation $\sqrt{x}+\sqrt{y}=\sqrt{x+y}$ is:

(A) a circle (B) a point (C) two lines (D) two rays (E) NOTA

8. The integers greater than 1 are arranged in five columns as seen below:

	2	3	4	5
9	8	7	6	
	10	11	12	13
17	16	15	14	
	18	19	20	21
⋮	⋮	⋮	⋮	⋮

In which column (ordered from left to right) will the number 2012 appear?

(A) first (B) second (C) third (D) fourth (E) NOTA

9. Find the smallest positive value of θ such that $\tan(6\theta)=\dfrac{\sin(\theta)-\sin(7\theta)}{\cos(7\theta)-\cos(\theta)}$.

(A) $\dfrac{\pi}{20}$ (B) $\dfrac{\pi}{16}$ (C) $\dfrac{\pi}{12}$ (D) $\dfrac{\pi}{8}$ (E) NOTA

10. A single-elimination tournament is played with 2^n teams for some integer $n>1$. In such a tournament, all undefeated teams are paired together, and only the winning teams advance to the next round. This process is repeated until only one undefeated team remains. Which of the following expressions correctly represents the total number of games played in such a tournament?

(A) $\displaystyle\sum_{k=0}^{n}\binom{n}{k}$ (B) $\displaystyle\sum_{k=1}^{n-1}2^k$ (C) $\displaystyle\sum_{k=1}^{n}2^{k-1}$ (D) $\displaystyle\sum_{k=1}^{n}2^k$ (E) NOTA

11. The six roots of $f(x)=x^6-729$ are graphed in the complex plane. The roots are labeled $x_1, x_2, ..., x_6$ in counter-clockwise order starting with $x_1=3$. What is the value of $|x_3-x_6|$?

(A) $\frac{3\sqrt{3}}{2}$ (B) 3 (C) $3\sqrt{3}$ (D) 6 (E) NOTA

12. Find the minimum value of $f(x)=\frac{\sin^4(x)+\cos^4(x)}{\sin^2(2x)}$. *Hint:* $\left(\sin^2(x)+\cos^2(x)\right)^2=1$.

(A) $\frac{1}{4}$ (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) 1 (E) NOTA

13. A ladder of length 12 feet is placed at a $40°$ angle with the ground such that it just reaches the top of a building that forms a right angle with the ground. A longer ladder is placed directly behind the first ladder at a $20°$ angle with the ground so that it also exactly reaches the top of the building at the same point. Assuming that the ground is flat, how many feet on the ground behind the shorter ladder was the longer ladder placed? If necessary, use that $\cos(20°)=.94$, $\cos(40°)=.77$, $\sin(20°)=.34$, and $\sin(40°)=.64$. Round your answer to the nearest tenth.

(A) 7.7 (B) 11.3 (C) 11.9 (D) 12.0 (E) NOTA

14. Solve the equation $\sin(\theta)+\cos(\theta)=\frac{5}{4}$, where $0<\theta<\frac{\pi}{2}$. What is the value of (2θ)?

(A) $\sin^{-1}\left(\frac{1}{4}\right)$ (B) $\sin^{-1}\left(\frac{9}{32}\right)$ (C) $\sin^{-1}\left(\frac{9}{16}\right)$ (D) $\sin^{-1}\left(\frac{5}{8}\right)$ (E) NOTA

15. Evaluate $\lim_{m\to\infty}\left(\sum_{n=1}^{m}\frac{1}{n^2+n}\right)$.

(A) 0.75 (B) 0.8 (C) 0.9 (D) 0.95 (E) NOTA

16. Let $\Re_n(x) \in \{1,2,3,...,n\}$ be the element such that $x \equiv (\Re_n(x)) (\mathrm{mod}\, n)$; that is, $(x - \Re_n(x))$ is divisible by n. Find the number of positive integers x less than 1000 such that $\Re_{50}(x) = \Re_{100}(x)$.

(A) 450 (B) 500 (C) 950 (D) 999 (E) NOTA

17. Consider the system of equations:

$$\begin{cases} \log_{16}(a^2+b^2) + \log_4(ab) = 2012 \\ \log_4(a-b) + \log_{16}(a^2b^2) = 2012 \end{cases}$$

When a and b are non-negative real numbers where $a > b$, the system of equations is:

(A) consistent (B) dependent (C) inconsistent (D) independent (E) NOTA

18. Find the value of S such that $S = \sum_{n=1}^{S} \frac{n}{12}$.

(A) 5 (B) 11 (C) 12 (D) 23 (E) NOTA

19. The domain of $f(3x-6)$ is $[0,6]$. Find the domain of $g(x) = 2f(x)$.

(A) $[0,2]$ (B) $[0,4]$ (C) $[0,12]$ (D) $[2,4]$ (E) NOTA

20. Let z be a complex number such that $|z| = 1$ and $|z+1| = \sqrt{3}$. When graphed in the complex plane, z is in the first quadrant; that is, $\mathrm{Re}(z) > 0$ and $\mathrm{Im}(z) > 0$. Find the value of $|z+i|$.

(A) $\sqrt{3}$ (B) $\sqrt{2+\sqrt{3}}$ (C) 2 (D) $2+\sqrt[4]{12}$ (E) NOTA

Use the following information for questions 21 and 22:

The perimeter of a regular $n-$gon inscribed in a circle with radius r is given as $2nr\sin\left(\frac{\pi}{n}\right)$.

21. Find the perimeter of a regular octagon inscribed in a circle of radius 2.

(A) $8\sqrt{2-\sqrt{2}}$ (B) $16\sqrt{2-\sqrt{2}}$ (C) $16\sqrt{2}$ (D) $16\sqrt{2+\sqrt{2}}$ (E) NOTA

22. Evaluate $\lim_{n\to\infty}\left(n\sin\left(\frac{\pi}{n}\right)\right)$.

(A) $-\infty$ (B) 0 (C) π (D) ∞ (E) NOTA

23. Find the amplitude of the graph of $f(\theta)=\sin\left(\theta+\frac{\pi}{3}\right)+\cos\left(\theta+\frac{\pi}{6}\right)$.

(A) 1 (B) $\sqrt{2}$ (C) $\sqrt{3}$ (D) 2 (E) NOTA

24. Find the sum of all values of $\theta\in(0,2\pi)$ such that either $\sin\theta=\frac{\sqrt{7}-\sqrt{3}}{4}$ or $\cos\theta=\frac{\sqrt{7}-\sqrt{3}}{4}$.

(A) π (B) 2π (C) 3π (D) 4π (E) NOTA

25. The shortest distance between the line $y=x+10$ and the circle $x^2+2x+y^2=2$ is closest to:

(A) 3 (B) 5 (C) 6 (D) 11 (E) NOTA

26. Eli has 9 blue socks and 12 black socks. He randomly chooses two socks without replacement. If the odds, in simplified form, of Eli's two socks being the same color is $a:b$, find $b-a$.

(A) -1 (B) 1 (C) 18 (D) 24 (E) NOTA

27. Let $f(x)$ be a cubic polynomial with integral coefficients. If $f(1)=-1,\ f(2)=f(3)=-5,$ $f(4)=5,$ and $f(5)=31,$ find the value of $f(6)$.

(A) 48 (B) 67 (C) 79 (D) 81 (E) NOTA

28. Let $f(x)=1+\dfrac{1}{x}$ and denote the nth iteration of f as $f^{[n]}(x)=\underbrace{f\Big(f\Big(f\cdots\big(f(x)\cdots\big)\Big)\Big)}_{n \text{ times}}$.

Evaluate $\lim\limits_{n\to\infty}\left(f^{[n]}(n)\right)$.

(A) 1 (B) $\dfrac{1+\sqrt{5}}{2}$ (C) 2 (D) e (E) NOTA

29. Solve the equation $2\cos^2(\theta)+\cos(2\theta)=\sqrt{3}$. If $0<\theta<\dfrac{\pi}{2}$, what is the value of $\sin(2\theta)$?

(A) $\dfrac{\sqrt{3}-1}{2}$ (B) $\dfrac{3-\sqrt{3}}{4}$ (C) $\dfrac{\sqrt[4]{3}}{2}$ (D) $\dfrac{\sqrt[4]{12}}{2}$ (E) NOTA

30. If $\det\left(\sum\limits_{n=1}^{\infty}\begin{bmatrix}0 & x\\ 1 & 0\end{bmatrix}^n\right)=-3,$ find the value of x.

(A) −0.75 (B) 0.25 (C) 0.75 (D) 1.50 (E) NOTA