ANSWERS: DEBAC BDDAC DCDEE BCDDB BCCCB BCBDC

1. On this interval, which is equivalent to $\pi < \theta < 2\pi$, $\sin\theta < 0$. So, $\sin\theta = -\sqrt{1-\cos^2\theta} = -\frac{2\sqrt{2}}{3}$. Thus, $\cos\left(\theta + \frac{\pi}{2}\right) = -\sin\theta = \frac{2\sqrt{2}}{3}$. **D**

2. Rewrite the quadratic as $\left(n^2+18n+17\right)+1995$. Since 35 divides 1995, we need only that 35 divide $(n+17)(n+1)$. The most simple solutions to find occur when one of the terms is divisible by 35. Letting $(n+17)=35$ gives $n=18$, which is smaller than all of the answer choices. In fact, the smallest such value is $n=13$. **E**

3. There are 73 students between 7 and 81; therefore, there must also be 73 students between 81 and 7. Six of these are students 1 through 6, while the remaining 67 are students 82 through 148. **B**

4. Using the formulas for the sum and product of the roots, we have $\frac{10}{3} - c = \frac{8}{3c}$, which gives $c = \left\{\frac{4}{3}, 2\right\}$. Since $c \in \mathbb{Z}$, $c = 2$, and $ab = \frac{8}{3(2)} = \frac{4}{3}$. **A**

5. We know that M will not be invertible if $\det(M) = 0$. Taking the determinant through various methods gives $\det(M) = \tan^2(\theta)+1+\csc^2(\theta)-\sec^2(\theta)-1-1 = \csc^2(\theta)-2 = 0$. So, $\frac{1}{\sin^2(\theta)} = 2$, which occurs when $\sin(\theta) = \pm\frac{\sqrt{2}}{2}$. The desired sum is $\frac{\pi}{4}+\frac{3\pi}{4}+\frac{5\pi}{4}+\frac{7\pi}{4} = 4\pi$. **C**

6. One of the angles is a right angle, and the other two are complementary. We can choose A and B as the acute angles to obtain $\sin^2(A)+\cos^2(A)+\sin^2(90°) = 1+1 = 2$. **B**

7. Square both sides to obtain $x+y+2\sqrt{xy} = x+y$, so $xy = 0$. This occurs when either $x=0$ or $y=0$. However, the domain for both variables is strictly non-negative, so the solution set is two rays (starting point at the origin for the positive x and y axes). **D**

8. The multiples of 4 will clearly occur in the second and fourth columns; a bit of investigation shows that the product of 4 and an odd number, such as $2012 = 4\cdot 503$, occur in the fourth. **D**

9. Write each argument on the right-hand-side as $(4\theta \pm 3\theta)$ and expand each term using the proper addition/subtraction formula and simplify terms to obtain $\frac{\cos(4\theta)}{\sin(4\theta)}$. Cross multiplication gives $\sin(6\theta)\sin(4\theta) = \cos(6\theta)\cos(4\theta)$, or $\cos(6\theta + 4\theta) = \cos(10\theta) = 0$. The smallest positive solution will occur when $10\theta = \frac{\pi}{2}$, so $\theta = \frac{\pi}{20}$. **A**

10. There are several ways to approach this; one is to consider that in each game, one team will lose, and there must be a total of $2^n - 1$ losses (since one team will win, and every other team will be eliminated by losing 1 game). This is well known to be $1 + 2 + 4 + \ldots + 2^{n-1} = \sum_{k=1}^{n} 2^{k-1}$. **C**

11. In the complex plane, these points form a regular hexagon with side length 3. The desired value is the distance from x_3 to x_6, which is a diagonal of the hexagon with length 6. **D**

12. Use the hint to obtain $\sin^4(x) + 2\sin^2(x)\cos^2(x) + \cos^4(x) = \sin^4(x) + \frac{\sin^2(2x)}{2} + \cos^4(x) = 1$, so we can rewrite the fraction as $\frac{1 - \frac{\sin^2(2x)}{2}}{\sin^2(2x)} = \frac{1}{\sin^2(2x)} - \frac{1}{2}$. Since $\sin^2(2x) \in [0,1]$, $\frac{1}{\sin^2(2x)} \in [1, \infty)$. Thus, the minimum value is $1 - \frac{1}{2} = \frac{1}{2}$. **C**

13. Drawing a diagram shows that the length of the shorter ladder and the distance between the bases of the ladders must be equal, as they are the congruent sides of an isosceles triangle with vertex angle 140 degrees, so the answer is 12.0. **D**

14. Square both sides to obtain $1 + \sin(2\theta) = \frac{25}{16}$. So, $\sin(2\theta) = \frac{9}{16}$. However, for the solution in $\left(0, \pi/4\right)$ which gives $2\theta = \sin^{-1}\left(\frac{9}{16}\right)$, there is also second solution in $\left(\pi/4, \pi/2\right)$ which will give $2\theta = \pi - \sin^{-1}\left(\frac{9}{16}\right)$. Thus, the value of 2θ is not uniquely determined. **E**

15. The sum is $\sum_{n=1}^{\infty} \frac{1}{n^2 + n} = \sum_{n=1}^{\infty}\left(\frac{1}{n} - \frac{1}{n+1}\right) = \left(\frac{1}{1} - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) + \ldots = 1$. **E**

16. Since 100 is a multiple of 50, integers have the same remainder with respect to 50. However, when $\Re_{100}(x) > 50$, it will not be equal to $\Re_{50}(x)$, but instead it will be $\Re_{50}(x)+50$. All solutions for x are found in the sets $[1,50]\cup[101,150]\cup[201,250]\ldots$ for a total of $50(10)=500$. **B**

17. To find solutions, we set $\log_{16}(a^2+b^2)+\log_4(ab)=\log_{16}(a^2b^2)+\log_4(a-b)$. Note that $\log_{16}(a^2b^2)=2\left(\frac{1}{2}\right)\log_4(ab)=\log_4(ab)$, so the equation simplifies to $\log_{16}(a^2+b^2)=\log_4(a-b)$. This is equivalent to $\frac{1}{2}\log_4(a^2+b^2)=\log_4(a-b)$, so $a^2+b^2=(a-b)^2=a^2-2ab+b^2$, so $ab=0$. However, $ab\neq 0$ because of the domain restriction on the logarithm function. Thus, there are no solutions to the system, so it is called inconsistent. **C**

18. We have $S=\sum_{n=1}^{S}\frac{n}{12}=\frac{\frac{S(S+1)}{2}}{12}$, so $S=\frac{S(S+1)}{24}$, and $S+1=24$, giving $S=23$. **D**

19. The domain of $2f(x)$ is the domain of $f(x)$, and since the domain of $f(3x-6)$ is $[0,6]$, the domain of $f(x)$ is $[2,4]$. **D**

20. Let $z=a+bi$. Then, $a^2+b^2=1$ and $(a+1)^2+b^2=3$, so subtracting the two equations gives $a=\frac{1}{2}$, so $b=\frac{\sqrt{3}}{2}$. Thus, $|z+i|=\sqrt{\left(\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}+1\right)^2}=\sqrt{2+\sqrt{3}}$. **B**

21. This value is $2(8)(2)\sin\left(\frac{\pi}{8}\right)=32\sqrt{\frac{1-\sin\left(\frac{\pi}{4}\right)}{2}}=32\sqrt{\frac{2-\sqrt{2}}{4}}=16\sqrt{2-\sqrt{2}}$. **B**

22. Note that as n becomes large, the shape approaches the shape of the circle. We know that the perimeter of a circle is $2\pi r$, so $\lim_{n\to\infty}\left(2nr\sin\left(\frac{\pi}{n}\right)\right)=2\pi r$. Thus, $\lim_{n\to\infty}\left(n\sin\left(\frac{\pi}{n}\right)\right)=\pi$. **C**

23. Expanding gives $f(\theta)=\sin(\theta)\cos\left(\frac{\pi}{3}\right)+\sin\left(\frac{\pi}{3}\right)\cos(\theta)+\cos(\theta)\cos\left(\frac{\pi}{6}\right)-\sin(\theta)\sin\left(\frac{\pi}{6}\right)$, so $f(\theta)=\sqrt{3}\cos(\theta)$, which has amplitude $\sqrt{3}$. **C**

24. Let the first quadrant solutions to the two equations be α and β, respectively. Then, by the properties of sine and cosine, the other solutions are $\pi-\alpha$ and $2\pi-\beta$, for a sum of 3π. **C**

25. The distance from the center of the circle $(x+1)^2+y^2=3$ to the line $x-y+10=0$ is $\frac{|(-1)(1)+(0)(-1)+10|}{\sqrt{(1)^2+(-1)^2}}=\frac{9\sqrt{2}}{2}\approx 6.4$. Subtracting the radius of $\sqrt{3}\approx 1.7$ gives 4.7. **B**

26. The probability is $\left(\frac{9}{21}\right)\left(\frac{8}{20}\right)+\left(\frac{12}{21}\right)\left(\frac{11}{20}\right)=\frac{17}{35}$. Thus, the odds are $17:18$, and $b-a=1$. **B**

27. Since this is a cubic function (degree 3), taking differences of consecutive terms 3 times will result in equal differences. Alternatively, a system of equations can be used to solve for the coefficients of the polynomial, but that is more time consuming.

-1	-5	-5	5	31	$f(6)$
-4	0	10	26	$f(6)-31$	
4	10	16	$f(6)-57$		
6	6	$f(6)-73$			

Thus, $f(6)-73=6$, so $f(6)=79$. **C**

28. We are looking for the value of $1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{\vdots}}}$. Let $x=1+\frac{1}{x}$ to obtain $x=\frac{1\pm\sqrt{5}}{2}$, and since x is clearly positive, the answer is $\frac{1+\sqrt{5}}{2}$. Note that the fact that the input value for f was $n\to\infty$ will not affect the value of the limit. **B**

29. There are a few ways that this can be approached. One is to write $\cos(2\theta)=2\cos^2(\theta)-1$, so $4\cos^2(\theta)=\sqrt{3}+1$. Then, $\cos(2\theta)=\sqrt{3}-\frac{\sqrt{3}+1}{2}=\frac{\sqrt{3}-1}{2}$. Thus, $\sin(2\theta)=\sqrt{1-\cos^2(2\theta)}$

$$=\sqrt{1-\left(\frac{\sqrt{3}-1}{2}\right)^2}=\sqrt{1-\left(\frac{2-\sqrt{3}}{2}\right)}=\sqrt{\frac{\sqrt{3}}{2}}=\frac{\sqrt[4]{3}}{\sqrt{2}}=\frac{\sqrt[4]{12}}{2}.$$ **D**

30. By various methods (most easily found by writing out a few terms, despite not being very rigorous), we find that $\sum_{n=1}^{\infty}\begin{bmatrix}0 & x\\ 1 & 0\end{bmatrix}^n=$

$$\begin{bmatrix}0 & x\\ 1 & 0\end{bmatrix}+\begin{bmatrix}x & 0\\ 0 & x\end{bmatrix}+\begin{bmatrix}0 & x^2\\ x & 0\end{bmatrix}+\begin{bmatrix}x^2 & 0\\ 0 & x^2\end{bmatrix}+\begin{bmatrix}0 & x^3\\ x^2 & 0\end{bmatrix}+\begin{bmatrix}x^3 & 0\\ 0 & x^3\end{bmatrix}+\ldots=\begin{bmatrix}x+x^2+\ldots & x+x^2+\ldots\\ 1+x+\ldots & x+x^2+\ldots\end{bmatrix}.$$

Using sum formulas for infinite geometric series (assuming $|x|<1$) this matrix becomes

$\begin{bmatrix} \frac{x}{1-x} & \frac{x}{1-x} \\ \frac{1}{1-x} & \frac{x}{1-x} \end{bmatrix}$. Hence, $\det\begin{bmatrix} \frac{x}{1-x} & \frac{x}{1-x} \\ \frac{1}{1-x} & \frac{x}{1-x} \end{bmatrix} = \frac{x^2-x}{(1-x)^2} = \frac{x}{x-1} = -3$, so $-3x+3=x$, and $x=.75$. **C**