2012 MAΘ National Convention — Alpha Bowl: Question 1

If $f(x)$ is an even function, $g(x)$ is an odd function, and $h(x)=\frac{f(x)}{g(x)}$, find $A+B+C+D$.

	$f(x)$	$g(x)$	$h(x)$
$x=-2$	3	D	???
$x=-1$	4	1	A
$x=1$	???	C	???
$x=2$	B	???	2

2012 MAΘ National Convention — Alpha Bowl: Question 1

If $f(x)$ is an even function, $g(x)$ is an odd function, and $h(x)=\frac{f(x)}{g(x)}$, find $A+B+C+D$.

	$f(x)$	$g(x)$	$h(x)$
$x=-2$	3	D	???
$x=-1$	4	1	A
$x=1$	???	C	???
$x=2$	B	???	2

2012 MAΘ National Convention — Alpha Bowl: Question 2

Consider the vector $\mathbf{v} = \langle -2, 3, k \rangle$ and find the value of $\dfrac{AB}{CD}$.

$A = \|\mathbf{v}\|$, if $k = 6$

$B = k$, if $\mathbf{v} \cdot \langle 1,2,1 \rangle = 9$

$C = k$, if $\mathbf{v}$ is orthogonal to $\langle 6, -k^2, 12 \rangle$

$D = (j + k)$, if $\mathbf{v}$ is parallel to $\langle 1, j, -7 \rangle$

2012 MAΘ National Convention — Alpha Bowl: Question 2

Consider the vector $\mathbf{v} = \langle -2, 3, k \rangle$ and find the value of $\dfrac{AB}{CD}$.

$A = \|\mathbf{v}\|$, if $k = 6$

$B = k$, if $\mathbf{v} \cdot \langle 1,2,1 \rangle = 9$

$C = k$, if $\mathbf{v}$ is orthogonal to $\langle 6, -k^2, 12 \rangle$

$D = (j + k)$, if $\mathbf{v}$ is parallel to $\langle 1, j, -7 \rangle$

2012 MAΘ National Convention **Alpha Bowl: Question 3**

Consider the following information and give the value of $\det\begin{bmatrix} A & B \\ C & D \end{bmatrix}$.

$A =$ the number of distinct arrangements of the letters 'ALPHABOWL'

$B =$ the number of distinct groupings of eight students into four pairs of two students

$C =$ the solution to the equation $C_7 = 70_9$ where the subscripts indicate bases*

$D =$ the probability of obtaining a sum of less than 5 but greater than 2 when two fair six-sided dice are rolled

*Note that C_7 is a base 7 number, but you must use the base-10 digits of C in the final calculation. For example, if you obtain a solution of 24_7, then $C = 24$.

2012 MAΘ National Convention **Alpha Bowl: Question 3**

Consider the following information and give the value of $\det\begin{bmatrix} A & B \\ C & D \end{bmatrix}$.

$A =$ the number of distinct arrangements of the letters 'ALPHABOWL'

$B =$ the number of distinct groupings of eight students into four pairs of two students

$C =$ the solution to the equation $C_7 = 70_9$ where the subscripts indicate bases*

$D =$ the probability of obtaining a sum of less than 5 but greater than 2 when two fair six-sided dice are rolled

*Note that C_7 is a base 7 number, but you must use the base-10 digits of C in the final calculation. For example, if you obtain a solution of 24_7, then $C = 24$.

2012 MAΘ National Convention — Alpha Bowl: Question 4

Find the sum of the numbers next to the identities that are always true when defined.

(1) $\cos(2\theta) = 2\sin^2(\theta) - 1$

(3) $\dfrac{1+\cos(\theta)}{1-\cos(\theta)} = 2\cot^2(\theta) + 1$

(5) $2\sin^2(\theta) = 1 - \sin(2\theta)$

(7) $\sin^4(\theta) + \cos^2(\theta) = \cos^4(\theta) + \sin^2(\theta)$

(9) $\sin\left(\theta - \dfrac{\pi}{2}\right) = \cos(\pi - \theta)$

(11) $\sin^2(\alpha) + \cos^2(\beta) = 1$

2012 MAΘ National Convention — Alpha Bowl: Question 4

Find the sum of the numbers next to the identities that are always true when defined.

(1) $\cos(2\theta) = 2\sin^2(\theta) - 1$

(3) $\dfrac{1+\cos(\theta)}{1-\cos(\theta)} = 2\cot^2(\theta) + 1$

(5) $2\sin^2(\theta) = 1 - \sin(2\theta)$

(7) $\sin^4(\theta) + \cos^2(\theta) = \cos^4(\theta) + \sin^2(\theta)$

(9) $\sin\left(\theta - \dfrac{\pi}{2}\right) = \cos(\pi - \theta)$

(11) $\sin^2(\alpha) + \cos^2(\beta) = 1$

2012 MAΘ National Convention — Alpha Bowl: Question 5

Consider the following information and give the value of $A+B$.

$A =$ the sum of all numbers of the form 2^p such that p is an integer and $2^p < 2012$

$B =$ the sum of all numbers of the form $2^j 3^k$ such that j and k are positive integers less than 5

2012 MAΘ National Convention — Alpha Bowl: Question 5

Consider the following information and give the value of $A+B$.

$A =$ the sum of all numbers of the form 2^p such that p is an integer and $2^p < 2012$

$B =$ the sum of all numbers of the form $2^j 3^k$ such that j and k are positive integers less than 5

2012 MAΘ National Convention — Alpha Bowl: Question 6

Find the value of $(x+y)$ given that $\begin{bmatrix} A & B \\ C & D \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} 230 \\ 43 \end{bmatrix}$, where

$A =$ the solution to $4^a - 4^{a-1} = 24$

$B = {}_6P_2$

$C =$ the solution to $\log_3(b+1) = \log_3(b-1) + 2$

$D =$ the number of real solutions to $x^2 = 2^x$

2012 MAΘ National Convention — Alpha Bowl: Question 6

Find the value of $(x+y)$ given that $\begin{bmatrix} A & B \\ C & D \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} 230 \\ 43 \end{bmatrix}$, where

$A =$ the solution to $4^a - 4^{a-1} = 24$

$B = {}_6P_2$

$C =$ the solution to $\log_3(b+1) = \log_3(b-1) + 2$

$D =$ the number of real solutions to $x^2 = 2^x$

2012 MAΘ National Convention — Alpha Bowl: Question 7

Consider the following information and compute $ABCD$.

Let r_1, r_2, and r_3 be the roots of the function $f(x) = 2x^3 - 4x^2 + 4x - 1$.

$A = r_1 + r_2 + r_3$ $\quad$ $B = r_1 r_2 r_3$ $\quad$ $C = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}$ $\quad$ $D = \frac{r_2 + r_3}{r_1} + \frac{r_3 + r_1}{r_2} + \frac{r_1 + r_2}{r_3}$

2012 MAΘ National Convention — Alpha Bowl: Question 7

Consider the following information and compute $ABCD$.

Let r_1, r_2, and r_3 be the roots of the function $f(x) = 2x^3 - 4x^2 + 4x - 1$.

$A = r_1 + r_2 + r_3$ $\quad$ $B = r_1 r_2 r_3$ $\quad$ $C = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}$ $\quad$ $D = \frac{r_2 + r_3}{r_1} + \frac{r_3 + r_1}{r_2} + \frac{r_1 + r_2}{r_3}$

2012 MAΘ National Convention — Alpha Bowl: Question 8

Consider the following information and find the exact value of $A+B-C+D$.

$$A=\sin\left(\frac{-\pi}{24}\right) \qquad B=\sin\left(\frac{15\pi}{24}\right) \qquad C=\cos\left(\frac{3\pi}{24}\right) \qquad D=\cos\left(\frac{11\pi}{24}\right)$$

2012 MAΘ National Convention — Alpha Bowl: Question 8

Consider the following information and find the exact value of $A+B-C+D$.

$$A=\sin\left(\frac{-\pi}{24}\right) \qquad B=\sin\left(\frac{15\pi}{24}\right) \qquad C=\cos\left(\frac{3\pi}{24}\right) \qquad D=\cos\left(\frac{11\pi}{24}\right)$$

2012 MAΘ National Convention — Alpha Bowl: Question 9

Consider the following information regarding polar equations and find $\frac{A}{\pi}+\frac{B}{1006}$.

$A=$ the area bounded by graph of $r^2 = r\sin\theta + 4$

$B=\sum_{n=1}^{2012} P(n)$, where $P(n)$ is the number of petals in the graph of $r=2012\cos(n\theta)$

2012 MAΘ National Convention — Alpha Bowl: Question 9

Consider the following information regarding polar equations and find $\frac{A}{\pi}+\frac{B}{1006}$.

$A=$ the area bounded by graph of $r^2 = r\sin\theta + 4$

$B=\sum_{n=1}^{2012} P(n)$, where $P(n)$ is the number of petals in the graph of $r=2012\cos(n\theta)$

2012 MAΘ National Convention — Alpha Bowl: Question 10

Consider the following parametric equations when graphed in the $xy-$plane and find ABC.

$A=$ the area bounded by the graph of $x(t)=4\sin(\theta)$ and $y(t)=9\cos(\theta)$

$B=$ the number of letters in the name of the conic defined by $x(t)=t$ and $y(t)=t^2-2t+4$

$C=$ the value of r where $(x(r),y(r))=(x(s),y(s))$ and $r>s,$ given that $x(t)=t^2-t$ and $y(t)=t^3-4t$

2012 MAΘ National Convention — Alpha Bowl: Question 10

Consider the following parametric equations when graphed in the $xy-$plane and find ABC.

$A=$ the area bounded by the graph of $x(t)=4\sin(\theta)$ and $y(t)=9\cos(\theta)$

$B=$ the number of letters in the name of the conic defined by $x(t)=t$ and $y(t)=t^2-2t+4$

$C=$ the value of r where $(x(r),y(r))=(x(s),y(s))$ and $r>s,$ given that $x(t)=t^2-t$ and $y(t)=t^3-4t$

2012 MAΘ National Convention — Alpha Bowl: Question 11

Consider the following information and find the value of $A-B$.

$A=$ the sum of all values of $\theta\in(0,2\pi)$ such that $2\cos^2(3\theta)=\cos(3\theta)$

$B=$ the sum of all values of $\theta\in(0,2\pi)$ such that $\cos(4\theta)+3\sin(2\theta)=2$

2012 MAΘ National Convention — Alpha Bowl: Question 11

Consider the following information and find the value of $A-B$.

$A=$ the sum of all values of $\theta\in(0,2\pi)$ such that $2\cos^2(3\theta)=\cos(3\theta)$

$B=$ the sum of all values of $\theta\in(0,2\pi)$ such that $\cos(4\theta)+3\sin(2\theta)=2$

2012 MAΘ National Convention **Alpha Bowl: Question 12**

Consider the following information and find the value of $\left(\tan\left(A+B\right)+\tan\left(C-D\right)\right)$.

$A =$ the acute angle formed when the line $x-2y=8$ intersects the $x-$axis

$B =$ the obtuse angle formed when $3x+4y=10$ intersects the $x-$axis

$C =$ the acute angle formed when $3x+4y=10$ intersects the $x-$axis

$D =$ the obtuse angle formed when the line $x-2y=8$ intersects the $x-$axis

2012 MAΘ National Convention **Alpha Bowl: Question 12**

Consider the following information and find the value of $\left(\tan\left(A+B\right)+\tan\left(C-D\right)\right)$.

$A =$ the acute angle formed when the line $x-2y=8$ intersects the $x-$axis

$B =$ the obtuse angle formed when $3x+4y=10$ intersects the $x-$axis

$C =$ the acute angle formed when $3x+4y=10$ intersects the $x-$axis

$D =$ the obtuse angle formed when the line $x-2y=8$ intersects the $x-$axis

2012 MAΘ National Convention — Alpha Bowl: Question 13

The mathematical constant $e \approx 2.72$ can be defined in several ways. Two of them are as follows:

$$e = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n \qquad e = \frac{1}{0!}+\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+\ldots$$

Use these definitions to evaluate the following expressions, and give the value of AB.

$$A = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{4n} \qquad B = \frac{0}{1!}+\frac{1}{2!}+\frac{2}{3!}+\frac{3}{4!}+\ldots$$

2012 MAΘ National Convention — Alpha Bowl: Question 13

The mathematical constant $e \approx 2.72$ can be defined in several ways. Two of them are as follows:

$$e = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n \qquad e = \frac{1}{0!}+\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+\ldots$$

Use these definitions to evaluate the following expressions, and give the value of AB.

$$A = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{4n} \qquad B = \frac{0}{1!}+\frac{1}{2!}+\frac{2}{3!}+\frac{3}{4!}+\ldots$$

2012 MAΘ National Convention   Alpha Bowl: Question 14

Define three sequences as follows: A is an arithmetic sequence with nth term A_n, G is a geometric sequence with nth term G_n, and X is a sequence such that its nth term is $X_n = A_n + G_n$. If $A_1 = 0$, $X_1 = X_2 = 1$, and $X_3 = 2$, find the difference between the maximum and minimum values of $\sum_{n=1}^{10} X_n$.

2012 MAΘ National Convention   Alpha Bowl: Question 14

Define three sequences as follows: A is an arithmetic sequence with nth term A_n, G is a geometric sequence with nth term G_n, and X is a sequence such that its nth term is $X_n = A_n + G_n$. If $A_1 = 0$, $X_1 = X_2 = 1$, and $X_3 = 2$, find the difference between the maximum and minimum values of $\sum_{n=1}^{10} X_n$.