ANSWERS:

1. $\frac{9}{2}$ or 4.5	**2.** $\frac{7}{5}$ or 1.4	**3.** 0	**4.** 16
5. 5648	**6.** 26	**7.** 20	**8.** 0
9. $\frac{12097}{4}$ or 3024.25	**10.** $24\pi + 24\pi\sqrt{13}$	**11.** $\frac{15}{2}\pi$ or 7.5π	**12.** $\frac{20}{11}$
13. e^4	**14.** 978		

1. **9/2 or 4.5**

$A = h(-1) = \frac{f(-1)}{g(-1)} = \frac{4}{1} = 4.$ $\quad B = f(2) = f(-2) = 3.$ $\quad C = g(1) = -g(-1) = -1.$

$D = g(-2) = \frac{f(-2)}{h(-2)} = \frac{f(-2)}{-h(2)} = \frac{3}{-2}.$ $\quad A + B + C + D = 4 + 3 - 1 - \frac{3}{2} = \frac{9}{2}$ or 4.5.

2. **7/5 or 1.4**

$A = \sqrt{2^2 + 3^2 + 6^2} = \sqrt{49} = 7.$ $\quad \langle -2,3,k\rangle \bullet \langle 1,2,1\rangle = -2 + 6 + k = 4 + k = 9,$ so $B = k = 5.$

$\langle -2,3,k\rangle \bullet \langle 6,-k^2,12\rangle = -12 - 3k^2 + 12k = 0,$ so $3(k-2)^2 = 0,$ and $C = k = 2.$

$\frac{-2}{1} = \frac{3}{j} = \frac{k}{-7},$ so $D = (j+k) = -\frac{3}{2} + 14 = \frac{25}{2}.$ $\quad \frac{AB}{CD} = \frac{(7)(5)}{2(25/2)} = \frac{(7)(5)}{25} = \frac{7}{5}$ or 1.4.

3. **0**

$A = \frac{9!}{2!2!}$ $\quad B = \frac{\binom{8}{2}\binom{6}{2}\binom{4}{2}\binom{2}{2}}{4!}$, dividing by 4! because the numerator counts the pairs as having order – that is, it distinguishes between if 2 people were the first pair or the third pair, etc. when this order is in fact not considered distinguishable to the final arrangement of pairs.

$70_9 = 7(9^1) = 63 = 49 + 14 + 0 = 120_7,$ so $C = 5!$ $\quad D = \frac{2}{36} + \frac{3}{36} = \frac{5}{36}$ (sum of 3 + sum of 4)

Thus, $\det\begin{bmatrix} A & B \\ C & D \end{bmatrix} = AD - BC = \frac{9!}{2!2!}\cdot\frac{5}{36} - \frac{8!}{2!2!2!2!4!}\cdot 5! = 0.$

4. **16**

(1) This is false; it should be $2\cos^2(\theta)$.

(3) This is false; take $\theta = \pi/4$.

(5) This is false; take $\theta = 0$.

(7) This is true. If $|\sin\theta| = |\cos\theta|$, it is trivial. Otherwise, rearrange and divide by $\sin^2\theta - \cos^2\theta$ to obtain $\sin^2\theta + \cos^2\theta = 1$.

(9) This is true. Both are equivalent to $-\cos\theta$.

(11) This is false; it should be $\cos^2(\alpha)$.

The sum is $7 + 9 = 16$.

5. **5648**

$A = 2^{10} + 2^9 + \cdots + 2^0 + 2^{-1} + \cdots = \dfrac{2^{10}}{1 - 1/2} = 2048.$

$B = (2+4+8+16)(3+9+27+81) = (30)(120) = 3600.$ $A + B = 5648.$

6. **26**

$A = 5/2$, since $4^{a-1}(4-1) = 24 \Rightarrow 4^{a-1} = 8 \Rightarrow a - 1 = 3/2$.

$B = \dfrac{6!}{(6-2)!} = \dfrac{6!}{4!} = 30.$ $C = 5/4$, since $\log_3\left(\dfrac{b+1}{b-1}\right) = 2 \Rightarrow \dfrac{b+1}{b-1} = 9 \Rightarrow b + 1 = 9b - 9 \Rightarrow 8b = 10.$

$D = 3$ (draw a rough graph to see this)

So, $\dfrac{5}{2}x + 30y = 230$ and $\dfrac{5}{4}x + 3y = 43$, giving $(x, y) = (20, 6)$. Thus, $x + y = 26$.

7. **20**

$A = \dfrac{-(-4)}{2} = 2.$ $B = \dfrac{-(-1)}{2} = \dfrac{1}{2}.$ $C = \dfrac{-(4)}{-1} = 4.$

$D = \dfrac{2 - r_1}{r_1} + \dfrac{2 - r_2}{r_2} + \dfrac{2 - r_3}{r_3} = 2(C) - 3 = 8 - 3 = 5.$ Thus, $ABCD = 20$.

8. **0**

There are quite a few ways to evaluate this. One of the more simple ways is as follows:

$A=-\sin\left(\frac{\pi}{24}\right)=-\cos\left(\frac{\pi}{2}-\frac{\pi}{24}\right)=-\cos\left(\frac{11\pi}{24}\right)=-D\Rightarrow A+D=0.$

$B=\sin\left(\frac{15\pi}{24}\right)=\cos\left(\frac{\pi}{2}-\frac{15\pi}{24}\right)=\cos\left(-\frac{3\pi}{24}\right)=\cos\left(\frac{3\pi}{24}\right)=C\Rightarrow B-C=0.$ The answer is thus 0.

9. **12097/4 or 3024.25**

$A=\frac{17\pi}{4},$ since this is the graph of $x^2+y^2=y+4\Rightarrow x^2+\left(y-\frac{1}{2}\right)^2=\frac{17}{4}.$

$B=2(2+4+\ldots+2012)+(1+3+\ldots+2011)=2\cdot1006\cdot1007+1006^2.$

The answer is $\frac{17}{4}+2\cdot1007+1006=\frac{12097}{4}$ or 3024.25.

10. **24π(1+sqrt(13)) or equivalent**

$A=6\pi,$ since it forms the ellipse $\frac{x^2}{4}+\frac{y^2}{9}=1.$ $B=8,$ since it forms the parabola $y=x^2-2x+4.$

C. We have that $s^2-s=r^2-r\Rightarrow s^2-r^2=s-r\Rightarrow s+r=1$ and
$s^3-4s=r^3-4r\Rightarrow(s-r)(s^2+sr+r^2)=4(s-r)\Rightarrow s^2+sr+r^2=4.$ Substituting in gives
$(1-r)^2+(1-r)r+r^2=r^2-r+1=4\Rightarrow r=\frac{1+\sqrt{13}}{2}.$ The product is $24\pi\left(1+\sqrt{13}\right).$

11. **15π/2 or 7.5π**

A. $\cos(3\theta)=0\Rightarrow3\theta=\frac{\pi}{2},\frac{3\pi}{2},\frac{5\pi}{2},\frac{7\pi}{2},\frac{9\pi}{2},\frac{11\pi}{2},$ so $\theta=\frac{\pi}{6},\frac{3\pi}{6},\frac{5\pi}{6},\frac{7\pi}{6},\frac{9\pi}{6},\frac{11\pi}{6};$

OR $\cos(3\theta)=\frac{1}{2}\Rightarrow3\theta=\frac{\pi}{3},\frac{5\pi}{3},\frac{7\pi}{3},\frac{11\pi}{3},\frac{13\pi}{3},\frac{17\pi}{3},$ so $\theta=\frac{\pi}{9},\frac{5\pi}{9},\frac{7\pi}{9},\frac{11\pi}{9},\frac{13\pi}{9},\frac{17\pi}{9}.$

B. $\cos^2(2\theta)-\sin^2(2\theta)+3\sin(2\theta)=1-2\sin^2(2\theta)+3\sin(2\theta)=2\Rightarrow\sin(2\theta)=\frac{1}{2},1.$

So, $2\theta=\frac{\pi}{2},\frac{5\pi}{2}\Rightarrow\theta=\frac{\pi}{4},\frac{5\pi}{4}.$ OR $2\theta=\frac{\pi}{6},\frac{5\pi}{6},\frac{13\pi}{6},\frac{17\pi}{6}\Rightarrow\theta=\frac{\pi}{12},\frac{5\pi}{12},\frac{13\pi}{12},\frac{17\pi}{12}.$

Thus, $\mathrm{A}-\mathrm{B}=\left[3(2\pi)+3(2\pi)\right]-\left[\frac{3\pi}{2}+2\left(\frac{3\pi}{2}\right)\right]=12\pi-\frac{9}{2}\pi=\frac{15}{2}\pi=7.5\pi.$

12. **20/11**

$\frac{\tan A+\tan B}{1-\tan A\tan B}=\frac{\frac{1}{2}-\frac{3}{4}}{1-\left(\frac{1}{2}\right)\left(-\frac{3}{4}\right)}=-\frac{2}{11}.$ $\frac{\tan C-\tan D}{1+\tan C\tan D}=\frac{\frac{3}{4}+\frac{1}{2}}{1+\left(\frac{3}{4}\right)\left(-\frac{1}{2}\right)}=2.$ The sum is $\frac{20}{11}.$

13. **e^4**

$A = \left[\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n\right]^4 = e^4.$

$B = \frac{1-1}{1!}+\frac{2-1}{2!}+\frac{3-1}{3!}+\ldots = \left(\frac{1}{1!}+\frac{2}{2!}+\frac{3}{3!}+\ldots\right)-\left(\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+\ldots\right)=\left(\frac{1}{0!}+\frac{1}{1!}+\frac{1}{2!}+\ldots\right)-\left(\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+\ldots\right)=1.$

The answer is $AB = e^4$.

14. **978**

Since $X_1 = 1 = a + 0$, $a = 1$, so the geometric series is $1, r, r^2, \ldots$ Since $A_1 = 0$, the arithmetic series can be written as $0, x, 2x, 3x, \ldots$ We then have $X_2 = 1 = r + x \rightarrow x = 1 - r$, so $X_3 = 2 = r^2 + 2(1-r)$ which gives $r = 2$, so $x = -1$. The desired sum is $\left(1 + 2 + \ldots + 2^9\right) + \left(0 - 1 - 2 - \ldots - 9\right)$ for a total of $\left(2^{10} - 1\right) - \frac{9 \cdot 10}{2} = 1023 - 45 = 978.$