A unit circle is cut into 2012 pieces (all equal in arc length) by 2012 distinct points. Three of them are chosen at random. The probability that these three points form a right triangle is \( \frac{m}{n} \) where \( m \) and \( n \) are relatively prime integers. What is \( m + n \)?

- **(A) 673491**
- **(B) 673686**
- **(C) 2014**
- **(D) 2013**
- **(E) NOTA**

Consider the equation of circle \( \omega \) given by: \((x + 1)^2 + (y + 1)^2 = 1\). Let \( u = ab \) where the point \((a, b)\) lies on the circle. Find the sum of the largest and smallest value of \( u \).

- **(A) \(3 + 2\sqrt{2}\)**
- **(B) \(\frac{3}{2} + \sqrt{2}\)**
- **(C) \(\sqrt{2}\)**
- **(D) \(\frac{3}{2}\)**
- **(E) NOTA**

There are two distinct lines tangent to circle \( \omega \) (see question (3)) which pass through \((5, 3)\). The larger of the slopes of these two lines can be expressed in the form \( \frac{x + \sqrt{y}}{z} \) where \( x, y, z \in \mathbb{Z} \) and where \( y \) isn’t divisible by the square of any prime. Find the sum of \( x, y, \) and \( z \).

- **(A) 120**
- **(B) 110**
- **(C) 100**
- **(D) 90**
- **(E) NOTA**

What is the area of the triangle formed by connecting the points \((0, 0, 0), (1, 1, 2), \) and \((3, 4, 2)\)?

- **(A) \(\sqrt{61}\)**
- **(B) \(\sqrt{53}\)**
- **(C) \(\sqrt{61}/2\)**
- **(D) \(\sqrt{53}/2\)**
- **(E) NOTA**

The vectors \(\langle 1, 1 \rangle\) and \(\langle 7, k \rangle\) will be orthogonal for which value(s) of \( k \)?

- **(A) 7**
- **(B) −7**
- **(C) 7 and −7**
- **(D) −1**
- **(E) NOTA**

What is the maximum number of regions 2011 lines divide a plane into (you may assume that all the lines lie in the given plane)?

- **(A) \(\frac{2011^2 + 2009}{2}\)**
- **(B) \(2011^2 + 2011\)**
- **(C) \(\frac{2011^2 + 2011}{2}\)**
- **(D) \(\frac{2011^2 + 2013}{2}\)**
- **(E) NOTA**

Consider two integers, \(a\) and \(b\), which are both randomly chosen from the interval \((0, 2012)\). What is the probability that the sum of the slopes of the asymptotes of the hyperbola shown below is zero?

\[
\frac{(x - b)^2}{a^2} - \frac{y^2}{4b^2} = 1
\]

- **(A) \(\frac{1}{2011}\)**
- **(B) \(\frac{1}{2012}\)**
- **(C) \(\frac{1}{\binom{2011}{2}}\)**
- **(D) \(\frac{1}{\binom{2010}{2}}\)**
- **(E) NOTA**

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The three points \((1, 1), (k, 3),\) and \((2, k)\) will all lie on a circle except for \(t\) (an integer) values of \(k\). Find the sum of the possible values of \(k\) and the value of \(t\).

\[(A) \ 4 + \sqrt{2} \quad (B) \ 4 \quad (C) \ 2 + \sqrt{2} \quad (D) \ 1 + \sqrt{2} \quad (E) \ \text{NOTA}\]

For Questions (9), and (10) let \(\vec{i} = (1,0,0), \vec{j} = (0,1,0),\) and \(\vec{k} = (0,0,1)\).

Let \(\vec{a} = i \times (j \times k)\). What is \(\|\vec{a}\|\) ?

\[(A) \ 0 \quad (B) \ 1 \quad (C) \ \sqrt{3} \quad (D) \ 3 \quad (E) \ \text{NOTA}\]

Let \(\vec{v} = i \times (j \times (i \times (j \times k \times (i \times (j \times (j \times k)))))\)). Find \(\|\vec{v}\|\).

\[(A) \ 0 \quad (B) \ 1 \quad (C) \ \sqrt{3} \quad (D) \ 3 \quad (E) \ \text{NOTA}\]

Consider the two sets: \(A = \{(x, y) \mid x^2 + y^2 = 4\}\), and \(B = \{(x, y) \mid (x - 3)^2 + (y - 4)^2 = r^2\}\). If the intersection of sets \(A\) and \(B\) only has one element, what is the sum of the possible values of \(r\) (assuming they are positive)?

\[(A) \ 3 \quad (B) \ 4 \quad (C) \ 7 \quad (D) \ 10 \quad (E) \ \text{NOTA}\]

By definition, if \(\vec{a}\) and \(\vec{b}\) are vectors, and \(\theta\) is the angle between them, then \(\|\vec{a} \times \vec{b}\|\) is equivalent to . . .

\[(A) \ \sin \theta \quad (B) \ \cos \theta \quad (C) \ \|\vec{a}\|\|\vec{b}\| \cos \theta \quad (D) \ \|\vec{a}\|\|\vec{b}\| \sin \theta \quad (E) \ \text{NOTA}\]

Let \(\theta\) be the angle, in degrees, between the two vectors \(\langle 1, 1, 1 \rangle\) and \(\langle 2, 4, 3 \rangle\). In which of the following intervals does \(\theta\) lie? \(\text{NOTE: all intervals are in degrees}\).

\[(A) \ (0, 20) \quad (B) \ (20, 40) \quad (C) \ (40, 60) \quad (D) \ (60, 80) \quad (E) \ \text{NOTA}\]

Consider the plane which passes through the points \((1, 2, 1), (0, 3, 1),\) and \((2, 5, 7)\) What is the shortest distance between this plane and the point \((-1, -2, 2)\)?

\[(A) \ \frac{5\sqrt{11}}{11} \quad (B) \ \frac{10\sqrt{11}}{11} \quad (C) \ \frac{5\sqrt{22}}{11} \quad (D) \ \frac{10\sqrt{22}}{11} \quad (E) \ \text{NOTA}\]

Vectors, \(\vec{a}\) and \(\vec{b}\) are said to be orthogonal if . . . (in the following answer choices, \(\theta\) is the smaller angle between the two vectors).

\[(A) \ \sin \theta = 0 \quad (B) \ \cos \theta = 0 \quad (C) \ \tan \theta = 0 \quad (D) \ \tan \theta\ \text{is undefined} \quad (E) \ \text{NOTA}\]
Find the area of the polar graph: \( r = \frac{5}{3 + 2\cos \theta} \).

(A) \( 3\pi \sqrt{5} \) \hspace{1cm} (B) \( 3\pi \sqrt{6} \) \hspace{1cm} (C) \( 3\pi \sqrt{7} \) \hspace{1cm} (D) \( 6\pi \sqrt{2} \) \hspace{1cm} (E) NOTA

What is the eccentricity of the graph of: \( 2012! \cdot y^2 = \frac{1}{2011!} (x - 2010!) \)?

(A) 0 \hspace{1cm} (B) 1 \hspace{1cm} (C) \( 2012! \cdot 2011! \) \hspace{1cm} (D) \( \frac{1}{2011! \cdot 2012!} \) \hspace{1cm} (E) NOTA

In a certain ellipse, one of its foci lies at \((2012, 2012)\). If the point \((0, 0)\) is on the graph, and the ellipse has major axis length \(2012^2 \sqrt{2}\), then which of the following might be the coordinate of the other focus?

(A) \((2012, -2012)\) \hspace{1cm} (B) \((-2012, 2012)\) \hspace{1cm} (C) \((-2011^2 \cdot 2012^2, 0)\)

(D) \((2011^2 \cdot 2012^2 \cdot 2, 0)\) \hspace{1cm} (E) NOTA

What is the slope of the line tangent to the circle \((x - 2)^2 + (y - 3)^2 = 2\) at the point with \(x\)-coordinate 2 and \(y\)-value greater than 3?

(A) \(3 + \sqrt{2}\) \hspace{1cm} (B) \((3 + \sqrt{2})/2\) \hspace{1cm} (C) 0 \hspace{1cm} (D) \(3 - \sqrt{2}\) \hspace{1cm} (E) NOTA

The polar point \((2, \frac{\pi}{3})\) in rectangular coordinates is . . .

(A) \((1, \sqrt{3})\) \hspace{1cm} (B) \((\sqrt{3}, 1)\) \hspace{1cm} (C) \((2, 2\sqrt{3})\) \hspace{1cm} (D) \((2\sqrt{3}, 2)\) \hspace{1cm} (E) NOTA

How many distinguishable permutations of the words \textit{analytic} exist?

(A) 40,320 \hspace{1cm} (B) 20,160 \hspace{1cm} (C) 10,080 \hspace{1cm} (D) 5040 \hspace{1cm} (E) NOTA

The points \((1, 1), (5, 3),\) and \((a, b)\) form an equilateral triangle. Find the minimum value of \(ab\).

(A) \(-4 - 3\sqrt{3}\) \hspace{1cm} (B) \(-4\sqrt{3}\) \hspace{1cm} (C) \(3\sqrt{3} - 4\) \hspace{1cm} (D) 1 \hspace{1cm} (E) NOTA

What is the angle between the space diagonal of a cube and one of its face diagonals?

(A) \(\sqrt{3}/2\) \hspace{1cm} (B) \(\sqrt{6}/2\) \hspace{1cm} (C) \(\arccos \left( \frac{\sqrt{6}}{3} \right)\) \hspace{1cm} (D) \(\arccos \left( \frac{\sqrt{3}}{3} \right)\) \hspace{1cm} (E) NOTA
24 Let $R$ be the circumradius and $r$ be the in-radius of a generic triangle $ABC$. The law of sines says that:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} = \frac{1}{X}$$

Which of the following is $X$?

(A) $R$  
(B) $2R$  
(C) $r$  
(D) $2r$  
(E) NOTA

25 Consider the graph $f$ defined parametrically by: $x = \sin(2t)$ and $y = \cos t$. What kind of graph is this?

(A) Parabola  
(B) Circle  
(C) Ellipse  
(D) Hyperbola  
(E) NOTA

26 The area of one of the “petals” on the graph of the rose $r = 12 \cos(6\theta)$ is $6\pi$. What is the area of the entire graph?

(A) $12\pi$  
(B) $24\pi$  
(C) $36\pi$  
(D) $72\pi$  
(E) NOTA

27 The circle which passes through the points $(1,1)$, $(0,2)$, and $(4,0)$ can be expressed in the form $x^2 + y^2 + Ax + By + C = 0$. Compute $ABC$.

(A) $-1280$  
(B) $-128$  
(C) $128$  
(D) $1280$  
(E) NOTA

28 The solutions to $x^3 - 1 = 0$, when graphed in the complex plane, form an equilateral triangle. Find this triangle’s area.

(A) $3\sqrt{3}/4$  
(B) $\sqrt{3}/4$  
(C) $3/4$  
(D) $\sqrt{3}$  
(E) NOTA

29 The graph of $\pm(x^2 + y^2)^{3/2} = 2xy$ is that of a . . .

(A) rose  
(B) lemniscate  
(C) cardioid  
(D) spiral  
(E) NOTA

30 Suppose the centroid of triangle $EFG$, where the vertices have coordinates $E(1,1)$, $F(2,3)$, and $G(4,7)$, is $C(m,n)$. Find the sum $m + n$.

(A) 9  
(B) $17/2$  
(C) $17/3$  
(D) 6  
(E) NOTA