**MAO National Convention 2012 – Precalc Hustle Solutions**

1. The amplitude is 5, the period is , and the phase shift is also so their sum is .
2. Computing the determinant by expanding minors from the first column (or by noting that the matrix is triangular, and thus the determinant is the product of the entries in the main diagonal) yields the short expression:.
3. The domain of is [-1,1], the range of the domain of which is , the range of the domain of which is. The union of these three sets is: .
4. He needs to choose two guys from four and two girls from four: .
5. For a number to be the cube root of ***i***, it must satisfy the equation: Note that the sum of ***a, b, c*** corresponds to the sum of the roots of this polynomial; using Vieta’s Formula for the sum of the roots of a polynomial,
6. Inverse trig functions return angles, so what we have is a sine angle addition: .
7. The line will periodically intersect with sin(x) until the upper bound of the sine function is exceeded; so, after , the functions cannot intersect. Given the graph of the sine function and linear functions, during each period of the sine function and while , then line will intersect the sine function twice (once when sine is decreasing and once when sine is increasing); the only exception to this is during the period because an extra intersection occurs at the origin. Thus, these two graphs will intersect times, yielding **35** solutions.
8. To see the period algebraically we want to use some technique that will remove the exponent from the trig expression. Using the half angle formula produces a clean trig expression without exponents: the period of f(x) is .
9. .
10. The area of triangle ABC can be calculated using the sine formula for area: But a triangle’s area is also equal to .
11. Multiplying both sides by *r* and converting to rectangular coordinates, we get the equation: – which is the equation for a **circle**.
12. There are two quick ways to evaluate the expression. One way is to use the half angle formulas for sine and cosine to get the numerical value. The second is as follows: Let . Since both sine and cosine are positive in the first quadrant, the negative solution is extraneous and therefore the value is **.**
13. While is always greater than 0, is negative on the interval . Furthermore, knowledge of exponential and logarithmic growth tell us that for positive values of x, is greater than and will grow faster than ; so the graphs do not intersect over the reals and therefore no solutions exist. **0/no solutions**.
14. The technique used to solve this problem is the exact same technique used to derive the formula for a series. Let , then . Subtracting from both sides of the first equation yields:

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1. Using substitution: (since x is equal to a square root, it cannot be negative).
2. Given the limitations on the angle and the value of we can determine the value of by solving a right triangle that satisfies the value of and then using the measurements of this right triangle to solve for ; because the trig functions are ratios, we are guaranteed that the value we get will be the only possible value of since the triangle we solve for will be similar to any triangle that satisfies the same set of requirements. Here we solve the right triangle with hypotenuse length 1 and one leg of length to get (given the restrictions on angle theta, *x* cannot equal 1 so we don’t need to restrict the domain of *x* in our answer).
3. Let , then **.**
4. We have two equations: . Using substitution yields .
5. First use De Moivre’s Theorem and then compute the magnitude of the complex number: .
6. Since we have a 2x2 matrix that maps vector v to a 2x1 vector, we know that vector v must be a 2x1 vector; thus we can solve the for vector v by multiplying out the matrix and solving the resulting system of 2 equations. Alternatively, we could multiply both sides by the inverse of the 2x2 matrix as follows: (note that the positioning of the inverse matrix in the multiplication on the right hand side is important and mandatory). Solving this matrix equation for v yields the vector **<5, 2>**.
7. The constant term will occur when the power of the first term is twice that of the second term; thus the constant term is: .
8. We can create a system of equations and solve for A and B by multiplying both sides by the common denominator and then plugging in two distinct values for *x*; this will leave A and B as the two unknown variables in a system of two equations. Alternatively, we can use the heavy-side method: – using the same process or simply substituting the value of A back into the equation yields B = -3, so *A+B =* **-1**.
9. If Michelle runs at radians/sec, then she travels half the circle’s circumference per second; so, her linear speed is .
10. After the first drop of 80 feet, the distance is equal to twice the value of an infinite geometric series with first term 48 and common ratio 3/5 (twice because the ball bounces up and down, resulting in double distance). Thus the total vertical distance is: .