

Applicant to the Sacred Society and Honorable Order of Νυ Αλφα Θητα,

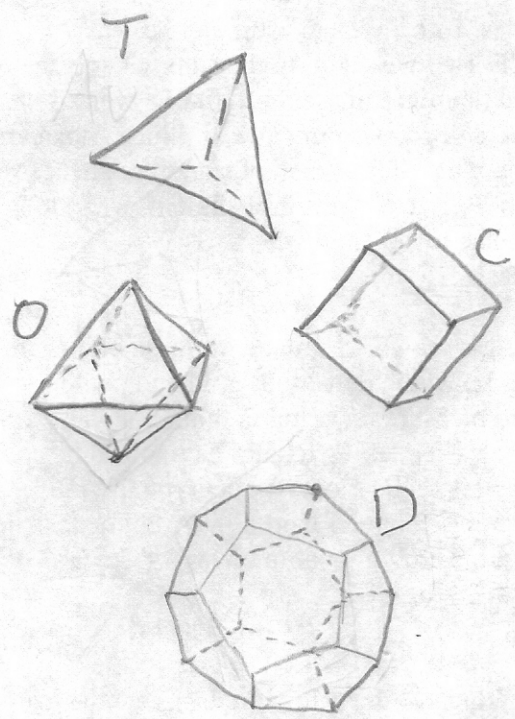
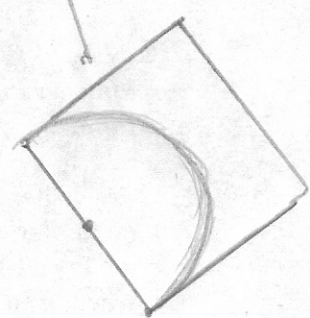
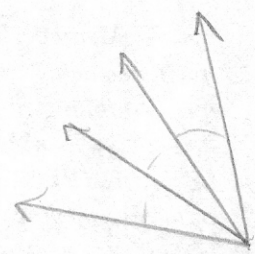
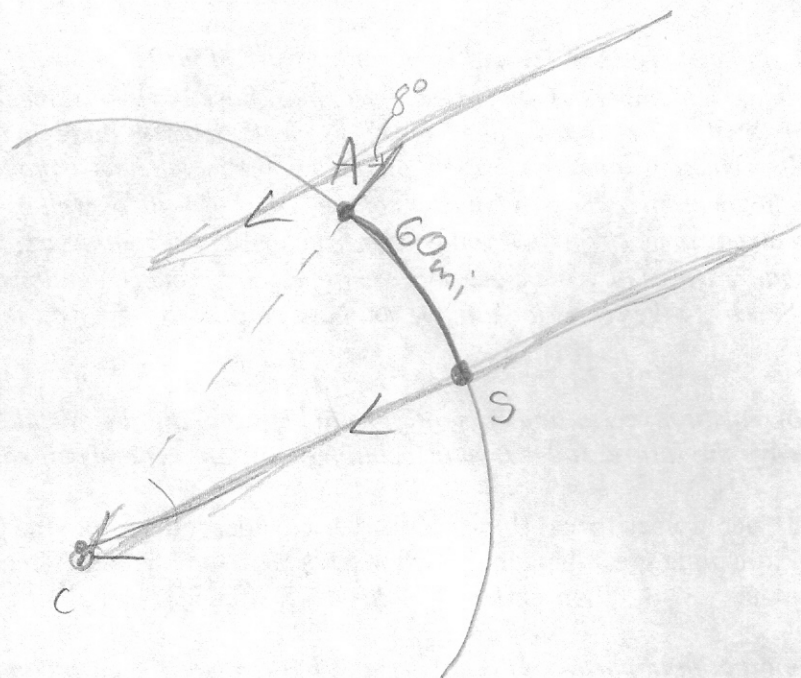
Membership in the Sacred Society and Honorable Order of ναθ is reserved for only the most deserving applicants. Members of the Order of ναθ must have the utmost mental acuity, quick-thinking ability, and well-rounded knowledge of disparate fields or study. In order to prove your worthiness, you must pass the test laid out below. When you have completed your task, you will know the location of the headquarters of the Order of ναθ, as well as a sequence of letters serving as an entrance key. When you gain entrance to the headquarters, the elders will reveal the secrets and treasures of ναθ before your very eyes. But proceed cautiously: the bold do not often blaze the way to the secret location. If you have made it this far, you should not need luck. But: good luck.

First, you must know history: recent and ancient, for only by studying the mistakes of the past can we look to improve the future. Indeed, your location at present depends on your own past.

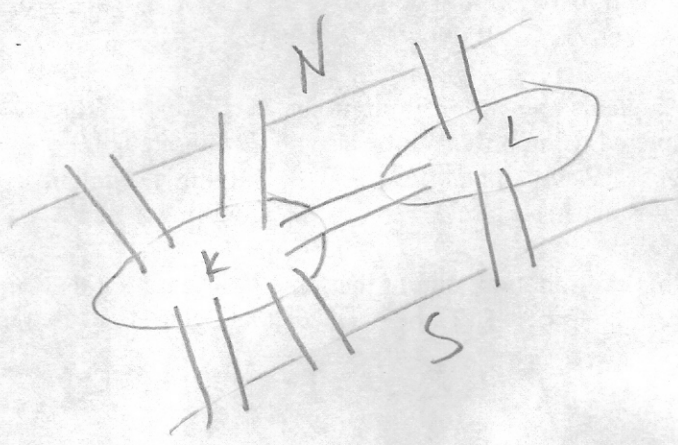
1. The name of our Sacred Society and Honorable Order includes letters from the Greek alphabet. Of the following great thinkers, which one was born and lived in Greece?
 A. Brahmagupta B. Fibonacci C. Cicero D. Plato E. None of these

Here begins the computational portion. It is not certain, but contemporary ancient thinkers Archimedes and Eratosthenes are believed to have been friends and members of ναθ. Solve the following famous problems unraveled by these great minds:

2. Eratosthenes used properties of parallel lines, angles, and circles to estimate the circumference of the earth in the third century BCE. He knew that on the summer solstice in the town of Syene, the sun shone directly overhead (sunlight hit the bottom of a well), whereas in Alexandria, the sun was measured to be 8 degrees from vertical. Since Alexandria is 60 miles due north of Syene, and the sun's rays are parallel to each other, Eratosthenes was able to approximate the circumference of the Earth. Relive his accomplishment: what do those measurements predict the Earth's circumference to be?
 A. 480 mi B. 2600 mi C. 2700 mi D. 10320 mi E. NOTA
3. Archimedes once was asked to determine if the king's crown was made of pure gold, or if the gold had been diluted with another metal. Since he knew the density of gold and could measure the mass of the crown, he needed a way to measure its volume, though it was unfortunately an irregular shape. The story goes that when taking a bath, he realized he could measure volume by water displacement. Excited by his realization, he apocryphally ran naked through the town shouting "EUREKA" (meaning "I have found it"). Archimedes knew that the crown weighed $\frac{1}{2}$ stone, and measured that the crown's volume was $\frac{3}{8}$ sextarii. Was the crown made of pure gold?
 A. Yes B. No E. NOTA
4. The ancients knew their limitations, as must you. Which of the following tasks can be completed using only a compass and a straightedge?
 A. Trisecting an angle B. Halving a segment
 C. Squaring a circle D. Doubling a cube E. NOTA
5. On this examination, one of the famed Platonic Solids is mentioned. Which one is it?
 A. Tetrahedron B. Cube C. Octohedron D. Dodecahedron E. NOTA



$$\frac{\text{Au Dens.}}{19.3 \frac{\text{g}}{\text{cm}^3}} \cdot \frac{\text{LS stone}}{6350.7 \text{ mg}} \cdot \frac{546 \text{ cm}^3}{150 \text{ grains}} = 1.66 \text{ stone/golden}$$



The art and science of mathematics is the most essential of all studies. Answer the following with your knowledge of mathematics.

6. Which of the following numbers is greatest?

A. $\sqrt[3]{2}$

B. $\sqrt[5]{3}$

C. $\sqrt[10]{10}$

D. $\sqrt[15]{31}$

E. NOTA

7. Consider the polynomial $f(x) = Ax^5 + Bx^4 + Cx^3 + Dx^2 + Ex + F$. If the product of the roots of $f(x)$ is $9/28$, then what is the product of the roots of $g(x) = Fx^5 + Ex^4 + Dx^3 + Cx^2 + Bx + A$?

A. $9/28$

B. $28/9$

C. 9

D. 28

E. NOTA

8. Given: $A = \frac{2^{41} - 2^{40}}{2}$ and $B = 2^{39}$, which is greater, A or B ?

A. $A > B$

B. $B > A$

C. $A = B$

D. Cannot be determined

E. NOTA

9. There are two sequences, a_n and b_n , where a_n is an arithmetic sequence and b_n is geometric. If $a_1 = b_1$, $a_2 = b_2$, $a_4 = b_3$, and $b_4 = 20$, then what is a_1 ?

A. 1

B. 2

C. 2.5

D. 4

E. NOTA

10. How many ways can three of the integers from 2 to 7, inclusive, be chosen such that no pair of the numbers have a common divisor besides one?

A. Four

B. Six

C. Eight

D. Ten

E. NOTA

11. Two fair eight-sided dice are rolled, and the results are added. What is the probability that the sum is a prime number?

A. $23/64$

B. $1/2$

C. $5/12$

D. $1/3$

E. NOTA

12. Five distinct points A, B, C, D , and E lie on a circle. How many concave polygons can be formed by choosing vertices from points A, B, C, D , and E ?

A. 0

B. 4

C. 8

D. 16

E. NOTA

13. Five distinct points A, B, C, D , and E lie on a circle. How many convex polygons can be formed by choosing vertices from points A, B, C, D , and E ?

A. 0

B. 4

C. 8

D. 16

E. NOTA

14. A glass full of water is placed on a scale and weighs $1/5$ stone. Half the water is poured out, and the half-full glass then weighs $1/7$ stone. How much does the empty glass weigh?

A. $5/49$ stone

B. $1/9$ stone

C. $3/35$ stone

D. $2/35$ stone

E. NOTA

15. For which positive integer values of n is the equation $a^n + b^n = c^n$ true for at least one ordered triplet (a, b, c) where $abc \neq 0$?

A. None

B. $\{1, 2\}$

C. Infinite

D. Cannot be determined

E. NOTA

16. What comes next in the sequence?



10, 6, 3, 2

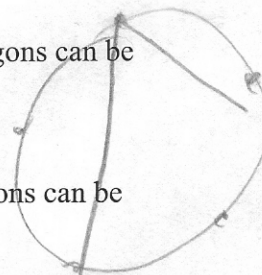


FLIP = FLIP

$a_1 + 3d = b_1 \cdot r^2$

2, 3, 2^2, 5, 2^3, 7

1 → 16



I have discovered a truly marvelous proof of #14, which this margin is too

vaθ expects its members to travel far and wide, in body and on the written page.

17. A river runs through the city of Königsberg, with two islands in the middle. There are seven bridges between the landmasses: two from the north bank to Kneiphof Island, two from the south bank to Kneiphof Island, one from Kneiphof Island to Lomse Island, one from Lomse Island to the north bank, and one from Lomse Island to the south bank. In order to walk each bridge exactly once, where should a pedestrian start?

(The islands cannot be reached by any route other than the bridges, and every bridge must be crossed completely every time; one cannot walk halfway onto the bridge and then turn around and later cross the other half from the other side.)

- A. North Bank B. South Bank
C. Kneiphof Island D. Lomse Island E. NOTA

Need map

18. If one bridge between Kneiphof and the north bank, and one bridge between Kneiphof and the south bank are removed, then where should a pedestrian start in order to walk each bridge exactly once?

- A. north or south bank
B. either of the islands
C. only Kneiphof
D. it is possible starting from anywhere
E. NOTA

19. A rectangular manuscript with dimensions 8.5in by 11in and corners labeled W , X , Y , and Z , where WX and YZ are the 11in sides. Let M be the midpoint of XZ . Follow these steps to fold the paper: WY is folded over onto the YZ , bisecting angle Y . Then Y is folded down onto point Z . Last, X is folded onto M . How many sides does the resulting figure have?

- A. Three B. Four C. Five D. Six E. NOTA

20. Find the area of the shape formed by the folding described in the previous question.

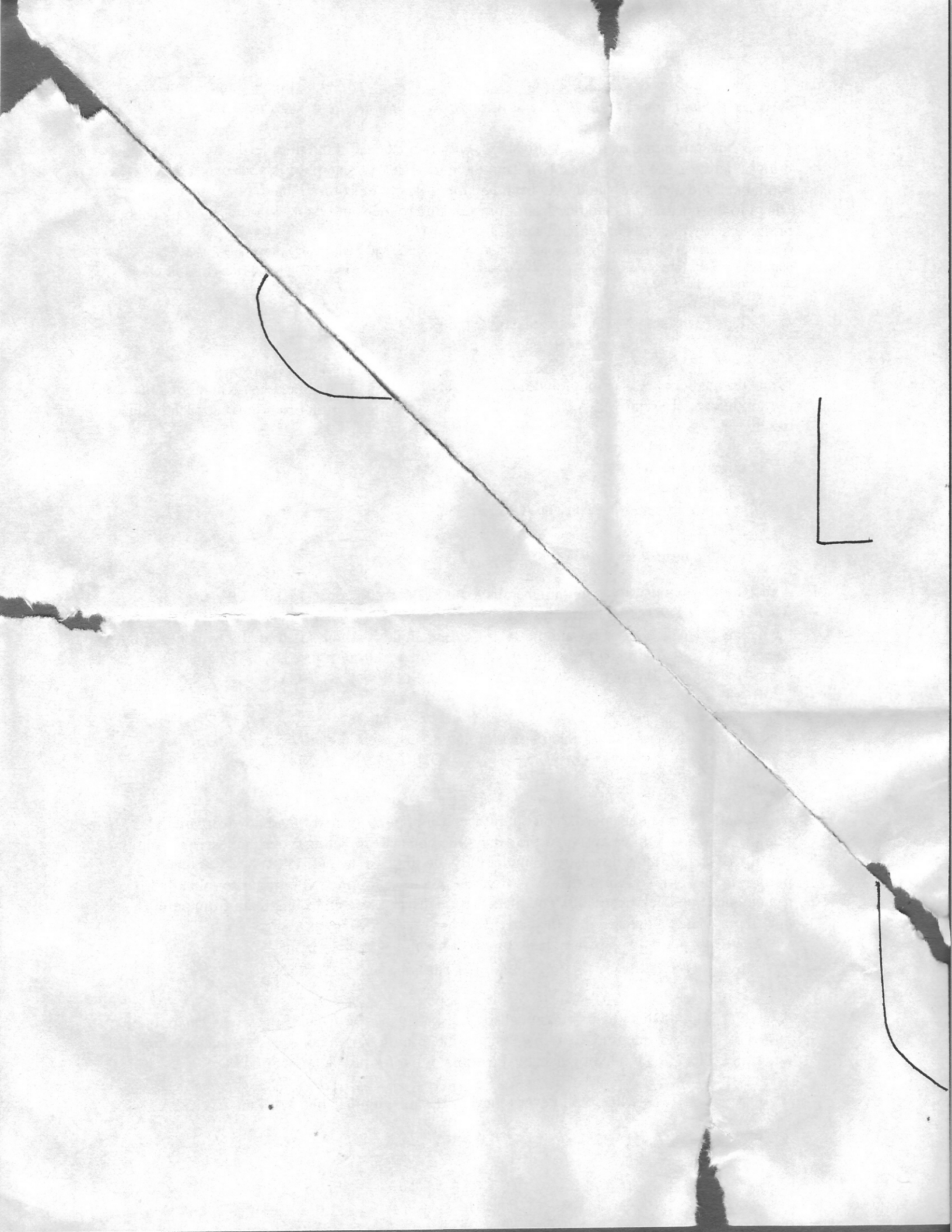
- A. $187/4$ B. $169/4$ C. 5 D. $9/2$ E. NOTA

21. Your boat travels at one speed. You put into a river at point A, and travel downstream to point B, which is 8 miles away. At the same time you start traveling, your manuscript drops into the river and floats downstream under only the force of the current. When you reach point B at 2:07pm, you immediately notice that you have dropped your manuscript and turn around, heading back upstream. You meet the manuscript two miles upstream from point B at 2:52pm. What is the speed of the current?

- A. 2 miles per hour B. 4 miles per hour
C. 6 miles per hour D. $20/3$ miles per hour E. NOTA

22. A circular wagon wheel has 15 identical spokes spaced evenly apart. If the wheel can only be turned by exactly 21 degrees at a time, what is the minimum number of turns necessary to return the wheel to an indistinguishable position to that from which it started?

- A. 105 B. 24
C. 8 D. wheel will never return to an indistinguishable position E. NOTA



You must prepare for any challenges that may cross your path. The first rule of a strange land is to follow the letter of the laws, however treacherous they may be.



Euler?

23. You and two compatriots have been captured by villainous foes. They line the three of you up, one behind the other, facing toward a wall. You are in the front and can see neither friend; the man behind you can see only you, the man behind him can see both of you. Your foes inform you that there are five hats: two white and three black. You and each of your compatriots will have a hat placed on his head, though he will not be able to see the color of his own hat. If any of the three of you can tell what color hat he is wearing, then all will go free. The compatriot in the rear, who can see both you and the second compatriot, is asked first. He says "I do not know what color hat I wear." Compatriot number two considers this response, then also says "I do not know what color hat I wear." What color hat do you wear?

A. White B. Black C. Cannot be determined D. No Hat E. NOTA

24. You are captured once again by more villainous foes. These foes have a peculiar method of execution. The victim (you, in this case) is given 51 marbles: 26 black and 25 white. The victim is also given two indistinguishable urns. The victim is to place all of the marbles into the urns in any way that he wishes, so long as neither urn is empty. Then, the executioner will choose an urn at random and draw a marble randomly from within it. If the marble is black, the victim is executed. If the marble is white, the victim lives. What is the best probability of survival that you, the victim, can achieve?

A. 25/51 B. 1/2 C. 2/3 D. 37/50 E. NOTA



25. Alas! Captured by villainous foes once more! This time you are blindfolded and given a deck of 52 cards. Ten of them—although you do not know which ten—are facing upward; the rest face downward. You are to start from the top of the deck and flip over successive cards, forming a second stack. You must stop when the second stack contains the same number of upward-facing cards as the first. How many cards are in the second stack?

A. Five B. Ten C. Twenty D. Twenty-six E. NOTA



26. Answer the Riddle of the Sphinx: "Which creature walks on four legs at dawn, two legs in day, and three legs at twilight?"

A. Dog B. Dove C. Bear D. Man E. NOTA

27. *There's one of me in five and ten, but two in three and seven.
There's also one in any prime, whilst three lie in eleven.
When I'm written as a numeral, my digits fill all space.
For your answer write the digit lying in my units place.*

A. 1 B. 2 C. 3 D. 4 E. NOTA



28. What is the increase?

A. twofold B. threefold C. sevenfold D. tenfold E. NOTA

29. A. ♥ B. ♣ C. ♦ D. ♠ E. NOTA

30. A. Athens B. Boston C. Cairo D. Rome E. Elsewhere