1. A circle has a radius of \( \frac{x - 2}{\sqrt{x} - \sqrt{2}} \), where defined. Determine the area of the circle in terms of \( x \).

(A) \( \pi (\sqrt{x} - \sqrt{2}) \)  
(B) \( \pi (\sqrt{x} + \sqrt{2}) \)  
(C) \( \pi (x - \sqrt{2x} + 2) \)  
(D) \( \pi (x + \sqrt{2x} + 2) \)  
(E) NOTA

For Problems 2, 3, and 4, use the following information:

Parallel lines \( \Omega \) and \( \rho \) are drawn below. A transversal \( \Sigma \) is drawn passing through \( \omega \) and \( \rho \) and angles \( \alpha_i \) for \( 1 \leq i \leq 8 \) are identified below.

2. Let \( \alpha_1 = 70^\circ \). Compute the measure of \( \alpha_7 \), in degrees.

(A) 35\(^\circ\)  
(B) 70\(^\circ\)  
(C) 110\(^\circ\)  
(D) 145\(^\circ\)  
(E) NOTA

3. Let \( \alpha_1 = 70^\circ \). Consider two positive integers \( i, j \) such that \( 1 \leq i, j \leq 8 \) where \( i \) is even and \( j \) is odd. Compute the maximum possible value of \( \alpha_i + \alpha_j \).

(A) 70\(^\circ\)  
(B) 140\(^\circ\)  
(C) 180\(^\circ\)  
(D) 220\(^\circ\)  
(E) NOTA

4. Let \( 0 < \alpha_1 < 90^\circ \). It is clear that \( \sum_{i=1}^{8} \alpha_i = 720^\circ \). Consider, however, the sum \( S = \left( \sum_{i=1}^{8} \alpha_i \right) - \alpha_k \) where \( k \) is a positive integer randomly chosen from the set \( \{1, 2, \ldots, 8\} \). What is the probability that \( S > 630^\circ \)?

(A) 1/8  
(B) 1/4  
(C) 1/2  
(D) 3/4  
(E) NOTA
5. Consider a line segment $AB$. Point $C$ is chosen on $AB$ such that $\frac{AC}{AB} = \frac{1}{5}$ and point $D$ is randomly chosen on $AB$ between $C$ and $B$. The segments $AC$, $CD$, and $DB$ are used to make a triangle $T$. Compute the probability that $T$ is nondegenerate.

(A) $\frac{1}{5}$  (B) $\frac{1}{4}$  (C) $\frac{3}{4}$  (D) $\frac{4}{5}$  (E) NOTA

6. Rectangle $ABCD$ has area 96. Parallelogram $ACEF$ is drawn such that $EF$ passes through $D$. What is the area of $ACEF$?

(A) 48  (B) 72  (C) 96  (D) 108  (E) NOTA

7. Let a unit polygon be a regular polygon with an area of 1. Compute the perimeter of the unit triangle.

(A) $\frac{1}{\sqrt{3}}$  (B) $\frac{2}{\sqrt{3}}$  (C) $\frac{4}{\sqrt{3}}$  (D) $\frac{6}{\sqrt{3}}$  (E) NOTA

8. Consider the unit polygon defined in question 7. As the number of sides of the unit polygon increases, the perimeter of the unit polygon approaches a certain number. Specifically, let $n = 10,000,000,000$. Which of the following is closest to the perimeter of a unit polygon with $n$ sides?

(A) $\frac{1}{\sqrt{\pi}}$  (B) $\frac{2}{\sqrt{\pi}}$  (C) $\sqrt{\pi}$  (D) $2\sqrt{\pi}$  (E) NOTA

9. Consider the function $f(x) = |x + |x||$. Determine the area of the region above the $x$-axis, below $f(x)$, and between $x = 1$ and $x = -1$.

(A) 1  (B) 2  (C) 3  (D) 4  (E) NOTA

10. Consider the sequence $a_n = \{0, -1, -1, \ldots\}$ defined by $a_n = \frac{n(n-3)}{2}$ for $n \geq 0$. There exists a positive integer $z$ such that $a_z = 275$. Determine the number of diagonals in a regular polygon with $z$ sides.

(A) 25  (B) 26  (C) 27  (D) 28  (E) NOTA

11. Determine the area of a rhombus inscribed in a circle with radius 6.

(A) 36  (B) 72  (C) 144  (D) Need More Info  (E) NOTA
12. Consider a nondegenerate triangle $T$ and denote the *trisection points* of this triangle as the three sets of two points which trisect each side of the triangle. If we connect the trisection points in clockwise order we obtain a hexagon $H$. Compute the value of $\frac{[H]}{[T]}$, where $[R]$ denotes the area of the region $R$.

(A) $\frac{1}{9}$  (B) $\frac{1}{6}$  (C) $\frac{1}{3}$  (D) $\frac{2}{3}$  (E) NOTA

13. A circle is drawn through the points $(2,5)$, $(0,-1)$, and $(-3,1)$. If the center of the circle can be written as $\left(\frac{a}{c}, \frac{b}{c}\right)$, where $a, b, c$ are positive integers, $\gcd(a, b) = 1$ and $\gcd(b, c) = 1$, compute the value of $a + b + c$.

(A) 74  (B) 75  (C) 76  (D) 77  (E) NOTA

14. A chord $\overline{AB}$ is drawn in a circle of radius 4 such that its midpoint is at a distance of 2 from the center. A circle is then inscribed within the ensuing circular segment. Compute the area of region $R$, the shaded region shown below.

(A) $\frac{13\pi}{6} - 4\sqrt{3}$  (B) $\frac{13\pi}{6} - 2\sqrt{3}$  (C) $\frac{13\pi}{3} - 4\sqrt{3}$  (D) $\frac{13\pi}{3} - 2\sqrt{3}$  (E) NOTA

15. The lengths of the sides of a certain nondegenerate triangle are in the ratio $2:4:5$. The lengths of the altitudes of this same triangle can be written as $a:b:c$, where $a, b, c$ are positive integers such that their greatest common factor is 1. Compute $a + b + c$.

(A) 17  (B) 18  (C) 19  (D) 20  (E) NOTA
16. What is the area of the region defined by $|x + y| < 2$ and $0 < x < 2$?

(A) 2  (B) 4  (C) 6  (D) 8  (E) NOTA

17. Square $ABCD$ has a side length of 4 and two equilateral triangles $ABE$ and $ABF$ are drawn such that $E$ is on the interior of $ABCD$ and $F$ is on the exterior of $ABCD$. Determine the area of triangle $CFE$.

(A) $\sqrt{3}$  (B) $2\sqrt{3}$  (C) $4\sqrt{3}$  (D) $8\sqrt{3}$  (E) NOTA

18. The height of a right circular cone with a volume of 1 is three times the length of its radius. Compute the length of the radius of the cone.

(A) $\frac{1}{\sqrt[1/2]{\pi}}$  (B) $\frac{1}{\sqrt[1/3]{\pi}}$  (C) $\frac{3}{\sqrt[1/2]{\pi}}$  (D) $\frac{3}{\sqrt[1/3]{\pi}}$  (E) NOTA

19. A parallelogram has 3 of its vertices at $(1, 2)$, $(3, 8)$, and $(4, 1)$. There are 3 possible points for the fourth vertex. Let $k$ be the sum of the coordinates of the fourth point. Determine the sum of all possible $k$.

(A) 18  (B) 19  (C) 20  (D) 21  (E) NOTA

20. Consider square $ABCD$ with side length 1. Isosceles triangle $ABE$ where $AE = BE$ is drawn on the exterior of the square such that the lengths $CD$, $AC$, and $CE$ are in increasing geometric progression. Determine the area of $ABE$.

(A) $\frac{\sqrt{15}}{4} - \frac{1}{2}$  (B) $\frac{\sqrt{15}}{2} - 1$  (C) $\frac{\sqrt{15}}{4}$  (D) $\frac{\sqrt{15}}{2}$  (E) NOTA

21. Compute the volume of a cylinder with a radius of $r$ and a height of $\frac{1}{r}$, where defined.

(A) $\pi$  (B) $\frac{\pi}{r}$  (C) $\pi r$  (D) $\pi r^2$  (E) NOTA

22. Circle $\omega$ is circumscribed about triangle $ABC$ with $AB = 4$, $BC = 6$, and $AC = 3$. Point $D$ is chosen on $\omega$ such that chord $AD$ intersects $BC$ at its midpoint $M$. Compute the length of $MD$.

(A) $\frac{9\sqrt{14}}{r}$  (B) $\frac{11\sqrt{14}}{r}$  (C) $\frac{13\sqrt{14}}{r}$  (D) $\frac{15\sqrt{14}}{r}$  (E) NOTA
23. A square is inscribed in a $60^\circ$ sector of a circle with radius 6 such that two of its consecutive vertices lie on the arc of the sector. Compute the area of the square.

(A) $9(2 - \sqrt{3})$  (B) $18(2 - \sqrt{3})$  (C) $27(2 - \sqrt{3})$  (D) $36(2 - \sqrt{3})$  (E) NOTA

24. Determine the area of a triangle with a perimeter of 18 and an inradius of 8.

(A) 36  (B) 72  (C) 96  (D) 108  (E) NOTA

25. The region $R$ is comprised of square $ABCD$ with side length 4 and four equilateral triangles drawn on the exterior of $ABCD$ such that each triangle shares one distinct side with the square. Determine the area of the smallest square which contains $R$.

(A) $16 - 8\sqrt{3}$  (B) $32 - 16\sqrt{3}$  (C) $16 + 8\sqrt{3}$  (D) $32 + 16\sqrt{3}$  (E) NOTA

26. Define $F(n)$ as the sum of the exterior angles (one at each angle and in degrees) of an $n$-sided convex polygon. Compute $F(3) + F(4) + \cdots + F(n)$ in terms of $n$.

(A) $360n - 1080$  (B) $360n - 720$  (C) $360n - 360$  (D) $360n$  (E) NOTA

27. Right triangle $ABC$ has $AB = 9$, $BC = 12$, and $AC = 15$. Points $E$, $F$, and $G$ are chosen on sides $AB$, $BC$ and $AC$ respectively such that $BE = 3$, $BF = 8$, and $CG = 5$. Compute the area of triangle $EFG$.

(A) 6  (B) 8  (C) 12  (D) 16  (E) NOTA

28. Let $S_1$ be a square with side length 10. We form $S_2$ by connecting the midpoints of the sides of $S_1$ and continue this process; thus the square $S_n$ is formed by connecting the midpoints of the sides of $S_{n-1}$. Compute $\sum_{n=1}^{\infty} [S_n]$. Note: $[H]$ denotes the area of the region $H$.

(A) 125  (B) 150  (C) 175  (D) 200  (E) NOTA
For Problems 29 and 30, use the following information:

**British Flag Theorem.** Consider some rectangle $ABCD$ and a point $P$ chosen on the same plane. Then the following is satisfied:

$$AP^2 + CP^2 = BP^2 + DP^2.$$ 

29. Consider a square $ABCD$ and a point $P$ chosen on its interior. If $AP = 5$, $BP = 3$, and $CP = 4$, compute the length of $DP$.

(A) $2\sqrt{2}$  
(B) $3\sqrt{2}$  
(C) $4\sqrt{2}$  
(D) $5\sqrt{2}$  
(E) NOTA

30. Consider some rectangle $ABCD$ such that $AB = 8$ and $BC = 6$. A square $EBDF$ is drawn such that it has diagonal $BD$ as a side and contains point $A$. Compute the value of $(EA - FA)(EA + FA)$.

(A) 22  
(B) 24  
(C) 26  
(D) 28  
(E) NOTA