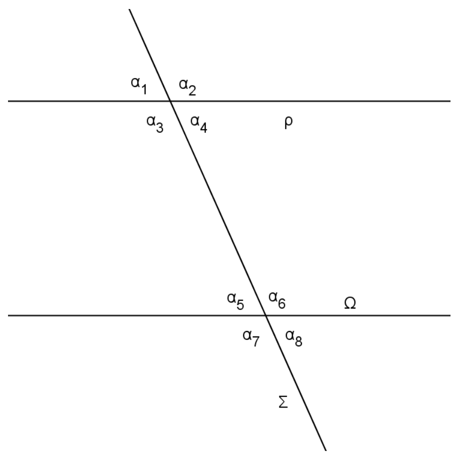
Let [H] denote the area of a region H. If none of the above answers are correct, choose NOTA. Good luck!

1. A circle has a radius of $\dfrac{x-2}{\sqrt{x}-\sqrt{2}}$, where defined. Determine the area of the circle in terms of x.

(A) $\pi(\sqrt{x}-\sqrt{2})$ **(B)** $\pi(\sqrt{x}+\sqrt{2})$ **(C)** $\pi(x-\sqrt{2x}+2)$ **(D)** $\pi(x+\sqrt{2x}+2)$ **(E)** NOTA

For Problems 2, 3, and 4, use the following information:

Parallel lines Ω and ρ are drawn below. A transversal Σ is drawn passing through ω and ρ and angles α_i for $1 \leq i \leq 8$ are identified below.



2. Let $\alpha_1 = 70°$. Compute the measure of α_7, in degrees.

(A) 35° **(B)** 70° **(C)** 110° **(D)** 145° **(E)** NOTA

3. Let $\alpha_1 = 70°$. Consider two positive integers i, j such that $1 \leq i, j \leq 8$ where i is even and j is odd. Compute the maximum possible value of $\alpha_i + \alpha_j$.

(A) 70° **(B)** 140° **(C)** 180° **(D)** 220° **(E)** NOTA

4. Let $0 < \alpha_1 < 90°$. It is clear that $\sum_{i=1}^{8} \alpha_i = 720°$. Consider, however, the sum $S = \left(\sum_{i=1}^{8} \alpha_i\right) - \alpha_k$ where k is a positive integer randomly chosen from the set $\{1, 2, \ldots, 8\}$. What is the probability that $S > 630°$?

(A) 1/8 **(B)** 1/4 **(C)** 1/2 **(D)** 3/4 **(E)** NOTA

5. Consider a line segment $\overline{AB}$. Point C is chosen on $\overline{AB}$ such that $\frac{AC}{AB} = \frac{1}{5}$ and point D is randomly chosen on $\overline{AB}$ between C and B. The segments $\overline{AC}$, $\overline{CD}$, and $\overline{DB}$ are used to make a triangle T. Compute the probability that T is nondegenerate.

   (A) 1/5 **(B)** 1/4 **(C)** 3/4 **(D)** 4/5 **(E)** NOTA

6. Rectangle $ABCD$ has area 96. Parallelogram $ACEF$ is drawn such that $\overline{EF}$ passes through D. What is the area of $ACEF$?

   (A) 48 **(B)** 72 **(C)** 96 **(D)** 108 **(E)** NOTA

7. Let a *unit polygon* be a **regular** polygon with an area of 1. Compute the perimeter of the unit triangle.

   (A) $\dfrac{1}{\sqrt[4]{3}}$ **(B)** $\dfrac{2}{\sqrt[4]{3}}$ **(C)** $\dfrac{4}{\sqrt[4]{3}}$ **(D)** $\dfrac{6}{\sqrt[4]{3}}$ **(E)** NOTA

8. Consider the unit polygon defined in question 7. As the number of sides of the unit polygon increases, the perimeter of the unit polygon approaches a certain number. Specifically, let $n = 10,000,000,000$. Which of the following is closest to the perimeter of a unit polygon with n sides?

   (A) $\frac{1}{\sqrt{\pi}}$ **(B)** $\frac{2}{\sqrt{\pi}}$ **(C)** $\sqrt{\pi}$ **(D)** $2\sqrt{\pi}$ **(E)** NOTA

9. Consider the function $f(x) = |x + |x||$. Determine the area of the region above the x-axis, below $f(x)$, and between $x = 1$ and $x = -1$.

   (A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** NOTA

10. Consider the sequence $a_n = \{0, -1, -1, \ldots\}$ defined by $a_n = \dfrac{n(n-3)}{2}$ for $n \geq 0$. There exists a positive integer z such that $a_z = 275$. Determine the number of diagonals in a regular polygon with z sides.

    (A) 25 **(B)** 26 **(C)** 27 **(D)** 28 **(E)** NOTA

11. Determine the area of a rhombus inscribed in a circle with radius 6.

    (A) 36 **(B)** 72 **(C)** 144 **(D)** Need More Info **(E)** NOTA

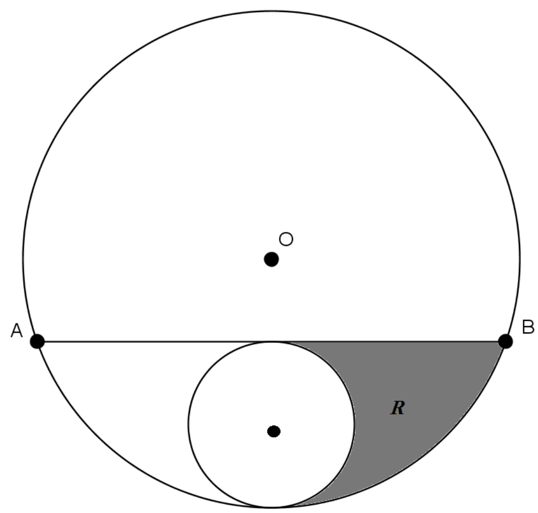
12. Consider a nondegenerate triangle T and denote the *trisection points* of this triangle as the three sets of two points which trisect each side of the triangle. If we connect the trisection points in clockwise order we obtain a hexagon H. Compute the value of $\frac{[H]}{[T]}$, where $[R]$ denotes the area of the region R.

(A) 1/9 **(B)** 1/6 **(C)** 1/3 **(D)** 2/3 **(E)** NOTA

13. A circle is drawn through the points $(2,5)$, $(0,-1)$, and $(-3,1)$. If the center of the circle can be written as $\left(\frac{a}{c}, \frac{b}{c}\right)$, where a, b, c are positive integers, $\text{gcf}(a,b) = 1$ and $\text{gcf}(b,c) = 1$, compute the value of $a + b + c$.

(A) 74 **(B)** 75 **(C)** 76 **(D)** 77 **(E)** NOTA

14. A chord $\overline{AB}$ is drawn in a circle of radius 4 such that its midpoint is at a distance of 2 from the center. A circle is then inscribed within the ensuing circular segment. Compute the area of region R, the shaded region shown below.



(A) $\frac{13\pi}{6} - 4\sqrt{3}$ **(B)** $\frac{13\pi}{6} - 2\sqrt{3}$ **(C)** $\frac{13\pi}{3} - 4\sqrt{3}$ **(D)** $\frac{13\pi}{3} - 2\sqrt{3}$ **(E)** NOTA

15. The lengths of the sides of a certain nondegenerate triangle are in the ratio $2:4:5$. The lengths of the altitudes of this same triangle can be written as $a:b:c$, where a, b, c are positive integers such that their greatest common factor is 1. Compute $a + b + c$.

(A) 17 **(B)** 18 **(C)** 19 **(D)** 20 **(E)** NOTA

16. What is the area of the region defined by $|x+y| < 2$ and $0 < x < 2$?

(A) 2 **(B)** 4 **(C)** 6 **(D)** 8 **(E)** NOTA

17. Square $ABCD$ has a side length of 4 and two equilateral triangles ABE and ABF are drawn such that E is on the interior of $ABCD$ and F is on the exterior of $ABCD$. Determine the area of triangle CFE.

(A) $\sqrt{3}$ **(B)** $2\sqrt{3}$ **(C)** $4\sqrt{3}$ **(D)** $8\sqrt{3}$ **(E)** NOTA

18. The height of a right circular cone with a volume of 1 is three times the length of its radius. Compute the length of the radius of the cone.

(A) $\frac{1}{\pi^{1/2}}$ **(B)** $\frac{1}{\pi^{1/3}}$ **(C)** $\frac{3}{\pi^{1/2}}$ **(D)** $\frac{3}{\pi^{1/3}}$ **(E)** NOTA

19. A parallelogram has 3 of its vertices at $(1,2)$, $(3,8)$, and $(4,1)$. There are 3 possible points for the fourth vertex. Let k be the sum of the coordinates of the fourth point. Determine the sum of all possible k.

(A) 18 **(B)** 19 **(C)** 20 **(D)** 21 **(E)** NOTA

20. Consider square $ABCD$ with side length 1. Isosceles triangle ABE where $AE = BE$ is drawn on the exterior of the square such that the lengths CD, AC, and CE are in increasing geometric progression. Determine the area of ABE.

(A) $\frac{\sqrt{15}}{4} - \frac{1}{2}$ **(B)** $\frac{\sqrt{15}}{2} - 1$ **(C)** $\frac{\sqrt{15}}{4}$ **(D)** $\frac{\sqrt{15}}{2}$ **(E)** NOTA

21. Compute the volume of a cylinder with a radius of r and a height of $\frac{1}{r^2}$, where defined.

(A) π **(B)** $\frac{\pi}{r}$ **(C)** πr **(D)** πr^2 **(E)** NOTA

22. Circle ω is circumscribed about triangle ABC with $AB = 4$, $BC = 6$, and $AC = 3$. Point D is chosen on ω such that chord $\overline{AD}$ intersects $\overline{BC}$ at its midpoint M. Compute the length of MD.

(A) $\frac{9\sqrt{14}}{7}$ **(B)** $\frac{11\sqrt{14}}{7}$ **(C)** $\frac{13\sqrt{14}}{7}$ **(D)** $\frac{15\sqrt{14}}{7}$ **(E)** NOTA

23. A square is inscribed in a 60° sector of a circle with radius 6 such that two of its consecutive vertices lie on the arc of the sector. Compute the area of the square.

(A) $9(2-\sqrt{3})$ **(B)** $18(2-\sqrt{3})$ **(C)** $27(2-\sqrt{3})$ **(D)** $36(2-\sqrt{3})$ **(E)** NOTA

24. Determine the area of a triangle with a perimeter of 18 and an inradius of 8.

(A) 36 **(B)** 72 **(C)** 96 **(D)** 108 **(E)** NOTA

25. The region R is comprised of square $ABCD$ with side length 4 and four equilateral triangles drawn on the exterior of $ABCD$ such that each triangle shares one distinct side with the square. Determine the area of the smallest square which contains R.

(A) $16-8\sqrt{3}$ **(B)** $32-16\sqrt{3}$ **(C)** $16+8\sqrt{3}$ **(D)** $32+16\sqrt{3}$ **(E)** NOTA

26. Define $F(n)$ as the sum of the exterior angles (one at each angle and in degrees) of an n-sided convex polygon. Compute $F(3)+F(4)+\cdots+F(n)$ in terms of n.

(A) $360n-1080$ **(B)** $360n-720$ **(C)** $360n-360$ **(D)** $360n$ **(E)** NOTA

27. Right triangle ABC has $AB=9$, $BC=12$, and $AC=15$. Points E, F, and G are chosen on sides $\overline{AB}$, $\overline{BC}$ and $\overline{AC}$ respectively such that $BE=3$, $BF=8$, and $CG=5$. Compute the area of triangle EFG.

(A) 6 **(B)** 8 **(C)** 12 **(D)** 16 **(E)** NOTA

28. Let S_1 be a square with side length 10. We form S_2 by connecting the midpoints of the sides of S_1 and continue this process; thus the square S_n is formed by connecting the midpoints of the sides of S_{n-1}. Compute $\sum_{n=1}^{\infty}[S_n]$. *Note: [H] denotes the area of the region H.*

(A) 125 **(B)** 150 **(C)** 175 **(D)** 200 **(E)** NOTA

For Problems 29 and 30, use the following information:

British Flag Theorem. Consider some rectangle $ABCD$ and a point P chosen on the same plane. Then the following is satisfied:

$$AP^2 + CP^2 = BP^2 + DP^2.$$

29. Consider a square $ABCD$ and a point P chosen on its interior. If $AP = 5$, $BP = 3$, and $CP = 4$, compute the length of DP.

(A) $2\sqrt{2}$ **(B)** $3\sqrt{2}$ **(C)** $4\sqrt{2}$ **(D)** $5\sqrt{2}$ **(E)** NOTA

30. Consider some rectangle $ABCD$ such that $AB = 8$ and $BC = 6$. A square $EBDF$ is drawn such that it has diagonal $\overline{BD}$ as a side and contains point A. Compute the value of $(EA-FA)(EA+FA)$.

(A) 22 **(B)** 24 **(C)** 26 **(D)** 28 **(E)** NOTA