Lines, Angles, and Squares, Oh MY! Mu Alpha Theta 2012 National Convention

**Solutions**

1. **D** The measure of each exterior angle is 360/9 = 40°. Each interior angle, being supplementary to the exterior angle, has measure 140°. Seven interior angles: 7 x 140° = **980°.**
2. **B** The radii of the circles and the side length of the squares each form geometric sequences. Values of radii: 4, 2, 2, , …; values of side lengths: 4, 4, 2, 2, … In each case, the common ratio is , or. The 9th circle has radius ¼ , so its area is . The 13th square has side length , so its area is . Adding these two areas, we get 

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1. **A** The cone formed has *h* = 4, *r* = 3, and slant height, *l* = 5. Lateral area = π*rl* = **15π.**
2. **C** Let the prime factorization of n be 2P 3Q 5R. Since 2n is a perfect square, P must be odd, and Q and R must be even. Since 3n is a perfect cube, Q must be one less than a multiple of 3 and P and R must be multiples of 3. Since 5n is a perfect power of 5, R must be one less than a multiple of 5 and P and Q must be multiples of 5. So: P is a multiple of 5 and 3, (and therefore 15), and one less than a multiple of 2. So P = 15 is the least possible value. Q is a multiple of 2 and 5, (and thus 10) and is one less than a multiple of 3, so Q = 20 is the least value. R is a multiple of 2 and 3 (and thus 6) and one less than a multiple of 5; this gives us R = 24. So n = **215320524**.
3. **B** Perpendicular line has slope . So ; cross multiplying gives us 10 – 5*y* = 5, so *y* = **1**.

1. **A** The inscribed hexagon has perimeter of 36π, and apothem . Area = ½ ap, so A = **.**

1. **D** The two given sides are the legs of the right triangle. Area = 150; hypotenuse = 25, and altitude to hypotenuse = 12. Sum of these = **187**.
2. **B** The hour hand moves 1/3 of the way from the 12 to the one or 10 degrees in the 20 minutes past 12 o’clock. So the total angle between the hands of the clock is **110°**.

**THROWN OUT**

1. **C** The circle has radius = 6 and Area = 36π. Since the area of the square = area of the circle, the side of the square = ; the diagonal of the square = .

1. **C** Factoring we get: . The removable point of discontinuity is at *x* = 5/2. The horizontal asymptote is *y* = 1; there are no vertical asymptotes**.**
2. **B** Connect the centers of the 2 circles with a line segment. The radius of each circle that is draw to the tangent line are perpendicular to the tangent line at the point oftangency. From the center of the smaller circle, a segment drawn perpendicular to the radius of the larger circle forms the 4th side of a rectangle, as well as the hypotenuse of a right triangle that shares a side with the rectangle. This triangle then has legs of length 8 and 12. Since the distance between the centers of the circles is the hypotenuse, solving we get 64 + 144 = the square of the hypotenuse. The distance we seek = .

1. **C** Slope of first line = ; the perpendicular line must have slope and equal . Cross multiplying and solving, we get , so *k*  = 9 or *k* = -4. Of the choices provided, the only correct one is **-4.**

1. **D** (1000+1)2 – (1000-1)2 = [(1000+1)+(1000-1)][(1000+1)-(1000-1)] = [2000][2] = **4000**
2. **C**  By dividing, we get the slant asymptote is *y* = 2*x* – 3. The only point not on this line is **(0, -13)**
3. **D** Each hemisphere has surface area ½ (4π r2) + π r2, or 3π r2. The volume of the hemisphere is half the volume of the sphere, or ⅔π *r*3. Set up equation: 3π r2 = ⅔π *r*3; solving we get *r* = **4½** .
4. **E** Each point on the circle is equidistant to the center, or ; squaring and simplifying, gives ; then -8*n* = 16*m* + 16, so 2*m* + *n* = **-2**.
5. **A** The center of the hyperbola is the midpoint of the vertices (or of the foci), which is (4,5). The distance between the vertices, 2*a* = 8; the distance between the foci, 2*c* = 5. Therefore *a* = 4, *c* = 5, and *b* = 3.The equation of the asymptotes is: . Solving for the *x*-intercepts, we get  and , so **the *x*-intercepts are ¼ and 3¼**.
6. **B** The equation of the circle transforms to , so the radius of the circle is and the area of the circle is 12π. The side length of the square is twice the radius, or , so the area of the square is 48. The requested area is 
7. **B** All points equidistant from the two given points lie on the perpendicular bisector of the segment between the 2 given points; that is the vertical line *x* = 4. We can then say that the point we are looking for is (4, *y*), and it lies a distance of 10 from each of the 2 given points. We can set up an equation using either of the 2 points: the distance 10 = , which simplified gives us , so *y* = ± 8. We are looking for a point in quadrant IV, so *y* = -8 and the point is **(4, -8)**.
8. **C** The lengths of the segments are 15 and 5; the ratio is **1:3**.
9. **A** Start with the largest possible square and going down from there.

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Side length of square | 15 | 14 | 13 | 12 | 11 | 10 | 9 | 8 | 7 |
| Number  | 1 | 4 | 9 | 16 | 25 | 36 | 49 | 64 | 81 |

 Adding to find the total number of squares formed, we get **285.**

1. **E** Transforming the equation, we get , so the vertex is (-1, 3) and the axis of symmetry is *y* = 3. Since *p* = -2½, and the parabola opens sideways, the direcrix is the line *x* = - 3½ . These two lines intersect at the point (- 3½, 3), the sum of the coordinates is **-½.**
2. **D**  The angle of depression is 60°, its complement, 30°, is at the top of a right triangle formed by Harry’s vertical distance above the ground, the distance along the ground below Harry to the HP calculator, and his line of sight, which is the hypotenuse of the right triangle. This is a 30-60-90 triangle, so the hypotenuse is simply twice the short leg, or, which simplifies to 
3. **A**  Transforming the equation to  tells us the center of the circle is (-2, 3). The slope of the radius to the point of tangency is ¾; The radius to the point of tangency is perpendicular to the tangent line, so its slope is - 4/3. The equation of the tangent line is ; we find the *x*-intercept by setting *y* = 0, and solving for *x*. This gives us *x* = **19.**
4. **A** For the expression to be defined over the Reals, *b* must be non-negative; *a* may be positive or negative. By definition, a square root represents a non-negative number. Thus the expression simplifies to . The answer is ***a.***
5. **B** Slope, *m* = ; so |*k*| - *k* = 12/5. For this to be true, *k* must be negative, so |*k*| = -*k*. This gives us –*k* – *k* = 12/5. Therefore *k* = **- 6/5.**
6. **E** In 1⅔ minutes (100 seconds) they travel 100/120, or ⅚ of a complete revolution. ⅚(300°) = **300°**
7. **A** The linear distance traveled in one complete revolution is the circumference, or 72π. In 50 seconds they will travel 5/12 of a complete revolution. (5/12)( 72π) = **30**π ft.
8. **D** The volume of the entire cake is **** = 200π**.** The slice cut is 27/360 of the entire cake, or 3/40. Its volume is then 3/40(200π**)** or **15π.**
9. **C** If the graph is symmetric to the line *y* = *x*, then any pair of points (*a*, *b*) and (*b*, *a*) must both be on the graph. The point (*a*, *b*) is on the graph if *ba* = 6, likewise the point (*b*, *a*) is on the graph if *ab* = 6. Since these statements are equivalent,, the graph is symmetric with respect to *y* = *x*. Similarly, symmetry with respect to the line *y* = -*x* means that both (*a* , *b*) and (-*b*, -*a*) must both be on the graph. The point (*a*, *b*) is on the graph if *ab* = 6. The point (*-b*, *-a*) is on the graph if (-*b*)(-*a*) = 6, which is equivalent to *ab* = 6, to the function is symmetric with respect to the line *y* = *x*. So **the function is symmetric to both.**