Solutions

1. **Answer (C):** $\log_b 2012 = 2\log 4 + b$. Since $\log 2 = 1 - c$, the answer is $a - 2c + 2$.

2. **Answer (B):** The units digits of powers of 3^n repeat with period 4. 2012 is dismissible by 4, and hence, the units digit of 3^{2012} is simply 1. Making a similar argument, the units digit of 4^{2012} is 6. The sum of these two numbers is 7.

3. **Answer (A):** Each of those expressions is equal to one another (easily seen if you apply the change of base formula on the first two terms). Since there are n terms, the expression becomes $n\log_2 x = 8$ which implies that $2^8 = x^n$. There are 4 integers x which satisfy this. Namely, they are 2, 4, 16 and 256; the sum of which is 278.

4. **Answer (B):** The infinite square root implies that $y = x\sqrt{y}$, or that $y = x^2$ (we reject the case $y = 0$ of course). Hence we are looking for the sum of the first 2012 squares, divided by 2012, which is given by:

$$\frac{1}{2012} \cdot \frac{2012 \cdot 2013 \cdot 4025}{6} = \frac{2700775}{2}$$

5. **Answer (D):** $\log 5 = 1 - \log 2 \approx 1 - .301 = .699$.

6. **Answer (B):** We make the substitution: $s = \log_s 27$. Converting to exponential form we find that $s^s = 27$ which implies $s = 3$.

7. **Answer (A):** There are no repeated letters.

8. **Answer (D):** We are looking for points such that $e^x = x$. Clearly, for $x \le 0$ no such point exists, because for all $x \in \mathbb{R}$, $e^x > 0$ but when $x < 0$, the expression is negative. If you graph the two, or make the argument that e^x (an exponential function) grows faster than the linear one x, we see that no points of intersection exist. Hence, the answer is 0.

9. **Answer (C):** The equation $x^{17} - x + 1 = 0$ implies: $x^{16} = 1 - 1/x$ (note that because $x = 0$ isn't a root, we can do this). Then, we have:

$$\begin{aligned} r_1^{16} &= 1 - 1/r_1 \\ \cdots &\cdots \cdots \\ r_{17}^{16} &= 1 - 1/r_{17} \end{aligned}$$

Hence, $r_1^{16} + r_2^{16} + \cdots r_{17}^{16} = (\underbrace{1 + 1 + \cdots + 1}_{17\ times}) - (-\frac{-1}{1})$ (which is the sum of the reciprocals of the roots). Simplifying, 16 is the answer, as desired.

10. **Answer (B):** Using change of base on each term, the answer is given by: $\log_{256} 2$ which is equivalent to $\frac{1}{8}$.

11. **Answer (B):** We have $\sqrt{100000} = 100$, and that $\sqrt{100} = 10$. Then, $3 < \sqrt{10} < 4$, which means $\sqrt{3} < \sqrt{\sqrt{10}} < 2$. Hence, the answer is 1.

12. **Answer (E):** The number of digits is given simply by: $\lfloor 2012\log(125) + 1 \rfloor$.

13. **Answer (A):** Changing to exponential form three times in a row yields $3^{49} = x^{49}$ which implies that $x = 3$.

14. **Answer: (D)**

15. **Answer (C):** Using the log addition properties, the expression is equal to $\log(7^3)$, which is closest to 2.5 (note, se can use the same $\sqrt{10}$ approximations as before)

16. **Answer (D):** For $\log\left(\frac{b}{c}\right)$ to have a characteristic greater than three, $\frac{b}{c}$ must be greater than 1000. This means that $\log_{11} a$ must be between 1000 and 10,000. Answer choice (D) is the only one large enough.

17. **Answer (C):** Because $\ln e^2 = 2$, the answer is 2^3, or 8.

18. **Answer (C):** We wish to calculate:

$$\begin{aligned} \log_8\left(\sqrt{2}\cdot\sqrt[4]{2}\cdot\sqrt[4]{2}\cdots\right) &= \log_8\left(2^{\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\cdots}\right) \\ &= \log_8\left(2^{\frac{1/2}{1-1/2}}\right) \\ &= \log_8 2^1 \\ &= \frac{1}{3}, \end{aligned}$$

as desired.

19. **Answer (D):** Only the third answer choice is correct (the others aren't true if their components are negative). Hence, 1.

20. **Answer (E):** The expressions $3^{\log x}$ and $x^{\log 3}$ are equivalent. Thus, let $y = 3^{\log x}$. We have that $y^2 - 2y - 8 = 0$ which implies that $y = 4$ (because we must reject the negative solution). Then, $3^{\log x} = 4$, or $x = 10^{\log_3 4}$. The sum $10 + 3 + 4 = 17$ is what we desired, and we are done.

21. **Answer: (C)**

22. **Answer (E):** One has that $A^2 + B^2 = A^2B^2$, which implies that:

$$B = \sqrt{\frac{A^2}{A^2 - 1}} = \frac{|A|}{\sqrt{A^2 - 1}},$$

as desired.

23. **Answer (A):** The approximations $e \approx 2.71$ and $\pi \approx 3.14$ are sufficient enough to produce the correct answer of 23.

24. **Answer (E):** Taking the log of both sides, we have $3x^2 \log 7 + x \log 5 - \log 11 = 0$. The sum of x values which satisfy this equation, is thus

$$-b/a = \frac{-\log 5}{3\log 7}.$$

25. **Answer (B):** The third side must be less than $\log_2 5 + \log_2 3 = \log_2 15$, whose floor has a value of 3.

26. **Answer (E):** If x is negative, the entire expression is positive and hence there are an infinite number of values.

27. **Answer (C):** Raise each number to the $6 \cdot 10 \cdot 2 \cdot 7$ hower. We easily see that $\sqrt[40]{4}$ is the largest of the numbers.

28. **Answer (C):** This is simply $\log\left(\frac{1}{2} \cdot \frac{2}{3} \cdot \frac{3}{4} \cdots \frac{2011}{2012}\right) = \log\left(\frac{1}{2012}\right)$ for a sum of 2013.

29. **Answer (D):** We want $\log_{2011}(\log_{2010}(\log_{2009})) > 0$ which means $\log_{2010}(\log_{2009} x) > 1$ which implies that $x > 2009^{2010}$

30. **Answer: (C)**