

Question	Solution
0.	Cube both sides of the equation to obtain $(7^x)^3 = 4^3$ , or $343^x = 64$ .
1.	The series under the square root consists of 9 consecutive integers, making the mean equal to the median, or 9. Thus, we know that the numbers under the square root add up to $9 \times 9 = 81$ , so the answer is <b>9</b> .
2.	We have $ 65 - 72i  = \sqrt{65^2 + (-72)^2} = 97$ . Knowledge of Pythagorean Triples helps expedite the calculation.
3.	All arguments are in degrees. Since $9000 = (360)(25)$ , we have $\sin 9300 = \sin(300) = -\sqrt{3}/2$ , making the desired value equal to $100 \left(-\frac{\sqrt{3}}{2}\right)^2 = 75$ .
4.	Draw the figure and notice that the altitudes of triangles PAD and PBC—as drawn from point P—altogether make up the height of the parallelogram with respect to sides BC and AD. Thus, the area of the parallelogram is $2(12 + 6) = 36$ .
5.	The sequence in the problem can be thought of as one where the “common difference” increases linearly. Thus, we know that $a_n$ is a quadratic, say $a_n = An^2 + Bn + C$ . Since $a_n = 1$ , we have $A + B + C = 1$ , and furthermore, $a_{n+1} - a_n = A(n+1)^2 + B(n+1) + C - An^2 - Bn - C = 2An + A + B = 8n$ . Equating coefficients yields $2A = 8$ and $A + B = 0$ . Thus, $A = 4$ , $B = -4$ , and $C = 1$ , making $a_n = 4n^2 - 4n + 1 = (2n - 1)^2$ . The answer is $\sqrt{a_{2013}} = 2(2013) - 1 = 4025$ .
6.	Let $N = \sin \theta + \cos \theta$ . We have $N^2 = \sin^2 \theta + 2 \sin \theta \cos \theta + \cos^2 \theta = 1 + \sin(2\theta) = 1 + .69 = 1.69$ . Thus, since $\theta$ is an acute angle, $N = \sqrt{1.69} = 1.3$ .
7.	Perhaps the fastest way to solve this problem is to conjecture (correctly) that because the hypotenuse has length 17, the legs have lengths of 8 and 15, yielding

	an area of $\frac{1}{2}(8)(15) = \mathbf{60}$ .
8.	<p>We have</p> $x = \frac{72_8!}{18_2!} = \frac{(72)(64)(56)\dots(8)}{(18)(16)(14)\dots(2)} = \frac{(18 \times 4)(16 \times 4)(14 \times 4)\dots(2 \times 4)}{(18)(16)(14)\dots(2)} = \frac{4^9(18)(16)(14)\dots(2)}{(18)(16)(14)\dots(2)} = 4^9 = 2^{18},$ <p>making <math>\log_2 x = \mathbf{18}</math>.</p>
9.	<p>Let <math>\theta</math> be an angle in the first quadrant. Noting that <math>\sin^2 \theta = \sin^2(180 - \theta) = \sin^2(180 + \theta) = \sin^2(360 - \theta)</math>, the desired sum is simply four times the value of <math>\sum_{n=0}^{90} \sin^2 n = 45.5</math>, less the values at 90 degrees and 270 degrees because they got counted twice in the process. The answer is <math>4(45.5) - \sin^2 90^\circ - \sin^2 270^\circ = 182 - 1 - 1 = \mathbf{180}</math>.</p>
10.	<p>Let <math>x = 1</math> in the functional equation. We have <math>f(f(1)) = f(3) - 3</math>, or <math>f(4) = f(3) - 3</math>, so that <math>f(3) = f(4) + 3 = 3 + 3 = 6</math>. Next, let <math>x = 4</math> in the functional equation to obtain <math>f(f(4)) = f(6) - 3</math>, or <math>f(3) = f(6) - 3</math>, so that <math>f(6) = f(3) + 3 = 6 + 3 = 9</math>. Finally, let <math>x = 3</math> in the functional equation to obtain <math>f(f(3)) = f(5) - 3</math>, or <math>f(6) = f(5) - 3</math>, so that <math>f(5) = f(6) + 3 = 9 + 3 = \mathbf{12}</math>.</p>
11.	<p>The left-hand side of the equation has roots of <math>-2, -3</math>, and <math>-4</math> while the right-hand side has roots of <math>2, -5</math>, and <math>-6</math>...both sets of roots add up to the same value. Therefore, both polynomials have the same coefficient for the quadratic term; we only need to worry about the coefficients for the sum of the product of the roots taken two at a time and one at a time. For the equation on the left side, these are <math>(-2)(-3) + (-2)(-4) + (-3)(-4) = 26</math> and <math>(-2)(-3)(-4) = -24</math>. For the equation on the right side, these are <math>(2)(-5) + (2)(-6) + (-5)(-6) = 8</math> and <math>(2)(-5)(-6) = 60</math>. Thus, we have <math>26x + 24 = 8x - 60</math>, or <math>x = -14/3</math>, making</p>

	$3x = -14.$
12.	<p>Write out the product, use change-of-base, and observe that the terms cancel:</p> $\frac{\ln 3}{\ln 2} \times \frac{\ln 4}{\ln 3} \times \dots \times \frac{\ln n}{\ln(n-1)} \times \frac{\ln(n+1)}{\ln n} = \frac{\ln(n+1)}{\ln 2} = \log_2(n+1)$ <p>If <math>\log_2(n+1) = 2013</math>, then <math>n = 2^{2013} - 1</math>, which, when converted to binary, is simply a string of 2013 1s. Thus, <math>D(n) = \mathbf{2013}</math>.</p>