1. (78x91 -12x22) = **6834**
2. Out of a total of 60 distinct possible matrices, there are 24 matrices whose determinant is 0. This can be found by placing the zeroes in the same row or column, with 6 possible choices for the remaining two elements (6*4=24). 24/60 = $\mathbf{2/5}$
3. Matrix A row-reduces to the identity matrix.
4. Factor out a 4 and convert to a rotation matrix with θ equal to 120 degrees. When raised to the 5th power, theta becomes 600 degrees, which is coterminal to 240 degrees. Thus, we find the matrix $4^5\begin{bmatrix} cos\,{}^{4\pi}/_3 & -sin\,{}^{4\pi}/_3 \\ sin\,{}^{4\pi}/_3 & cos\,{}^{4\pi}/_3 \end{bmatrix}^1$, which simplifies to $\begin{bmatrix} \mathbf{-512} & \mathbf{512\sqrt{3}} \\ \mathbf{-512\sqrt{3}} & \mathbf{-512} \end{bmatrix}$.
5. Only matrix with nonzero determinant is **II**
6. By a theorem, the only eigenvalue of a nilpotent matrix is 0. Upon inspection, only choice IV has 0 as an eigenvalue.
7. Det(4A) = 4^n * det(A) = 4^6 * 8 = 2^15 = **32768**
8. A matrix is said to be positive semidefinite if its eigenvalues are all greater than or equal to 0. Only choice **II** satisfies this definition.
9. The adjoint of a matrix is found by computing the transpose of the matrix of cofactors. Recall, a cofactor, c_{ij}, is computed by multiplying the minor, m_{ij}, by $-1^{(i+j)}$.
10. Multiply through by the scalars and combine matrices $\begin{bmatrix} \mathbf{26} & \mathbf{55} & \mathbf{76} \\ \mathbf{-4} & \mathbf{66} & \mathbf{28} \\ \mathbf{-43} & \mathbf{127} & \mathbf{44} \end{bmatrix}$
11. If Joyce obtains the 7, no matter what her other two numbers are, her vector's magnitude will be larger than Nick's. Thus, there are 5 nCr 2 winning combinations out of a total of 6 nCr 3 combinations = $\mathbf{1/2}$
12. By definition of a Markov chain model, the eigenvalue is **1**
13. Expand by cofactors to calculate determinant, then combine like terms. $\boldsymbol{6e^3 + 23e^2 - 2e}$

14. The determinant of the matrix is the scaling factor of the area of the original region. Calculate the area of the original region (39) and multiply by the determinant (26) to obtain **1014**
15. Associativity and zero cancellation do not hold for cross products. **III, IV** only
16. Find the direction vector. Pick an initial point to find the vector form <2, 0, -3> + t <5, -5, -7>. Solve for t in terms of x, y, and z. $\boldsymbol{\frac{x-2}{5} = \frac{y}{-5} = \frac{z+3}{-7}}$
17. Form two vectors, and take the cross product to find (A, B, C) in Ax + By + Cz + D = 0. Plug in a point to find D. $\boldsymbol{30x - 13y + z - 31 = 0}$
18. Find vector TZ as <4, 5, 22>. The direction cosine is given by 22 / (Sqrt(16+25+484)) = **22 / 5 Sqrt(21)** (rationalize denominator)
19. Calculate u dot v over the magnitude of v, then multiply this on the left of the unit vector in the direction of v. $\boldsymbol{\frac{27}{26}\langle 3, 1, 4\rangle}$
20. Row rank of A transpose is the equivalent to the rank of A. After applying row operations to row-reduce A, count the pivots (linearly independent columns) to get **3**
21. Volume of tetrahedron is 1/6 * triple scalar product (a dot b cross c). Form three vectors, take the cross product, dot them, take absolute value and divide by 6 to obtain **274/3**
22. The number of paths of length 3 can be found by analyzing the adjacency matrix for the graph G. The cube of the adjacency matrix will yield paths of length 3, then we need only look at the entry in the row for node T and column for node S **(8)**.
23. Assume Taz uses the strategy [p, 1-p]. Note, the third column can be eliminated since Sheila has dominant options in column 1 and column 2. Multiplying Taz's strategy on the left of the reduced payoff matrix $\begin{bmatrix} 4 & -1 \\ -2 & 1 \end{bmatrix}$, we set the entries in the resulting matrix equal to each other (since Taz is trying to maximize his score). We obtain 8p = 2 or p = 0.25, thus his optimal strategy is **[0.25 0.75]**
24. The shortest distance between 2 skew lines is given by |(a-c) •(b x d)| / |b x d|, where the lines (l_1) and (l_2) are given by $a + bs$ and $c + dt$ respectively. After

plugging in the vectors, we find the distance is given by **5 / sqrt(6)** (rationalize denominator)

25. Applying Sylvester's rank inequality, given (A is m x n and B is n x k),
rank(A) + rank(B) – n <= rank(AB)
we find that the rank of AB must be >= 6. Thus **5** is not a possible rank.

26. The diagonalization of a matrix can be computed by finding the eigenvalues and eigenvectors of A. We find the eigenvalues (by computing the roots of det(A – λI)) as -1, 1, 3. We proceed by calculating the associated eigenvectors. P's columns are comprised of the 3 eigenvectors of A. Note, order typically does not matter as it will only alter the order of the eigenvalues in D, however, the question's constraint forces us to write them in increasing order along the main diagonal. **D**

27. We can compute L by performing row operations on A to turn it into a lower triangular matrix. $L = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 2 & \frac{3}{2} & 1 \end{bmatrix}$, and thus we compute $\frac{uvx}{wyz}$ as $\mathbf{16/3}$

28. Instead of calculating the determinant via a brute-force method which will involve a lot of sub-matrices, we can reduce the first column to something much easier to manage. Multiply 2*row4 and add to row2, and multiply 4*row4 and add to row 1 to obtain $\begin{vmatrix} 0 & 21 & -3 & 1 \\ 0 & 9 & 6 & 6 \\ 0 & 3 & 2 & -1 \\ -1 & 4 & 1 & 0 \end{vmatrix}$. We can now compute the determinant using the expansion by cofactors method on column 1. **-459**

29. Recall that $(AB)^T = B^T A^T$, so we need only calculate the transpose of A and multiply on the left by the transpose of D.

30. There is only one real eigenvalue, thus its algebraic multiplicity is 1. The corresponding eigenspace has dimension 1, so the geometric multiplicity is also 1. **1 + 1 = 2**