1. A
\[ \frac{n(n+1)}{2} \rightarrow 197 \times 198 / 2 = 19503 \]

2. A
\[ BC = \sqrt{AC^2 - AB^2} = 16 \]
\[ 12 / BD = \frac{20}{16 - BD} \rightarrow 192 - 12BD = 20BD \]
\[ 192 = 32BD \]
\[ BD = 6, \text{ so that } AD = \sqrt{12^2 + 6^2} = 6\sqrt{5}. \]

3. C
\[ X = \sqrt{34^2 - 16^2} = 30 \]
\[ Y = \sqrt{34^2 - 25^2} = 3\sqrt{59} \]

4. D
\[ AF = \sqrt{AF^2 + FD} = 3\sqrt{5} \]
\[ AB = \sqrt{AF^2 + BF^2} = 2\sqrt{6} \]

5. A
\[ m\angle ABD = 180 - m\angle R - m\angle ADB = 30 \]
\[ m\angle CBD = 75 - 30 = 45 \]
\[ DC = BC = 6\sqrt{6} \]
\[ AD = 6\sqrt{3} \quad AB = 9 \]
\[ AD + AB + BC = 9 + 6\sqrt{3} + 6\sqrt{6} \]

6. A
\[ S = 8 + 4\sqrt{2} + \frac{1}{2} \left( 8 + 4\sqrt{2} \right) + \frac{1}{4} \left( 8 + 4\sqrt{2} \right) \]
\[ = \frac{8 + 4\sqrt{2}}{1 - 1/2} = 16 + 8\sqrt{2} \]

7. A
\[ \sqrt{x^2 + 9y^2} \]
\[ x / 3y \]

8. A
\[ QA = \sqrt{4^2 + (4 - x)^2} = \sqrt{32 - x + x^2} \]
\[ AB = \sqrt{2x^2} \]
\[ AB = QA \]
\[ \sqrt{2x^2} = \sqrt{32 - x + x^2} \]
\[ x = -4 + 4\sqrt{3} \]
\[ AB = \sqrt{2} \left( -4 + 4\sqrt{3} \right) \]
\[ = 4\sqrt{2} + 4\sqrt{6} \]

9. C → Definition

10. E

\[
\begin{align*}
\sin \angle K\!L\!X + \csc \angle J\!L\!X \\
\frac{XK}{L\!K} + \frac{J\!K}{J\!L} \\
\frac{KL + J\!L + J\!K^2}{J\!L + J\!K} = \frac{15 + 8 + 17^2}{8 + 17} = \frac{379}{136}
\end{align*}
\]

11. E

Law of sines

\[ \frac{AB}{\sin C} = \frac{BC}{\sin A} = \sin \frac{75}{37} \]

12. E

\[ 12 = x + x\sqrt{3} \rightarrow x = \frac{12}{1 + \sqrt{3}} \]

Perimeter = \[ 12 + 2x + x\sqrt{6} \]

\[ = \frac{24}{1 + \sqrt{3}} + \frac{12\sqrt{6}}{1 + \sqrt{3}} \]

\[ = 6 + 3\sqrt{6} + 9\sqrt{2} \]

13. B

\[ \frac{12}{\sin \frac{\pi}{6}} = 2R \]

\[ R = 12 \]

\[ \frac{A}{2} = 144 \frac{\pi}{2} = 72 \pi \]

14. B – Definition

15. A

\[ XY = \sqrt{33^2 + 44^2} = 55 \]

Mother's total time = \[ (33 + 27 + 70 + 17)*5 = 735 \text{ hours} \]

\[ V = \frac{55}{734.5} = 110/1469 \]

16. D

\[ 0.5*8*8\sqrt{6} = 32\sqrt{6} \]

17. A

\[ \sqrt{\frac{1 - \cos A}{2}} = \sqrt{\frac{1 - c/b}{2}} = \sqrt{\frac{b-c}{2b}} \]

18. B

0 because the largest angle has to be across the largest side

19. B

Use the law of sines

\[ \frac{b \sin C}{\sin b} = \frac{15 \sin 57^\circ}{\sin 56^\circ} \]

20. A

Heron's formula

\[ \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{176400} \]

21. A
**22. D**

\[ h = \sqrt{12^2 + 5^2} = 13 \]

\[ \sin a = \frac{5}{13} = 0.385 \]

**23. A**

Bird A takes 6 hrs

Bird B takes 5.5 hrs

Direct distance = \( \sqrt{30^2 + 54^2} = 6 \sqrt{61} \)

\[ V = 6 \frac{\sqrt{61}}{5.5} = 33 \sqrt{61} \]

**24. D**

A = 0.5absin a

= 0.5 * 10 * 12 sin (30) = 30

**25. D**

\[ \frac{12}{x} = \cos 65 \]

\[ X = \frac{12}{\cos 65} \]

**26. C**

\[ h = x + y \]

\[ \tan 40 = \frac{x}{60} \]

\[ \tan 35 = \frac{y}{60} \]

\[ h = 60 \tan 40 + 60 \tan 35 \]

**27. A**

**28. C**

**29. D**

Cos BCX = cos CAB = 12/37

sec ACX = sec CBA = 37/35

\[ \frac{12}{37} + \frac{37}{35} = 1789 \]

**30. D**

\[ \sqrt{x^2 + (x + 150)^2} = 2x \]

\[ x^2 + (x + 150)^2 = 4x^2 \]

\[ 2x^2 - 300x + 22500 = 0 \]

By quadratic formula \( x = 289.78 \)
Mu Alpha Theta National Convention: San Diego, 2013
Alpha Triangles Test - Updated Solutions

3 (D). By the symmetry of isosceles triangles, all of I-IV are true. Item IV is true regardless, by the definition of a cevian.

9 (C).

14 (A).

18 (B). All arguments are in degrees. By the Law of Sines, \(20/\sin 30 = 16/\sin C\), or \(\sin C = .40\). Since \(\sin 30 = .50\), \(C < 30\) or \(C > 150\). It can’t be the latter because we already have an angle that is equal to 30 degrees. Thus, there is only 1 possible triangle.

21 (A). Since \(ABC\) is acute and scalene, all the points in the problem are distinct from each other. The nine-point circle only passes through \(p_1\) through \(p_6\), so the probability is \(6/9 = 2/3\).

22 (D). By Stewart’s Theorem, \((10)(6)(10) + (10)(2)(10) = (x)(8)(x) + (2)(8)(6)\), or \(x^2 = 88\).

23 (B). By Ceva’s Theorem, \((6/2)(10/5)(RT/TP) = 1\), or \(RT/TP = 1/6\). Triangles \(QTR\) and \(QTP\) have the same base as measured from vertex \(Q\). Thus, the ratio of their areas is the same as the ratio of their respective bases, or \(1/6\). Thus, \(m + n = 1 + 6 = 7\).

24 (D). All arguments are in radians. Let \(\alpha = m\angle A\) and \(\beta = m\angle ABD = m\angle DBC = (\pi - \alpha)/4\). Using the Law of Sines, we have \(BC = \frac{\sin \alpha}{\sin \frac{\pi}{2}}\), \(BD = \frac{\sin \alpha}{\sin \frac{\pi}{3}}\), and \(AD = \frac{\sin \beta}{\sin \frac{\pi}{3}}\). Since \(AD + DB = BC\), this yields the equation \(\sin(\pi - 4\beta) \sin(3\beta) = (\sin(\pi - 4\beta) + \sin \beta) \sin(2\beta)\), which turns into \(\sin(2\beta) \sin(5\beta) = \sin(2\beta) \sin(4\beta)\) after application of the sum-to-product identities. Thus, \(\sin(5\beta) = \sin(4\beta)\), making \(\beta = \pi/9\) and \(\alpha = 5\pi/9\), or 100 degrees.

25 (B). All angles are in degrees. We have \(m\angle C = 75\). The inscribed triangle that will yield the least perimeter is the orthic triangle, and its area is given by \(K_{orthic} = \frac{abc}{2R}\), where \(R\) is the circumradius of triangle \(ABC\). Compare this with the formula for the area of \(ABC\), \(K = \frac{abc}{4R}\), and we see that the desired ratio is equal to \(2|\cos A \cos B \cos C|\). The answer is \(2 \cos 45 \cos 60 \cos 75 = (\sqrt{3} - 1)/4\).

26 (C). In a triangle, \(\tan \alpha + \tan \beta + \tan \gamma = \tan \alpha \tan \beta \tan \gamma\). Thus, the function in the problem is identically 1, which is both the minimum and the maximum.

27 (B). A straightforward calculation shows that a disc of radius 3 and 4 can be fit into two corners of an equilateral triangle with side \(11\sqrt{3}\) as to just touch. The disc of radius 2 will easily fit into the third corner without overlapping with the other two discs. Thus, \(L = 11\sqrt{3}\) and \(L^2 = 121(3) = 363\), making for a digital sum of \(3 + 6 + 3 = 12\).
29 (D). All angles are in degrees. Interpret each equation as applying the Law of Cosines to a particular set of triangles that are “stuck” together via a common vertex. In particular, consider triangle $ABC$ and a point $P$ inside the triangle such that $AP = x$, $BP = y$, $CP = z$, $m\angle APB = 90$, $m\angle APC = 120$, and $m\angle CPB = 150$. The desired expression is simply 4 times the area of $ABC$ in terms of $x$, $y$, and $z$. In this case, $ABC$ happens to be a right triangle with legs of $\sqrt{7}$ and $\sqrt{21}$, so its area is $(1/2)(\sqrt{7})(\sqrt{21}) = 7\sqrt{3}/2$. The answer is $14\sqrt{3}$. 