Relay 1:

1. $a^2 - b^2 = (a+b)(a-b) = 2013 = 3 \times 11 \times 61 = 2013 \times 1 = 671 \times 3 = 183 \times 11 = 61 \times 33$, so $M = 2013$, $N = 61$, and $2000 - M + N = \boxed{48}$.

2. $T = 48 = 2^4 \times 3$. The sum of factors is $(1 + 2 + 2^2 + 2^3 + 2^4)(1 + 3) = \boxed{124}$.

3. Suppose $f(x) = ax^2 + bx + c$. Then we have a system of equations
$$\begin{cases} a + b + c = 48 \\ 4a + 2b + c = 79 \\ 9a + 3b + c = 124 \end{cases}$$
Solve and get $a = 7$, $b = 10$, $c = \boxed{31}$.

4. All the prime numbers less than 31: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29. Only 3, 11, 17, 23, 29 satisfy the condition, the answer is $\boxed{5}$.

5. Paths to reach $(5,5) = \frac{8!}{4! \cdot 4!} = 70$. Paths through $(2,2)$ to $(5,5) = \frac{2!}{1! \cdot 1!} \cdot \frac{6!}{3! \cdot 3!} = 40$.
$70 - 40 = \boxed{30}$.

6. From the first two conditions, possible numbers are 108, 138, 168, 198, 228, 258, and 288. The only one that satisfies the last condition is $\boxed{258}$.

Relay 2:

1. The distance is $\left. |5(1) - 12(1) - 32| \middle/ \sqrt{5^2 + 12^2} \right. = {}^{39}\!/_{13} = \boxed{3}$.

2. x_1 and x_2 are solutions to $x^2 - 3x - 6 = -x^2 + 9x - 20$, or $2x^2 - 12x + 14 = 0$. Therefore, $x_1 + x_2 = {}^{12}\!/_{2} = \boxed{6}$.

3. First, notice that $T \geq 5$ or else the sum won't be a four-digit number. $W = 3, U = 6$. Therefore, conclude that $2 \cdot O < 10$, otherwise $U = 7$; and O is even because $2T$ is even. This makes O either 2 or 4. If $O = 2$, then $T = 6$, violating the unique use of digits. When $O = 4$, $T = 7$, $F = 1$, and $R = 8$. The addition is $734 + 734 = 1468$. The digits that are not used are 0, 2, 5, 9, which add up to $\boxed{16}$.

4. $16! = 2^{15} \cdot 3^6 \cdot 5^3 \cdot 7^2 \cdot 11 \cdot 13$. ${}^{a}\!/_{c} + {}^{b}\!/_{d} = {}^{15}\!/_{3} + {}^{6}\!/_{2} = \boxed{8}$.

5. The friendly numbers are, regardless of b, $0_b, 1_b, 10_b, 11_b, 100_b, 101_b, 110_b, 111_b, \cdots$. The 8th 8-friendly number is $111_8 = \boxed{73}$.

6. $S = \lfloor\sqrt{73}\rfloor = 8$.

$$\sum_{n=1}^{8}(1+i)^n = \frac{(1+i)((1+i)^8-1)}{(1+i)-1} = \frac{1+i}{i}\cdot((2i)^4-1) = \frac{15+15i}{i} = 15-15i$$

So $a = 15$, $b = -15$, $a + b = \boxed{0}$.

Relay 3:

1. ΔACB and ΔBCD are similar because $\overline{AC}/\overline{BC} = \overline{BC}/\overline{DC}$ and they share the same angle.

   Therefore, $\angle CAB = \angle CBD = \frac{1}{4}\angle DBA = \boxed{15}$ degrees.

2. $\det(A - \lambda I) = 0 = (15-\lambda)(30-\lambda) - 18^2 = (\lambda-3)(\lambda-42)$, the answer is $\boxed{3}$.

3. When the side length of an equilateral triangle is 6, the area is $6^2 \cdot \sqrt{3}/4 = 9\sqrt{3}$. Square it and get $\boxed{243}$.
4. The strategy of winning this game is to keep the remaining number of coins as one more than a multiple of $T + 1$. This way, if Connie takes m coins, David will take $T + 1 - m$, which is always legal, . In this case, multiples of $T + 1 = 16$ are called *game-winning numbers*. On his first turn, David should take $\boxed{3}$ and leave 240 behind.

5. $a + c = 30$, $bd = 9$, $b + c = 6$, $ad = 225$. Then $(a + c) - (b + c) = a - b = 24$, and $ad - bd = (a - b)d = 216$. We get $d = 9$. Substitute back, $b = 1$, $c = 5$, $a = 25$. $a + d + bc = 25 + 9 + 5 = \boxed{39}$.

6. 2013^{39} ends in 7; 39^{2013}ends in 9. The final answer is $7 + 9 = 16 \rightarrow \boxed{6}$.

Relay 4:

1. Let the distance be $2d$. Then $2d/40 - 1 = d/40 + d/50$, $d/40 - 1 = d/50$, $50d - 2000 = 40d$, $d = 200$. The answer is $\boxed{400}$.

2. If she has n children, then n is a factor of both $400 + 7 = 407$ and $\sqrt{400} + 2 = 22$. n can be either 1 or 11, but she wouldn't have problems distributing with only one child. $\boxed{11}$

3. Can consider this by subtraction: the inner $9 \times 9 \times 9$ volume is not visible and the 8 corners have too many, so $11^3 - 9^3 - 8 = \boxed{594}$; or by addition: six 9×9 center cubes and twelve 9 edge cubes, so $6 \times 9 \times 9 + 12 \times 9 = \boxed{594}$.

4. The smallest multiple of 100 greater than 594 is 600. So $u = 600 - 594 = \boxed{6}$.

5. Solve and find that the intersections are $(6,0)$, $(-2,0)$, and $(-3,1.5)$. The area is $8 \cdot 1.5/2 = \boxed{6}$.

6. $\binom{39}{5}\binom{13}{1} : \binom{39}{6} = 13 : \frac{34}{6} = 39 : 17$. The answer is $\boxed{56}$.

Relay 5:

1. $425 = 20^2 + 5^2 = 19^2 + 8^2 = 16^2 + 13^2$. So the circle goes through 24 lattice points: $(\pm 20, \pm 5), (\pm 19, \pm 8), (\pm 16, \pm 3), (\pm 5, \pm 20), (\pm 8, \pm 19), (\pm 3, \pm 16)$. Answer is $\boxed{12}$.

2. The volume we're looking for can be considered as a large cube plus a small pyramid minus a large pyramid. So volume $= T^3 + \left(\frac{T}{2}\right)^2 T \cdot \frac{1}{3} - T^2(2T) \cdot \frac{1}{3} = \frac{5}{12}T^3$. Substitute $T = 12$, the answer is $\boxed{720}$.

3. $x = \sqrt[3]{T + \sqrt[3]{T + \sqrt[3]{T + \cdots}}}$, then $x = \sqrt[3]{720 + x}$. $x = \boxed{9}$.

4. The probability is $3/36 + 2/36 + 1/36 = 1/6$. The reciprocal is $\boxed{6}$.

5. The g.c.d. of 12 and 720 is 12; the l.c.m. of 9 and 6 is 18. $\sqrt{|12 - 18| + 3} = \boxed{3}$.

6. $f(x,y) = Ex^2 + Dxy + Cy^2 + Ax + B = 3x^2 + 6xy + 9y^2 + 12x + 720 = (x + 3y)^2 + 2(x+3)^2 + 702 \geq \boxed{702}$, which is when $x = -3$ and $y = 1$.