Relay 1:

1. \(a^2 - b^2 = (a + b)(a - b) = 2013 = 3 \times 11 \times 61 = 2013 \times 1 = 671 \times 3 = 183 \times 11 = 61 \times 33\), so \(M = 2013, N = 61,\) and \(2000 - M + N = \boxed{48}\).

2. \(T = 48 = 2^4 \times 3\). The sum of factors is \((1 + 2 + 2^2 + 2^3 + 2^4)(1 + 3) = 124\).

3. Suppose \(f(x) = ax^2 + bx + c\). Then we have a system of equations

\[
\begin{align*}
    a + b + c &= 48 \\
    4a + 2b + c &= 79 \\
    9a + 3b + c &= 124
\end{align*}
\]

Solve and get \(a = 7, b = 10, c = \boxed{31}\).

4. All the prime numbers less than 31: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29. Only 3, 11, 17, 23, 29 satisfy the condition, the answer is \boxed{5}\).

5. Paths to reach \((5, 5) = \frac{8!}{4!4!} = 70\). Paths through \((2, 2)\) to \((5, 5) = \frac{2!}{1!1!} \cdot \frac{6!}{3!3!} = 40\).

\[70 - 40 = \boxed{30}\]

6. From the first two conditions, possible numbers are 108, 138, 168, 198, 228, 258, and 288. The only one that satisfies the last condition is \boxed{258}\).

Relay 2:

1. The distance is \(\frac{|5(1) - 12(1) - 32|}{\sqrt{5^2 + 12^2}} = \frac{39}{13} = \boxed{3}\).

2. \(x_1\) and \(x_2\) are solutions to \(x^2 - 3x - 6 = -x^2 + 9x - 20,\) or \(2x^2 - 12x + 14 = 0\).

Therefore, \(x_1 + x_2 = \frac{12}{2} = \boxed{6}\).

3. First, notice that \(T \geq 5\) or else the sum won’t be a four-digit number. \(W = 3, U = 6\).

Therefore, conclude that \(2 \cdot O < 10,\) otherwise \(U = 7;\) and \(O \) is even because \(2T\) is even. This makes \(O\) either 2 or 4. If \(O = 2,\) then \(T = 6,\) violating the unique use of digits. When \(O = 4, T = 7, F = 1,\) and \(R = 8.\) The addition is \(734 + 734 = 1468.\) The digits that are not used are 0, 2, 5, 9, which add up to \boxed{16}.

4. \(16! = 2^{15} \cdot 3^6 \cdot 5^3 \cdot 7^2 \cdot 11 \cdot 13. \frac{a}{c} + \frac{b}{d} = \frac{15}{3} + \frac{6}{2} = \boxed{8}\).

5. The friendly numbers are, regardless of \(b, 0_b, 1_b, 10_b, 11_b, 100_b, 101_b, 110_b, 111_b, \cdots.\)

The 8th 8-friendly number is \(111_b = \boxed{73}\).
6. \( S = \sqrt[8]{73} = 8. \)

\[
\sum_{n=1}^{8} (1 + i)^n = \frac{(1 + i)((1 + i)^8 - 1)}{(1 + i) - 1} = \frac{1 + i}{i} \cdot ((2i)^4 - 1) = \frac{15 + 15i}{i} = 15 - 15i
\]

So \( a = 15, b = -15, a + b = 0. \)

Relay 3:

1. \( \Delta ACB \) and \( \Delta BCD \) are similar because \( \frac{AC}{BC} = \frac{BC}{DC} \) and they share the same angle.

Therefore, \( \angle CAB = \angle CBD = \frac{1}{4} \angle DBA = 15 \) degrees.

2. \( \det(A - \lambda I) = 0 = (15 - \lambda)(30 - \lambda) - 18^2 = (\lambda - 3)(\lambda - 42) \), the answer is 3.

3. When the side length of an equilateral triangle is 6, the area is \( 6^2 \cdot \sqrt{3}/4 = 9\sqrt{3} \). Square it and get 243.

4. The strategy of winning this game is to keep the remaining number of coins as one more than a multiple of \( T + 1 \). This way, if Connie takes \( m \) coins, David will take \( T + 1 - m \), which is always legal. In this case, multiples of \( T + 1 = 16 \) are called game-winning numbers. On his first turn, David should take 3 and leave 240 behind.

5. \( a + c = 30, bd = 9, b + c = 6, ad = 225 \). Then \( (a + c) - (b + c) = a - b = 24, \) and \( ad - bd = (a - b)d = 216 \). We get \( d = 9 \). Substitute back, \( b = 1, c = 5, a = 25 \).

\( a + d + bc = 25 + 9 + 5 = 39 \).

6. \( 2013^{39} \) ends in 7; \( 39^{2013} \) ends in 9. The final answer is 7 + 9 = 16 \( \rightarrow 6 \).

Relay 4:

1. Let the distance be \( 2d \). Then \( \frac{2d}{40} - 1 = \frac{d}{40} + \frac{d}{50} \). \( \frac{d}{40} - 1 = \frac{d}{50} \).

\( 50d - 2000 = 40d, d = 200 \). The answer is 400.

2. If she has \( n \) children, then \( n \) is a factor of both 400 + 7 = 407 and \( \sqrt{400} + 2 = 22. \) \( n \) can be either 1 or 11, but she wouldn’t have problems distributing with only one child.

3. Can consider this by subtraction: the inner \( 9 \times 9 \times 9 \) volume is not visible and the 8 corners have too many, so \( 11^3 - 9^3 - 8 = 594 \); or by addition: six \( 9 \times 9 \) center cubes and twelve \( 9 \) edge cubes, so \( 6 \times 9 \times 9 + 12 \times 9 = 594 \).

4. The smallest multiple of 100 greater than 594 is 600. So \( u = 600 - 594 = 6 \).
5. Solve and find that the intersections are \((6, 0), (-2, 0),\) and \((-3, 1.5)\). The area is 
\[
8 \cdot 1.5 / 2 = 6
\]

6. \[
\binom{39}{5} \binom{13}{1} : \binom{39}{6} = 13 : \frac{34}{6} = 13 : 5.6. 
\]
The answer is \(56\).

Relay 5:

1. \(425 = 20^2 + 5^2 = 19^2 + 8^2 = 16^2 + 13^2\). So the circle goes through 24 lattice points: 
\((\pm 20, \pm 5), (\pm 19, \pm 8), (\pm 16, \pm 3), (\pm 5, \pm 20), (\pm 8, \pm 19), (\pm 3, \pm 16)\). Answer is \(12\).

2. The volume we’re looking for can be considered as a large cube plus a small pyramid minus a large pyramid. So volume 
\[
= T^3 + \left(\frac{T}{2}\right)^2 \cdot \frac{1}{3} - T^2 \cdot \frac{1}{3} = \frac{5}{12} T^3.
\]
Substitute \(T = 12\), the answer is \(720\).

3. \(x = \sqrt[3]{T + \sqrt[3]{T + \sqrt[3]{T + \cdots}}}\), then \(x = \sqrt[3]{720 + x}\). \(x = 9\).

4. The probability is \(3/36 + 2/36 + 1/36 = 1/6\). The reciprocal is \(6\).

5. The g.c.d. of 12 and 720 is 12; the l.c.m. of 9 and 6 is 18. \(\sqrt{|12 - 18| + 3} = 3\).

6. \(f(x, y) = Ex^2 + Dxy + Cy^2 + Ax + B = 3x^2 + 6xy + 9y^2 + 12x + 720 = (x + 3y)^2 + 2(x + 3)^2 + 702 \geq 702\), which is when \(x = -3\) and \(y = 1\).