This test consists of ten extended problems to be solved by your entire school in one week. Answers must be exact, complete, simplified, and written in the appropriate blanks on the answer sheet. Each problem on this test is worth 10 points. If a problem consists of multiple parts, the score for the problem shall be $S = \left\lfloor 10\left(\frac{c}{t}\right)\right\rfloor$, where *c* is the number of parts the team gets correct, *t* is the total number of parts, and $\lfloor x \rfloor$ is the greatest integer less than or equal to *x*.

1. In an array of unit squares, there are many pairs of adjacent squares (sharing a side). One of these pairs is chosen randomly, and then another square is chosen randomly from those adjacent to one of those two squares. What is the probability that the three squares are in the same row or column if the grid measures:

a. 2x2 b. 2x3 c. 3x3 d. 3x4 e. 4x4 f. 4x5 g. 5x5 h. 5x∞ i. ∞×∞

2. In how many distinguishable ways can the letters in the "words" below be arranged so that no letter appears to be in the same location as it originally was?

a. ABCDEFG b. ABCDEFA c. ABCDEAA d. ABCDEAB e. ABCDABC f. ABCDAAB g. ABCAABB

3. In Hasbro's board game, Battleship:

a. If you play to minimize the maximum number of turns it could take you to eliminate your opponent's fleet, what is the largest number of turns this can take? Assume you are not defeated.

b. "Everyone knows" that you shouldn't have your ships next to each other, not even diagonally. What is the smallest area of a rectangular region of the board that can contain all of your ships subject to this constraint?

c. How many arrangements of the ships are possible within such a smallest-area rectangle? Do not count reflections or rotations of an arrangement as different.

d. Using the same constraints as in b, what is the smallest *perimeter* of such a rectangle?

e. How many arrangements of the ships are possible within such a smallest-perimeter rectangle? Do not count reflections or rotations of an arrangement as different.

4. In the grid to the right, cells can be shaded to form a continuous path that forms no loops and never contains a 2x2 square so that: the cells labeled “e” border (including themselves and diagonally) an even number of shaded cells, and the cells labeled “o” border an odd number of shaded cells. For each part (a-k), what is the maximum possible number of shaded cells surrounding each boldly-outlined cell? a is in the upper left, b is to its right, c is in the next row, proceeding in reading-order to k in the lower right. The bold outlines have no bearing on the shading, and are only used when calculating answers.

e	e	o	e	o	e	e	o
o	**e**	o	e	e	**e**	e	o
o	o	o	**o**	o	e	e	e
e	e	o	e	e	o	**o**	o
o	e	**e**	o	**o**	o	o	e
o	o	o	e	o	o	o	**e**
o	**e**	o	o	**e**	e	o	o
o	e	e	e	e	e	**o**	e

5. a-d. List four fractions with positive numerators and denominators less than 1000 that round to .13579.

6. **Modified Ordered Collections**

a. An ordered collection of positive integers can be *augmented* by having additional numbers inserted into the collection in any position (preserving the order of the original elements). If the collections {1, 1, 2, 0, 1, 0, 2}, {1, 2, 1, 0, 1, 2}, and {1, 2, 2, 1, 1, 0, 2, 2} are each the result after a particular collection was augmented in three different ways, what are the largest collections that might have been the original collection?

b. An ordered collection of positive integers can be *depleted* by having numbers removed from any position (preserving the order of the other elements). If the collections {1, 1, 2, 0, 1, 0, 2}, {1, 2, 1, 0, 1, 2}, and {1, 2, 2, 1, 1, 0, 2, 2} are each the result after a particular collection was depleted in three different ways, what are the smallest collections that might have been the original collection?

c. An ordered collection of positive integers can be *mixed* by removing numbers from any position (preserving the order of the other elements) and inserting those numbers in new positions (preserving the order of the other elements). The collections {2, 2, 1, 2, 2, 1, 0, 1}, {2, 1, 1, 0, 2, 2, 1, 2}, and {1, 2, 2, 1, 1, 0, 2, 2} are each the result after a particular collection was mixed in three different ways. If the maximum number of total mixings needed to create these collections is a minimum, what collections might have been the original collection?

7. When you look at a five-by-five city grid of building lots from the north, the tallest buildings you see in each column of buildings are 5, 4, 4, 5, and 2 tall. When you look from the east, the tallest buildings you see in each row of buildings are 5, 5, 4, 3,and 5 tall.

a. What is the smallest possible total height of all buildings (including potential zeros for empty lots)?

b. How many arrangements are possible for this minimum total?

c. What is the *largest* possible total height?

d. How many arrangements are possible for this maximum total?

e. How many arrangements are possible if there are exactly three buildings of each height (1, 2, 3, 4, and 5)?

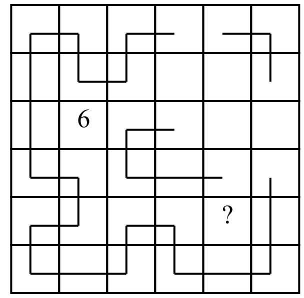
8. A positive integer is written …DCBA (if there were 26 digits, the first digit would be called Z), and has the property that the digit A is the remainder when the number is divided by 3, B is the remainder when the number is divided by 4, C is the remainder when the number is divided by 5, etc. What is the *n*th-smallest such number for each value of *n* below?

a. 1 b. 2 c. 3 d. 4 e. 5 f. 6 g. 7 h. 8 i. 9 j. 10 k. 11 l. 12

9. Digits are written repeatedly in the squares of a sheet of graph paper as shown in each part below. If you can use scissors to make any vertical or horizontal cut you like along the gridlines (you do not have to cut the entire width or height of the paper at once), what is the greatest number of distinguishable rectangular scraps of paper you can create? Assume that all numbers are distinctive enough that you cannot confuse them for one another, even when rotated. The back of the paper is blank.

a.	b.	c.	d.
12312	123412341	1264446264	0123456789012345678901234567890123456
12312	234123412	1235535535	0123456789012345678901234567890123456
12312	341234123	1634462644	0123456789012345678901234567890123456
12312	412341234	3254531113	0123456789012345678901234567890123456
12312	123412341	5234562262	0123456789012345678901234567890123456
	234123412	3614361135	0123456789012345678901234567890123456
	341234123	1216521264	0123456789012345678901234567890123456
	412341234	1212561235	0123456789012345678901234567890123456
	123412341	1636341634	0123456789012345678901234567890123456
		3254543254	… (14 more rows, for a total of 23) …

10. Each of the grids below can be filled with a closed path of line segments connecting the centers of the cells horizontally or vertically and not branching at all. The path will pass through all cells except those containing numbers or ?'s, and the cells (up to 8) around each cell with a number will contain that number of turns (e.g. a 6 will have 6 turns in the cells surrounding it). For each grid, what is the maximum number of turns in a solution? The example grid to the right has the parts of the path that can be deduced for certain, and there are four possible ways the path could be completed, with the maximum number of turns being 24.



a.

							3
	5			5			
				3	4		
	6						
						5	
		5					

b.

				4					
								6	
	7								
				5					
									?
		8					7		
					6				

c. In this part, none of the ?'s is a 5.

			?								
5				5				8			5
				5							
		?				5		5			
?											
			5				?		?		?
	?										
								?			