Shrtcts Specialty Test #801

1. Write your 6-digit ID# in the I.D. NUMBER grid, left-justified, and bubble. Check that each column has only one number darkened.
2. In the EXAM NO. grid, write the 3-digit Test # on this test cover and bubble.
3. In the Name blank, print your name; in the Subject blank, print the name of the test; in the Date blank, print your school name (no abbreviations).
4. Scoring for this test is 5 times the number correct + the number omitted.
5. You may not sit adjacent to anyone from your school.
6. TURN OFF ALL CELL PHONES OR OTHER PORTABLE ELECTRONIC DEVICES NOW.
7. No calculators may be used on this test.
8. Any inappropriate behavior or any form of cheating will lead to a ban of the student and/or school from future national conventions, disqualification of the student and/or school from this convention, at the discretion of the Mu Alpha Theta Governing Council.
9. If a student believes a test item is defective, select "(E) NOTA" and file a Dispute Form explaining why.
10. If a problem has multiple correct answers, any of those answers will be counted as correct. Do not select "(E) NOTA" in that instance.
11. Unless a question asks for an approximation or a rounded answer, give the exact answer.

Note: For all questions, answer "(E) NOTA" means none of the above answers is correct.

1. Compute $116 \cdot 124$.

   (A) 14364 (B) 14384 (C) 14744 (D) 14904 (E) NOTA

2. If the measure of an interior angle of a regular polygon is 17 times larger than the measure of its exterior angle, how many sides does this polygon have?

   (A) 18 (B) 19 (C) 35 (D)36 (E) NOTA

3. Suppose a right triangle has integer-valued sides of length 11, x, and y, where $y > x > 11$. What is the product of the digits of the hypotenuse of the triangle?

   (A) 0 (B) 6 (C) 18 (D) 21 (E) NOTA

4. Compute: $\sum_{k=1}^{2013} \frac{1}{k(k+1)}$

   (A) $\frac{2012}{2013}$ (B) $\frac{2013}{2014}$ (C) $\frac{2014}{2013}$ (D) $\frac{2013}{2012}$ (E) NOTA

5. What is the sum of the digits of the largest prime factor of 9999?

   (A) 2 (B) 4 (C) 6 (D) 8 (E) NOTA

6. I pick a random integer between 1 and 15 (inclusive) and you do the same (assume that we choose numbers independently, and all numbers are chosen with equal probability). We then compare numbers. Find the probability that my number is higher than yours.

   (A) $\frac{1}{15}$ (B) $\frac{112}{225}$ (C) $\frac{1}{2}$ (D) $\frac{8}{15}$ (E) NOTA

7. Find the last two digits of the sum 1+2+3+…+2013.

   (A) 56 (B) 78 (C) 82 (D) 91 (E) NOTA

8. Find the product of the LCM and GCD of 84 and 40.

   (A) 1680 (B) 3360 (C) 4200 (D) 5040 (E) NOTA

9. What is the sum of the digits of $111{,}111{,}111^2$?

   (A) 9 (B) 36 (C) 63 (D) 81 (E) NOTA

10. What is the sum of the reciprocals of the positive integer factors of 30?

(A) $\frac{7}{5}$ (B) 2 (C) $\frac{12}{5}$ (D) $\frac{71}{30}$ (E) NOTA

Questions 11 and 12 deal with rolling a dodecahedral (12-sided) die.

11. If I roll this die four times and sum the numbers appearing on each roll, which sum occurs with maximal probability?

(A) 25 (B) 26 (C) 27 (D) 28 (E) NOTA

12. If I roll this die twice, find the probability that the sum is greater than 5.

(A) $\frac{8}{9}$ (B) $\frac{67}{72}$ (C) $\frac{15}{16}$ (D) $\frac{23}{24}$ (E) NOTA

13. Let $P(x) = x^3 + ax^2 + bx + 1$ and let $Q(x) = x^3 + cx^2 + dx + 1$ such that whenever x is larger than 4, P(x) > Q(x); and P(4) = Q(4). Which of the following cannot be true?

(A) $|a| > |b|$ (B) $a + b = c + d$ (C) $|a| < |b|$ (D) $a = 0$ (E) NOTA

14. Let L_1 be the line $y = 2$, let L_2 be the line $y = 6$, and let L_3 be the line $y = 2x$. Let L_4 be the line $y = \alpha x + \beta$, with $\alpha < 0$ such that L_4 passes through the point where L_2 and L_3 intersect. Find a formula for the area of the triangle bounded by L_1, L_3, and L_4.

(A) $\frac{\alpha-2}{\alpha}$ (B) $\frac{2\alpha-4}{\alpha}$ (C) $\frac{4\alpha-8}{\alpha}$ (D) $\frac{8\alpha-16}{\alpha}$ (E) NOTA

15. Find the sum of all integers n such that $\frac{n^2+16}{n-4}$ is an integer and $n > 4$.

(A) 55 (B) 63 (C) 87 (D) 93 (E) NOTA

16. I'm thinking of a polynomial of degree 5. How many points on the Cartesian graph of this polynomial do you need in order to figure out the exact polynomial?

(A) 3 (B) 4 (C) 5 (D) 6 (E) NOTA

17. Let f be an even function and let g and h be odd functions. Which of the following functions are even? (The domain of all functions is the real numbers).

I. $f\left(g\left(f\left(f^{-1}(x)\right)\right)\right)$ II. $g\left(f\left(h(x) + g(x)\right)\right)$

III. $g\left(h^{-1}\left(f(x)\right)\right)$ IV. $g\left(f\left(g^{-1}\left(h(x) + g(x)\right)\right)\right) + f\left(h^{-1}(x)\right)$

(A) I, II only (B) III, IV only (C) II, III, IV only (D) I, II, III, IV (E) NOTA

18. Consider a rectangle of width 12 and height 6, with a circle of radius 12 centered at one of the vertices of the rectangle. Let the area of the region inside the rectangle but outside the circle be $A + B\sqrt{C} + D\pi$, where *A, B, C*, and *D* are integers and the radical is in simplest form. Find $\frac{AC^2}{BD}$.

(A) 3 (B) 8 (C) 12 (D) 16 (E) NOTA

19. Consider concentric circles of radii 1, 2, 3, … *n*. The innermost circle and each annulus is painted one of red, white, or blue. What is the smallest *n* for which it is possible for the sum of the areas of the red regions, the sum of the areas of the white regions, and the sum of the areas of the blue regions to all be equal?

(A) 6 (B) 9 (C) 12 (D) 15 (E) NOTA

20. Find the sum of the next two terms in the sequence: 1, 7, 19, 37, 61, …

(A) 182 (B) 194 (C) 218 (D) 244 (E) NOTA

21. Evaluate: $\left(\sqrt{5}+\sqrt{6}+\sqrt{7}\right)\left(-\sqrt{5}+\sqrt{6}+\sqrt{7}\right)\left(\sqrt{5}-\sqrt{6}+\sqrt{7}\right)\left(\sqrt{5}+\sqrt{6}-\sqrt{7}\right)$

(A)64 (B) 88 (C) 104 (D) 120 (E) NOTA

22. Let $N = 2^7 3^4$. How many positive integers less than *N* are factors of N^2 but not of *N*?

(A) 27 (B) 28 (C) 53 (D) 54 (E) NOTA

23. Let *N* be the smallest positive integer multiple of 15 such that all of *N*'s digits are either 4 or 0. Find the greatest integer less than $\frac{N}{50}$.

(A) 80 (B) 88 (C) 133 (D) 177 (E) NOTA

24. In the expansion of $(ax + b)^{1337}$, the coefficients on the x^2 and x^3terms are equal. If a and b are positive integers with $\text{GCD}(a, b) = 1$, find a+b.

(A) 372 (B) 446 (C) 842 (D) 1336 (E) NOTA

25. A circle is inscribed inside a square of radius 2. A line segment is drawn connecting one vertex of the square with the midpoint of an opposing side. This line segment is divided into two parts at a point in the interior of the square where it intersects with the circle. Find the length of the longer part (that is, the part of the line segment that serves as a chord of the circle).

(A)$\frac{3}{\sqrt{5}}$ (B) $\frac{7}{2\sqrt{5}}$ (C) $\frac{4}{\sqrt{5}}$ (D) $\frac{9}{2\sqrt{5}}$ (E) NOTA

26. Let $a_1 = \frac{1}{1-x}$ and for $n \geq 2$ let $a_n = \frac{1}{1-a_{n-1}}$. Find a_{2013} when $x = 2013$.

(A) $-\frac{1}{2012}$ (B) $\frac{1}{2013}$ (C) $\frac{2012}{2013}$ (D) 2013 (E) NOTA

27. If $\log_8(\log_2 x) = \log_2(\log_8 x)$, compute $\log_2 x$.

(A) $\sqrt{2}$ (B) 3 (C) $3\sqrt{3}$ (D) $4\sqrt{2}$ (E) NOTA

28. Suppose x and y are integers with $0 < x < y < 1000$. If the arithmetic mean of x and y is eight more than the geometric mean, how many possible values could y take on?

(A) 28 (B) 29 (C) 30 (D) 31 (E) NOTA

29. How many 9s are in the product 1234567×9999999 if you multiply it out?

(A) 0 (B) 1 (C) 7 (D) 13 (E) NOTA

30. Find the product of the values of x which satisfy the following equation:

$$\frac{1}{x^2 - 7x - 6} + \frac{1}{x^2 - 7x - 14} - \frac{2}{x^2 - 7x - 18} = 0$$

(A) -6 (B) -10 (C) -12 (D) -16 (E) NOTA