Round 1

Question 1
If today is March 1, 2014, how many days remain until February 1, 2015?

Answer: _________________________

Question 2
Given that
\[ \log_2 (\log_3 (\log_4 x)) = \log_3 (\log_2 (\log_4 y)) = \log_4 (\log_3 (\log_2 z)) = 0, \text{ find } x + y + z. \]

Answer: _________________________

Question 3
From a group of boys and girls, 15 girls leave. There are then two boys left for each girl. After this, 45 boys leave. There are then five girls for each boy. How many girls were there in the beginning?

Answer: _________________________

Question 4
The position of a particle moving along the x-axis is given by \[ x(t) = 3t^3 - 2t^2 + t - 1. \] What is the total distance traveled by the particle from \( t = 0 \) to \( t = 3 \)?

Answer: _________________________

Question 5
Of 6,000 apples harvested, every third apple is too small, every fourth apple is too green, and every tenth apple is bruised. The remaining apples are perfect. How many perfect apples are harvested?

Answer: _________________________

Question 6
The three, two-digit integers 30, 72, and \( N \) have the property that the product of any two of them is divisible by the third. What is the value of \( N \)?

Answer: _________________________

Question 7
How many times do the graphs of the parabolas \( y = 3x^2 + 5x + 2 \) and \( y = x^2 - 2x - 8 \) intersect?

Answer: _________________________

Question 8
Evaluate:
\[
\lim_{q \to 3} \frac{(3 + q)^2 - 3(3 + q) - (3^2 - 3 \cdot 3)}{q}
\]

Answer: _________________________

What is the sum of the answers for Questions 1-8?

**Sum = _________________________**
Round 2

**Question 1**
Evaluate: $245^2 - 235^2$

**Answer:** ________________

**Question 2**
For how many positive integer values of $n$ are both $\frac{n}{5}$ and $5n$ equal to positive four-digit integers?

**Answer:** ________________

**Question 3**
How many of the following series diverge?

i) $\sum_{n=1}^{\infty} \frac{1}{n}$

ii) $\sum_{n=2}^{\infty} \frac{n}{3n+4}$

iii) $\sum_{n=2}^{\infty} \left( \frac{1}{3} \right)^n$

iv) $\sum_{n=4}^{\infty} \frac{1}{n \ln n}$

**Answer:** ________________

**Question 4**
Find the sum of the first fifteen terms of the arithmetic sequence: $-4, 1, 6, 11, ...$

**Answer:** ________________

**Question 5**
In how many quadrants do points that satisfy $-3y - x < 5$ and $-3x < -4$ lie?

**Answer:** ________________

**Question 6**
How many positive three-digit numbers are perfect numbers, perfect squares, or perfect cubes?

**Answer:** ________________

**Question 7**
A dodecahedron has twelve pentagonal faces. A *face diagonal* is a line segment that lies along a face of a polyhedron and connects two nonadjacent vertices. How many face diagonals does a regular dodecahedron have?

**Answer:** ________________

**Question 8**
How many positive three-digit integers do not have 3 as a digit?

**Answer:** ________________

What is the sum of the answers for Questions 1-8?

**Sum=** ________________
Round 3

Question 1
Evaluate: $103^2 - 97^2 + 13^2 + 39^2$

Answer: ________________

Question 5
Evaluate $\int_0^{15} \lfloor x \rfloor \, dx$, where $\lfloor x \rfloor$ represents the greatest integer less than or equal to $x$.

Answer: ________________

Question 2
How many solutions $x$ to $2 \sin^2 x - \sin x = 1$ exist in the interval $x \in (0, 2\pi)$?

Answer: ________________

Question 6
What is the area of the region $|x| + |y| \leq 5$ in the Cartesian Plane?

Answer: ________________

Question 3
Given that $t$ and $g$ are differentiable functions, and that $t(3) = 7$, $t'(3) = 2$, $t(6) = 9$, $t'(6) = 3$, $g(2) = 5$, and $g'(2) = 4$; if $C(x) = t(3x)g(x)$, determine the value of $C'(2)$.

Answer: ________________

Question 7
What is the multiplicative inverse of 14 in modulo 41?

Answer: ________________

Question 4
Victor takes a test where he receives five points for a correct answer and loses three points for an incorrect answer. Assuming Victor answers 20 questions on the test, how many questions did he answer correctly if he earned 52 points?

Answer: ________________

Question 8
If the base 10 number 365,88A is a multiple of 11, what is the value of the digit A?

Answer: ________________

What is the sum of the answers for Questions 1-8?

Sum= ________________
Round 4

**Question 1**
How many solutions $x$ to $2 \sin^2 x + 3 \cos x = 3$ exist in the interval $x \in (0, 2\pi)$?

**Answer:**

**Question 5**
What is the largest integer value of $x$ such that $100!$ is divisible by $2^x$?

**Answer:**

**Question 2**
If the graph of $y = f(x) = -x^2 + 2x + q$ has a maximum ordinate value of 6 at $x = 1$, find the value of $f(2)$.

**Answer:**

**Question 6**
Express the following difference as a base 10 number:

$412_7 - 136_7$

**Answer:**

**Question 3**
Which Greek mathematician developed a fairly accurate estimate for the circumference of the earth and a "sieve" for determining prime numbers? Answer with the number 1, 2, 3, or 4, corresponding to the correct answer.

1) Archimedes
2) Aristarchus
3) Eratosthenes
4) Euclid

**Answer:**

**Question 7**
The number 210 can be written as a sum of consecutive positive integers in several ways. When written as the sum of the greatest possible number of consecutive positive integers, what is the largest of these integers?

**Answer:**

**Question 4**
In the expansion of $(2x - y)^{10}$ with like-terms combined, find the coefficient of the $x^2y^8$ term.

**Answer:**

**Question 8**
Evaluate: $\log_4(16^{500})$

**Answer:**

What is the sum of the answers for Questions 1-8?

**Sum=**
Round 5

Question 1
If $A$ is a $5 \times 5$ matrix with a determinant of 6, what is the determinant of the matrix $5A$?

Answer: _________________________

Question 2
Find the greatest common factor of 20! and 200000.

Answer: _________________________

Question 3
If $a + b = 4$ and $ab = 7$, evaluate: $a^2 + b^2$

Answer: _________________________

Question 4
If the cost $C$ of producing $x$ widgets is $C = 40\sqrt{x} + \frac{x^2}{400}$, how many widgets need to be produced to minimize the cost per widget. Assume that a fractional number of widgets is allowed.

Answer: _________________________

Question 5
How many integers $x$ satisfy the following inequality?

$\left| 3 - \frac{2x}{3} \right| < 20$

Answer: _________________________

Question 6
How many of the following series converge?

$\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$, $\sum_{n=1}^{\infty} \left( n + 1 \right) \left( \frac{2}{3} \right)^n$, $\sum_{n=1}^{\infty} \frac{1}{n^2 + n + 3}$

$\sum_{n=1}^{\infty} \frac{7^n}{3n+1}$, $\sum_{n=1}^{\infty} \frac{n}{\sqrt[3]{4n-3}}$

Answer: _________________________

Question 7
Evaluate: $(\log_2 25)(\log_5 8)(\log_3 49)(\log_7 243)$

Answer: _________________________

Question 8
Evaluate $\sum_{n=1}^{144} \frac{\sqrt{n}}{2}$ and round your answer to the nearest integer.

Answer: _________________________

What is the sum of the answers for Questions 1-8?

Sum= _________________________
Round 6

**Question 1**
If the value of the definite integral
\[
\int_2^3 \frac{x^4 - 2x^3 + 3x^2 - 2x + 1}{(x^3 - 3x + 1)^2} \, dx
\]
can be written as \(m/n\), where \(m\) and \(n\) are relatively prime positive integers, find \(m + n\).

**Answer:**

**Question 2**
For how many integer values of \(x\) does \(x^2 + 2x - 19\) have a negative value?

**Answer:**

**Question 3**
Solve for \(x\):
\[
2^{4x+8} \cdot 4^{2x+3} = 8^{2x+6}
\]

**Answer:**

**Question 4**
Evaluate:
\[
\begin{pmatrix}
3 & 1 & 0 & 2 \\
4 & -1 & -2 & 1 \\
2 & 2 & -2 & 1 \\
2 & 3 & 6 & 0
\end{pmatrix}
\]

**Answer:**

**Question 5**
If six fair, two-sided coins are flipped, the probability of obtaining more than two heads is \(m/n\), where \(m\) and \(n\) are relatively prime positive integers. Find \(m + n\).

**Answer:**

**Question 6**
If \(\frac{dy}{dx} = 3x^2\) and \(y(-1) = 2\), then find the value of
\[
\int_0^2 y(x) \, dx.
\]

**Answer:**

**Question 7**
How many ways can six indistinguishable pieces of candy be distributed among three children, provided there is no requirement that each child receive at least one piece of candy?

**Answer:**

**Question 8**
Find the sum of all positive, two-digit integers that are multiples of 4.

**Answer:**

What is the sum of the answers for Questions 1-8?

**Sum:**


Round 7

Question 1
A triangle with vertices at (1,4), (2,7), and (3,−1) is subjected to a linear transformation represented by the matrix \( \begin{pmatrix} 3 & 1 \\ 2 & 2 \end{pmatrix} \), resulting in a new triangle. What is the area of this new triangle?

Answer: _________________________

Question 5
What number, when added to the numerator and denominator of \( \frac{5}{8} \), results in a fraction whose value is equal to 0.40?

Answer: _________________________

Question 2
If \( \int_{0}^{4} (x^2 - 6x + 9)\,dx \) is approximated by four inscribed rectangles of equal width along the \( x \)-axis, what is the value of the approximation?

Answer: _________________________

Question 6
Find the remainder when \( 41^{100} \) is divided by 29.

Answer: _________________________

Question 3
If \( \log_{16} 2 = \frac{1}{4} \), find the value of \( \log_{8} 4096 \).

Answer: _________________________

Question 7
The Fibonacci Sequence \( F \) is defined by \( F_1 = F_2 = 1 \) and for integers \( n \), \( F_{n+2} = F_{n+1} + F_n \). Find the value of \( F_{-11} + F_6 \).

Answer: _________________________

Question 4
When listing out the digits of \( \lfloor 10^{2013} \pi \rfloor \) from left to right, which of the digits 0 to 9, inclusive, is the last one to make its first appearance?

Answer: _________________________

Question 8
Find the maximum value of the function \( f(x) = 2x^3 - 9x^2 + 12x - 1 \) on the interval \( x \in [-1,2] \).

Answer: _________________________

What is the sum of the answers for Questions 1-8?

Sum= _________________________
Round 8

Question 1
In how many ways can 36 be written as a product of the form $abcd$, where $a$, $b$, $c$, and $d$ are positive integers such that $a \leq b \leq c \leq d$?

Answer: _________________________

Question 2
The first three terms of an arithmetic sequence are $x + 3$, $3x - 1$, and $7x - 3$, in that order. What is the numerical value of the product of the first three terms of the sequence?

Answer: _________________________

Question 3
Evaluate:

$$\sum_{n=1}^{10} n^3$$

Answer: _________________________

Question 4
A ball was floating in a lake when the lake froze. The ball was removed without breaking the ice, leaving a hole 24 centimeters across the top and 8 centimeters deep. What was the radius of the ball? Express your answer in centimeters.

Answer: _________________________

Question 5
The arclength of the graph of $y = x^{3/2}$ on the interval $x \in \left[0, \frac{4}{3}\right]$ is equal to $\frac{m}{n}$, where $m$ and $n$ are relatively prime positive integers. Find $m + n$.

Answer: _________________________

Question 6
In the following system, find $x - y - z$, given that $x$, $y$, and $z$ are rational numbers.

\[
\begin{align*}
2x^3y^5z &= 7500 \\
3x^5y^2z &= 720 \\
5x^2y^3z &= 4050
\end{align*}
\]

Answer: _________________________

Question 7
For how many positive integers $n$ is $n! - 1$ divisible by $n$?

Answer: _________________________

Question 8
A cube is made up of 27 fair, six-sided dice. Each die’s opposite sides add up to 7. What is the smallest possible sum of all the values visible on the six faces of the large cube?

Answer: _________________________

What is the sum of the answers for Questions 1-8?

Sum= _________________________
Round 9

**Question 1**
Two tangent lines can be drawn from the point \((-2, -2)\) to the graph of \(y = f(x) = x^2 + 2x + 3\). Find the sum of the x-coordinates of the points where those lines are tangent to the graph of \(y = f(x)\).

**Answer:** ________________

**Question 5**
The repeating decimal \(0.475\) is equal to \(m/n\), where \(m\) and \(n\) are relatively prime positive integers. Find \(m + n\).

**Answer:** ________________

**Question 2**
What is the smallest positive integer that can be expressed as the sum of two positive perfect cubes in exactly two distinct ways?

**Answer:** ________________

**Question 6**
Evaluate: \(998^2 - 999^2\)

**Answer:** ________________

**Question 3**
In triangle \(ABC\), \(\tan A = 5/12\) and \(\sec B = 5/3\). If \(\sin C = m/n\), where \(m\) and \(n\) are relatively prime positive integers, find \(m + n\).

**Answer:** ________________

**Question 7**
If Kevin invests \(52000\) at a 4% annual interest rate compounded continuously, how many years will it take for Kevin’s investment to triple? Express your answer to the nearest year.

**Answer:** ________________

**Question 4**
A sequence of functions is defined as \(f_1(x) = x - 1\) and \(f_n(x) = 2f_{n-1}(x)\) for \(n > 1\). Find \(f_{10}(2)\).

**Answer:** ________________

**Question 8**
How many of the following series diverge?

\[
\sum_{n=1}^{\infty} \frac{2}{3^n}, \quad \sum_{n=1}^{\infty} \left(\frac{5}{3}\right)^n, \quad \sum_{n=1}^{\infty} \frac{2}{3n}, \quad \sum_{n=1}^{\infty} \frac{2}{n^3}, \quad \sum_{n=1}^{\infty} \frac{2n}{3^n}
\]

**Answer:** ________________

What is the sum of the answers for Questions 1-8?

**Sum:** ________________
**Question 1**
The ellipse \( x^2 + 4y^2 - 4x + 40y = 152 \) has major and minor axes lengths of \( R_1 \) and \( R_2 \), respectively. Find \( R_1 + R_2 \).

**Question 5**
Let \( x \) equal the sum of all two-digit positive prime numbers. Which of the following from 1 to 5 is correct regarding \( x \)? Answer with the number that corresponds to the correct answer.

1. \( x \) is a prime number.
2. \( x \) is a deficient number
3. \( x \) is an abundant number.
4. \( x \) is a perfect number.
5. None of 1 to 4.

**Answer:** ________________

**Question 2**
Let \( i = \sqrt{-1} \). Given that \((2i + 7)^3 = a + bi\), where \( a \) and \( b \) are real numbers, find the ratio of \( a \) to \( b \). Express your answer to the nearest integer.

**Answer:** ________________

**Question 3**
For real number \( x \), the expression

\[ L = \log_5(\log_4(\log_3(\log_2 x))) \]

is a well-defined real number when \( x > c \); otherwise \( L \) is not a well-defined real number. Find the value of \( c \).

**Answer:** ________________

**Question 4**
The area of the region in the first quadrant bounded by the graphs of \( y = \sqrt{x} \) and \( y = x^2 \) is equal to \( m/n \), where \( m \) and \( n \) are relatively prime positive integers. Find \( n^2 - m^2 \).

**Answer:** ________________

**Question 6**
A woman born in the first half of the nineteenth century was \( x \) years old in the year \( x^2 \). In what year was she born?

**Answer:** ________________

**Question 7**
A tire with a radius of length two feet travels one mile. How many complete revolutions does the tire make?

**Answer:** ________________

**Question 8**
The lines \( 2x - 3y = 4 \) and \( 4x + ky = 13 \) are parallel. Find the value of \( k \).

**Answer:** ________________

What is the sum of the answers for Questions 1-8?  

**Sum:** ________________