Note: For all questions, answer "(E) NOTA" means none of the above answers is correct.

1. A leaf at $(1, 0)$ experiencing a wind is moving in the xy-plane such that $\frac{dy}{dx} = 10(x + y)$. Use Euler's method with a step size of 0.01 to estimate the y-coordinate where it crosses $x = 1.03$.

   (A) .2351 (B) .3241 (C) .3351 (D) .3441 (E) NOTA

2. A student wants to enclose a rectangular area with a string of length 60cm using the wall in his room as one side. What is the largest possible area of that rectangle, in cm^2?

   (A) 400 (B) 420 (C) 432 (D) 450 (E) NOTA

3. The number of yeasts in a culture is growing at a rate of $1200e^{0.75t}$, where t represents time in seconds. Starting with 4800 bacteria (at $t = 0$), how long will it take for the number to double? Express your answer in seconds.

   (A) $\ln\frac{16}{3}$ (B) $\frac{4}{3}\ln 4$ (C) $\ln\frac{20}{3}$ (D) $\frac{4}{3}\ln 5$ (E) NOTA

4. Scientists have found an invincible bacteria species with a reproduction rate that's proportional to the population squared ($\frac{dP}{dt} = 0.001P^2$). If a lab scientist has isolated 100 such bacteria at $t = 0$, how many of these bacteria will he see at $t = 5$?

   (A) 200 (B) 250 (C) 300 (D) 500 (E) NOTA

5. The linear density of a thin rod that is 20 inches long varies with the distance r, in inches, from one end E, and can be modeled with the function $\rho(r) = r^2/400$, in lbs per inches. Find the distance, in inches, of the center of mass from E.

   (A) 12 (B) 15 (C) 16 (D) 18 (E) NOTA

6. (continuation of #5.) In physics, moment of inertia of an object is found with the expression $I = \int r^2 dm$, an integral over mass that's a distance r away from a fixed point. Calculate I, in $\text{lbs}\cdot\text{in}^2$, for the thin rod in Problem 5 with E as the fixed point. (for a thin rod, $dm = \rho(r)dr$, the ratio of linear mass dm to an infinitesimal length dr is $\rho(r)$)

   (A) $\frac{3200}{3}$ (B) $\frac{6400}{5}$ (C) 1600 (D) $\frac{6400}{3}$ (E) NOTA

7. What is the length of the curve $y = \ln(\sec x)$ between $(0, 0)$ and $\left(\frac{\pi}{4}, \ln\sqrt{2}\right)$?

   (A) $\ln(1 + \sqrt{2})$ (B) $\ln(2 + \sqrt{2})$ (C) $1 + \ln 2$ (D) $2 + \ln\sqrt{2}$ (E) NOTA

8. The region in the first quadrant bounded by the coordinate axes and the parabola $y = 4 - 2x^2$ is revolved about the y-axis to form a bullet. Find its volume.

(A) $\frac{8}{3}\pi$ (B) $2\sqrt{2}\pi$ (C) 4π (D) $\frac{16}{3}\pi$ (E) NOTA

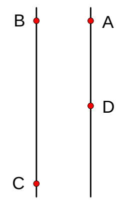
9. A car at the origin initially at rest is moving in a straight path. If its acceleration $a(t) = \begin{cases} t, & 0 \le t < 3 \\ 3, & t \ge 3 \end{cases}$ meters/sec^2, what is the average velocity, in meter per second, in the first 5 seconds?

(A) 3.9 (B) 4.5 (C) 19.5 (D) 22.5 (E) NOTA

10. A substance decaying exponentially has a half-life of 200 years. In 2000, the amount of the substance in a rock is 1 gram. In what decade approximately will there be only 0.01 grams remaining in the rock? Use the following approximations, and ***NOT*** more precise values: $\ln 2 \approx 0.7$; $\ln 10 \approx 2.3$.

(A) 3310s (B) 3330s (C) 3350s (D) 3370s (E) NOTA

11. A river is 1 mile wide ($\overline{AB} = 1$). A cable from a power plant at point A is installed along the river to point D then across the river to a factory at point C, which is 5 miles away from B. The price to install the cable on land is \$30/mile, and across the river is \$50/mile. What is $\overline{AD}$, in miles, that would yield the lowest cost?



(A) 11/3 (B) 4 (C) 17/4 (D) 4.5 (E) NOTA

12. The cost for Firm A to produce w_1 watches is $C(w_1) = {w_1}^2 + 100$. The selling price of a watch when there are w_2 watches in the market is $P(w_2) = 60 - w_2$. Suppose A is the only watch-making firm; how many watches should it produce to get the maximum profit possible?

(A) 10 (B) 12 (C) 15 (D) 16 (E) NOTA

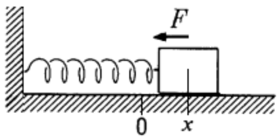
13. (Continuation of Problem #12.) Suppose another firm with the same cost function wants to enter the market, so now there are two firms competing. The market price function remains unchanged. What is the profit-maximizing output for each firm? And what is the market output (the total output)?

(A) 8, 16 (B) 10, 20 (C) 12, 24 (D) 14, 28 (E) NOTA

14. A force F acting on a particle is given by $F = xe^{3x}$. What is the work done on the particle from $x = 1$ to $x = 4$?

(A) $\frac{1}{3}e^{12} - \frac{1}{9}e^3$ (B) $\frac{7}{6}e^{12} - \frac{1}{6}e^3$ (C) $\frac{11}{9}e^{12} - \frac{2}{9}e^3$ (D) $\frac{11}{3}e^{12} - \frac{2}{3}e^3$ (E) NOTA

15. Force F acting on an object by a spring is kx towards its equilibrium position, k known as the spring constant. In addition, Newton's Law says $F = ma$, the product of mass and acceleration. If $k = 4m = 4$, initial position $x_0 = 5$, initial velocity $v_0 = 0$, where is the object at $t = 4$?



(A) $5\cos 4$ (B) $5\sin 4$ (C) $5\cos 8$ (D) $5\sin 8$ (E) NOTA

16. Allen has a job that pays \$8/hr. He divides his time between work and leisure, and spends all his income on consumption. His utility function (level of satisfaction) is $U(L, C) = L \cdot C$, where L and C are leisure time and consumption, respectively. How many hours should he work each day to maximize U?

(A) 8 (B) 9 (C) 10 (D) 11 (E) NOTA

17. At a robotics competition, robots have to complete a number of tasks, one of which is to go through a tunnel. The tunnel is the curve $z = 9 - x^2$, and let the ground level be the xy-plane. A team decides to make its robot in the shape of a trapezoidal prism with a height of 6. What is the largest possible volume of this robot?

(A) 150 (B)168 (C) 192 (D) 216 (E) NOTA

18. A young driver wants to assess her risk on the road. If driving at a speed of $10s$ mi/hr, she thinks the probability of a crash is $0.01s^2$, and she'd have to pay $5s^3$ dollars to repair her car. But the slower she drives, the more she spends on gasoline, which is $\$3600/s$. What is her cost-minimizing speed, in miles per hour?

(A) 45 (B) $10\sqrt[3]{120}$ (C) 50 (D) $10\sqrt[2]{120}$ (E) NOTA

19. (Continuation of Problem #18.) If she has insurance, the company would have to help her pay $5s^2$ dollars for repair. However, the company is only responsible for accidents happening between 30 and 60 mi/hr. How much does the company expect to pay, rounded to the nearest cent, for this young driver, assuming there is an equal chance of her driving at any speed in $[0, 100]$?

(A) \$7.53 (B) \$25.11 (C) \$31.50 (D) \$38.27 (E) NOTA

20. Alice has \$60 to spend on food and clothing. Her utility function $U(F, C) = 2\sqrt{F} + C$, where F and C represent the units of food and clothing, respectively. The price of food is \$4/unit and the price of clothing is \$6/unit. How many units of clothing should she buy to maximize her utility? (A fraction of a unit is allowed.)

(A) $C = \frac{22}{3}$ (B) $C = 7.5$ (C) $C = 8$ (D) $C = 9$ (E) NOTA

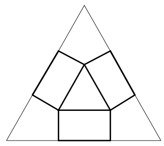
21. A regular octahedral crystal is forming naturally, retaining its octahedral shape as it grows. The side length is increasing at a constant rate of 0.25 in/min. When its volume is $9\sqrt{2}$ cubic inches, what is the rate of change of its surface area, in in^2/min?

(A) $\frac{3}{8}\sqrt{3}$ (B) $\frac{3}{2}\sqrt{3}$ (C) $2\sqrt{3}$ (D) $3\sqrt{3}$ (E) NOTA

22. A number generator returns a real value in $[0, 1]$, but unfortunately, it's not quite random. The probability that a number x is returned is proportional to $-2x^3 + 3x^2 + 6x + 1$. What is the expected value of this generator?

(A) $\frac{17}{30}$ (B) $\frac{11}{19}$ (C) $\frac{19}{30}$ (D) $\frac{61}{95}$ (E) NOTA

23. A high school senior is making a topless equilateral triangle box with clay. He's made a large equilateral triangle slab with side length of 18. He would have to cut off a kite-shaped quadrilateral from each vertex, but would also like to maximize the volume. What fraction of the original triangle should he remove?



(A) $\frac{1}{9}$ (B) $\frac{1}{4}$ (C) $\frac{1}{3}$ (D) $\frac{4}{9}$ (E) NOTA

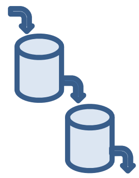
24. The natural growth of an Oryx population,$P(t)$, is proportional to the difference between $P(t)$ and M, the carrying capacity of the environment. However, tribesmen hunt a constant percentage, k, of oryx. So $\frac{dP}{dt} = r(M - P) - kP$ is the general expression that describes how the population changes over time. If $P(0) = 300, M = 1000, r = 0.8, k = 20\%$, what is $P(3)$?

(A) $-500e^{-0.6} + 800$ (B) $-500e^{-3} + 800$
(C) $-500e^{-0.6} + 1000$ (D) $-500e^{-3} + 1000$ (E) NOTA

25. A cylindrical container with base area of $2\sqrt{20}$ m^2 is filled with $200\sqrt{20}$ m^3 of water. Now with a hole at the bottom of the container, water is leaking out at a speed $v = \sqrt{2gh}$ m/sec (Torricelli's law), where g is the gravitational constant and h is the height of the water. Suppose the instantaneous change of volume of water in the container over time $\frac{dV}{dt} = -2v = -2\sqrt{2gh}$. How many seconds later will the container be empty? Use $g = 10$ m/sec^2.

(A) 10 (B) 15 (C) 20 (D) 40 (E) NOTA

26. Tank 1 initially contains 100 gallons of pure ethanol; tank 2 initially contains 100 gallons of pure water. Pure water flows into tank 1; the solution is perfectly mixed immediately then flows into tank 2, in which after another perfect mix the solution flows out. Suppose all three flow rates are 10 gallons/minute. What is the amount of ethanol in tank 2 after 5 minutes, in gallons?



(A) $25e^{-1}$ (B) $25e^{-0.5}$ (C) $50e^{-0.5}$ (D) $100e^{-0.5}$ (E) NOTA

27. Jesse needs to get from Town A to Town B which is 30 miles away by some combination of bike and taxi. He can ride a bike at 4mi/hr and the rental store charges him $\$t_b^2$, t_b being the time riding a bike. Taxi driver drives at 40mi/hr and charges $\$\left(5 + \frac{{d_t}^2}{5}\right)$, d_t being the distance taking a taxi. If Jesse wants to minimize the sum of his time and cost, how many miles should he bike?

(A) $\frac{15}{4}$ (B) $\frac{435}{41}$ (C) $\frac{157}{12}$ (D) $\frac{157}{7}$ (E) NOTA

28. A small steel ball is shot radially outward from the center of a spinning disk, and its path traces out a curve on a radar screen. If $\theta = 4t, r = e^{3t}$, find the length of the curve shown on the screen from $t = 0$ to $t = 1$.

(A) $\frac{3}{5}(e^3 - 1)$ (B) $\frac{4}{5}(e^3 - 1)$ (C) $\frac{5}{4}(e^3 - 1)$ (D) $\frac{5}{3}(e^3 - 1)$ (E) NOTA

29. Radial probability distribution (RPD) tells us the probability that an electron is a distance r away from the center of an atom. The RPD function for the 1s orbital of hydrogen is $P(r) = 4r^2 \cdot \left(\frac{1}{a_0}\right)^3 \cdot e^{-2r/a_0}$, and a_0 is a constant known as the Bohr radius. Find $\int_0^{a_0} P(r)dr$, the probability that a 1s electron of hydrogen is inside a spherical region with radius a_0.

(A) $1 - 5e^{-2}$ (B) $1 - 7e^{-2}$ (C) $1 - 9e^{-2}$ (D) $1 - 13e^{-2}$ (E) NOTA

30. Suppose $x = 1$ and $x = -1$ are the two riversides of a river. A swimmer at $(-1, 0)$ has constant velocity in the positive x-direction at 3 units/hour, while the water carries the swimmer at a speed of $9(1 - x^2)$ units/hour in the positive y-direction, where x is the x-coordinate of the swimmer's position. What is the y-coordinate of his position when he reaches the other side of the river?

(A) 3 (B) 4 (C) 5 (D) 6 (E) NOTA