MU ALPHA THETA

NATIONAL CONVENTION
SAN DIEGO 2013

Gemini

Mu

Test #911

1. In the Name blank, print the names of both members of the Gemini Team; in the Subject blank, print the name of the test; in the Date blank, print your school name (no abbreviations). If your Gemini Team consists of members from different schools, write down the names of both schools.
2. In the EXAM NO. grid, write the 3-digit Test # on this test cover and bubble.
3. In the I.D. NUMBER grid, write down the 6-digit ID# of one person in your Gemini Team, left-justified, and bubble. Check that each column has only one number darkened.
4. Scoring for this test is 5 times the number correct + the number omitted.
5. You may not sit adjacent to anyone from your school.
6. TURN OFF ALL CELL PHONES OR OTHER PORTABLE ELECTRONIC DEVICES NOW.
7. No calculators may be used on this test.
8. Any inappropriate behavior or any form of cheating will lead to a ban of the student and/or school from future national conventions, disqualification of the student and/or school from this convention, at the discretion of the Mu Alpha Theta Governing Council.
9. If a student believes a test item is defective, select “E) NOTA” and file a Dispute Form explaining why.
10. If a problem has multiple correct answers, any of those answers will be counted as correct. Do not select “E) NOTA” in that instance.
11. Unless a question asks for an approximation or a rounded answer, give the exact answer.

Note: For all questions, answer "(E) NOTA" means none of the above answers is correct.

1. How many x-values on the interval $[0, 2\pi]$ satisfy the equation $\cos x = \sin(2x)$?

(A) 2 (B) 3 (C) 4 (D) 5 (E) NOTA

2. Find the value of $\frac{C}{25} + 3A + \frac{4B}{\pi} + \text{D}$, ignoring units, given that:
A: The amplitude of the graph of $\text{y} = \text{f(x)} = 3\sin x + 4\cos x$,
B: The volume of a cone with slant height 5 and radius 3,
C: The value of the number 75^2, and D: The second smallest positive perfect number.

(A) 288 (B) 296 (C) 304 (D) 316 (E) NOTA

3. What is the local maximum value of the function $f(x) = 4x^3 + 3x^2 - 6x + 8$ on the interval $-2 < \text{x} < 2$?

(A) 13 (B) 17 (C) 24 (D) 40 (E) NOTA

4. Chloe has 16 identical candy bars. How many ways can she distribute the candy bars among herself and three other friends if she must keep at least three candy bars for herself and her other friends must each receive at least one candy bar?

(A) 125 (B) 286 (C) 364 (D) 415 (E) NOTA

5. Solve $13^{2013} \equiv x \ (mod\ 17)$, where x is a nonnegative integer less than 17.

(A) 4 (B) 5 (C) 12 (D) 13 (E) NOTA

6. Find the coefficient of x^4 in the expansion of $\left(2x + \frac{2}{x} + 3\right)^5$ with like-terms combined.

(A) 240 (B) 320 (C) 324 (D) 480 (E) NOTA

7. Let g(x) be the inverse of $\text{f(x)} = 3x^2 - 7x + 12$, where $\text{x} > 2$. Find $\text{g}'(18)$.

(A) $\frac{1}{101}$ (B) 101 (C) $\frac{1}{11}$ (D) 11 (E) NOTA

8. Find the area of the region bounded by the x-axis and the graph of $\text{y} = \text{f(x)} = 2x^3 - 8x$ on the interval $0 \leq \text{x} \leq 6$.

(A) 488 (B) 504 (C) 512 (D) 524 (E) NOTA

9. If today is July 25, 2013, what day will it be 2378 days from today? For example, one day from today would be July 26, 2013, two days would be July 27, 2013, etc.

(A) January 27, 2020 (B) January 28, 2020
(C) January 29, 2020 (D) January 30, 2020 (E) NOTA

10. Find the volume of the solid obtained by rotating the area bounded by the function $y = \text{f(x)} = x^4 - 2\text{x} + 3$ and the x-axis on the interval $0 \leq \text{x} \leq 2$ about the y-axis.

(A) $\frac{17\pi}{3}$ (B) $\frac{34\pi}{3}$ (C) $\frac{68\pi}{3}$ (D) $\frac{17\pi}{6}$ (E) NOTA

11. Compute the value of $\sum_{n=0}^{\infty} \frac{3^n}{n!}$.

(A) e^3 (B) $\frac{e}{3}$ (C) 3e (D) $\frac{3}{e}$ (E) NOTA

12. The approximation of sin 3 using the first three nonzero terms of the Maclaurin series of $\sin x$ is equal to $\frac{m}{n}$, where m and n are relatively prime positive integers. Compute the value of $m + n$.

(A) 34 (B) 37 (C) 54 (D) 61 (E) NOTA

13. Find the volume of a tetrahedron with vertices at A(1, -3, 5), B(4, 0, 3), C(2, -3, 4), and D(3, 1, 3).

(A) $\frac{2}{3}$ (B) $\frac{5}{6}$ (C) $\frac{1}{4}$ (D) $\frac{3}{4}$ (E) NOTA

14. Find the area enclosed by the polar curve $\text{r} = 3 + 2\cos\theta$.

(A) 11π (B) 22π (C) $\frac{11\pi}{3}$ (D) $\frac{22\pi}{3}$ (E) NOTA

15. Edwin and Jun are trying to meet up for dinner sometime between 7:00 P.M. and 8:00 P.M. However, both of them forget to tell the other party what time to meet up. Jun will arrive sometime between those two hours and wait ten minutes for Edwin and leave. Edwin will arrive sometime between those hours and wait a third of the time left till 8:00 P.M. and leave. What is the probability that they will meet and end up eating dinner together?

(A) $\frac{17}{36}$ (B) $\frac{49}{72}$ (C) $\frac{19}{36}$ (D) $\frac{23}{72}$ (E) NOTA

16. Evaluate: $\lim_{x\to 0}\frac{\cos x-1}{x}$

(A)1 (B) -1 (C)$\sqrt{2}$ (D) $\frac{1}{2}$ (E) NOTA

17. Find the derivative of $f(x) = x^2 \arcsin x$ evaluated at $x = \frac{\sqrt{2}}{2}$.

(A) $\frac{\sqrt{2}(3\pi+2)}{4}$ (B) $\frac{\pi+\sqrt{2}}{2}$ (C) $\frac{\pi+\sqrt{2}}{4}$ (D) $\frac{\sqrt{2}(\pi+2)}{2}$ (E) NOTA

18. Let $M\ln|x| + N\ln|x+5| + C$ equal the indefinite integral $\int \frac{2-x}{x^2+5x}dx$ for some open interval of x where the integrand is well-defined. Also, C, M, and N are constants with respect to x and y. Find the value of the product MN.

(A) $\frac{21}{5}$ (B) $\frac{-14}{25}$ (C) $\frac{-35}{2}$ (D) $\frac{25}{21}$ (E) NOTA

19. What numeral system did the Mayan calendar use?

(A) Vigesimal (B) Trigesimal (C) Tridecimal (D) Octodecimal (E) NOTA

20. What is the smallest number of vertices a planar graph can have if it has at least 60 edges?

(A) 11 (B) 12 (C) 16 (D) 22 (E) NOTA

21. Evaluate: $\lim_{n\to\infty}\sum_{i=1}^{n}\frac{7i}{n^2}$

(A) 0 (B) 1 (C) $\frac{7}{3}$ (D) 7 (E) NOTA

22. Six people placed their cell phones on a table. Each of them picks up a cell phone at random. In how many ways could this have been done so that no person picks up their own cell phone?

(A) 180 (B) 235 (C) 265 (D) 280 (E) NOTA

23. Let $0.\overline{mn}_5 = \frac{5}{6}$, where m and n are the digits of the base 5 repeating decimal. The fraction on the right-hand-side is in base 10. Find $2^m + 3^n$ in base 10.

(A) 17 (B) 41 (C) 43 (D) 67 (E) NOTA

24. Find the volume when the region bounded by the curves $\text{f}(\text{x}) = 2x^2 - 3x + 2$ and $\text{g}(\text{x}) = 4x - 3$ is revolved about the y-axis.

(A) $\frac{49\pi}{8}$ (B) $\frac{49\pi}{4}$ (C) $\frac{63\pi}{8}$ (D) $\frac{63\pi}{32}$ (E) NOTA

25. A rabbit is at the origin of the number line and moves according to the following rule: the rabbit jumps to the closest point with a greatest integer coordinate that is a multiple of 3, or to the closest point with a greatest integer coordinate that is a multiple of 17. A move sequence is a sequence of coordinates which correspond to valid moves, beginning with 0, and ending with 54. For example, 0, 3, 17, 34, 51, 54 is a move sequence. How many move sequences are possible for the rabbit?

(A) 245 (B) 255 (C) 389 (D) 459 (E) NOTA

26. Let n be the smallest integer greater than 4 such that $1^2 + 2^2 + 3^2 + 4^2 + \cdots + n^2$ is a multiple of 250. Let 100x + 10y + z = n, where x, y, and z are positive integers and z > y > x. Find the value of x^2y^3z.

(A)18 (B)32 (C)200 (D)500 (E) NOTA

27. Let $\text{y} = \frac{-e^{Lx}}{M} + Ce^{Nx}$ be the general solution to the differential equation $y' - 2y = e^{-x}$, where C, L, and M are constants with respect to x and y. Find L + M + N.

(A) -5 (B) 2 (C) 4 (D) 5 (E) NOTA

28. Evaluate: $1 + 1 - \frac{1}{2} + \frac{1}{4} + \frac{1}{3} + \frac{1}{9} - \frac{1}{4} + \frac{1}{16} - \frac{1}{25} + \cdots$

(A) $\ln 3 + \frac{\pi}{2}$ (B) $\ln 2 + \frac{\pi^2}{6}$ (C) $\frac{\sqrt{2}}{2} + \frac{\pi}{4}$ (D) diverges (E) NOTA

29. Evaluate: $\int_1^\infty \frac{4}{e^{x+1} + e^{3-x}} dx$

(A) $\ln 2 + e^2$ (B) $\frac{\pi^2 + e}{3}$ (C) $\frac{\pi}{2e^2}$ (D) $\frac{\pi^2}{6} + \ln 2$ (E) NOTA

30. Let S be the sum of all numbers of the form $\frac{m}{n}$ where m and n are relatively prime positive divisors of 2000. What is the greatest integer that does not exceed $\frac{S}{4}$?

(A) 1247 (B) 1463 (C) 1525 (D) 1789 (E) NOTA