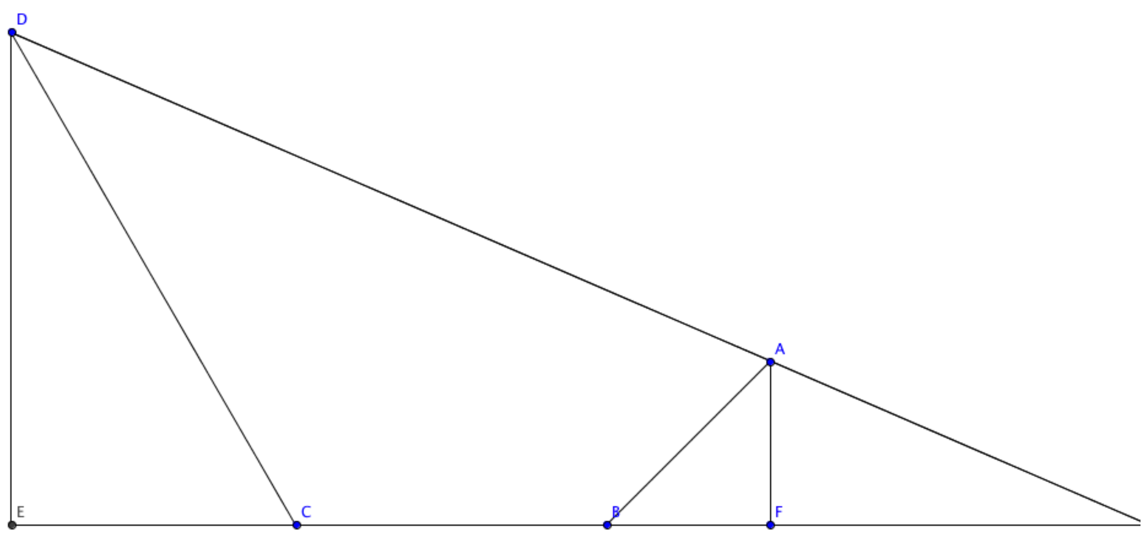
Question	Solution
P1.	Using the Change of Base formula backwards, the logarithm simplifies to $\log_{64} 1024 = \frac{10}{6} = \mathbf{\frac{5}{3}}$.
P2.	The integral is equal to $37^2 - 13^2 = (37+13)(37-13) = (50)(24) = \mathbf{1200}$.
P3.	Since 4004 is even, the smallest positive prime factor is **2**.
P4.	Since cosine and secant are reciprocals of each other, $y = 1$ and so the derivative is **0**.
P5.	We have $BAD + C = BA(0) + 2 = \mathbf{2}$.
1.	If $x = 3$, then $\|3-2\| < \|3-6\|$, or $1 < 3$, which is true. However, if $x = 4$, then $\|4-2\| < \|4-6\|$, or $2 < 2$, which is false. The largest integer solution is therefore $x = \mathbf{3}$.
2.	The period of $y = \sin\left(\frac{\pi x}{6}\right)$ is $\frac{2\pi}{\pi/6} = 12$. The absolute value cuts the period in half, since portions below the x-axis get reflected to positive values, so the graph starts the cycle quicker. The answer is **6.**
3.	By L'Hopital's Rule, $\lim_{x\to 0} \frac{\sin(2013x)}{2013x} = \lim_{x\to 0} \frac{2013\cos(2013x)}{2013} = \mathbf{1}$.
4.	Using the series $\sin^2 x = x^2 - \frac{x^4}{3} + \frac{2x^6}{45} - \frac{x^8}{315} + \cdots$, we have $\frac{1}{\sin^2 x} - \frac{1}{x^2} = \frac{x^2 - \sin^2 x}{x^2 \sin^2 x} = \frac{x^2 - \left(x^2 - \frac{x^4}{3} + \frac{2x^6}{45} - \frac{x^8}{315} + \cdots\right)}{x^2\left(x^2 - \frac{x^4}{3} + \frac{2x^6}{45} - \frac{x^8}{315} + \cdots\right)} = \frac{\frac{x^4}{3} - \frac{2x^6}{45} + \frac{x^8}{315} - \cdots}{\left(x^4 - \frac{x^6}{3} + \frac{2x^8}{45} - \frac{x^{10}}{315} + \cdots\right)} = \frac{\frac{1}{3} - \frac{2x^4}{45} + \frac{x^6}{315} - \cdots}{\left(1 - \frac{x^4}{3} + \frac{2x^6}{45} - \frac{x^8}{315} + \cdots\right)}$. As x approaches 0, this expression tends to $\mathbf{1/3}$.

5.	We have $ABCD^{-1} = (3)(6)(1)(1/3)^{-1} = \mathbf{54}$.
6.	Since $2^{12} < 5566 < 2^{13}$, the binary representation of 5566 is a 1 followed by 12 other digits for a total of **13** digits.
7.	For convenience's sake, let square $ABCD$ have a side length of 2. We know that triangles AED and DFC are congruent. Let $\phi = \angle EDA = \angle FDC$ so that $\theta = \frac{\pi}{2} - 2\phi$; thus, $\sin\theta = \sin\left(\frac{\pi}{2} - 2\phi\right) = \cos(2\phi) = 1 - 2\sin^2\phi$. Since $\sin\phi = 1/\sqrt{5}$, we have $\sin\theta = 1 - 2\sin^2\phi = 1 - 2\left(\frac{1}{\sqrt{5}}\right)^2 = 1 - \frac{2}{5} = \mathbf{\frac{3}{5}}$.
8.	The numerator is a constant and the denominator grows without bound, so the limit is **0**.
9.	By L'Hopital's Rule, $\lim_{x\to\infty} \frac{12x-5}{4+3x} = \lim_{x\to\infty} \frac{12}{3} = \mathbf{4}$.
10.	We have $A^2 - BC + D^2 = 13^2 - B(0) + 4^2 = 169 + 16 = \mathbf{185}$.
11.	If $L(x) = mx + b$, then $I(x) = \frac{x-b}{m}$. So we have $mx + b = \frac{4(x-b)}{m} + 3$, or $mx + b = \frac{4x}{m} + 3 - \frac{4b}{m}$. Set corresponding coefficients equal to each other to obtain the equations $m = 4/m$ and $b = 3 - \frac{4b}{m}$. Since the slope is positive, $m = 2$. Plug this into the second equation to get $b = 3 - \frac{4b}{2} = 3 - 2b$, or $b = 1$. Thus, $L(10) = 2(10) + 1 = \mathbf{21}$.
12.	By the Power-Reducing formula $\cos^2 x = \frac{1+\cos(2x)}{2}$, we have $\cos^2 x = \frac{1+\frac{3}{7}}{2} = \frac{5}{7}$, so that $m + n = \mathbf{12}$.
13.	The slope when $x = 7$ is $y'(7) = 2(7) - 11 = 3$. The equation of the line is $y - 25 = 3(x - 7)$, or $y = 3x + 4$. Thus, $m^2 + b^2 = \mathbf{25}$.

14.	A function whose second derivative is identically zero is a linear function. Let $f(x) = mx + b$. From the given information, $b = 20$ and $m = \frac{20-13}{0-1} = -7$. Thus, $f(17) = -7(17) + 20 = \mathbf{-99}$.
15.	We have $-A + B - C + D = -21 + 12 - 25 + -99 = \mathbf{-133}$.
16.	By inspection, $x = \mathbf{1}$.
17.	We have $\sin 20° (\tan 10° + \cot 10°) = 2 \sin 10° \cos 10° \left(\frac{\sin 10°}{\cos 10°} + \frac{\cos 10°}{\sin 10°}\right) =$ $2(\sin^2 10° + \cos^2 10°) = \mathbf{2}$.
18.	The volume of the sphere is $\frac{4}{3}\pi(6)^3 = 288\pi$. The volume of the desired region can be obtained by revolving the region bounded by $y = \sqrt{36 - x^2}$ and the x-axis on the interval $[-6, 3]$ about the x-axis. This volume is $\pi \int_{-6}^{3} \left(\sqrt{36 - x^2}\right)^2 dx = 243\pi$, making the ratio equal to $\frac{243}{288} = \mathbf{\frac{27}{32}}$.
19.	By the Chain Rule, $\left(f\left(g(x)\right)\right)' = f'\left(g(x)\right)g'(x)$. Evaluated at $x = 5$, we have $f'\left(g(5)\right)g'(5) = f'(3)g'(5) = 4 \times 7 = \mathbf{28}$.
20.	We have $\frac{(A+B)^3}{C} + \frac{D}{2} = \frac{(1+2)^3}{27/32} + \frac{28}{2} = 32 + 14 = \mathbf{46}$.
21.	Hexagonal numbers have a second-level common difference of $6 - 2 = 4$ and octagonal numbers have a second-level common difference of $8 - 2 = 6$. Thus, the hexagonal numbers are $1, 6, 15, 28, \ldots$ and the octagonal numbers are $1, 8, 21, 40, \ldots$. The answer is $15 + 40 = \mathbf{55}$.
22.	Let $x = \cos t$ and $y = \sin t$; this is a legal substitution since $x^2 + y^2 = 1$. Note that $x + y = \cos t + \sin t = \sqrt{2} \sin\left(t + \frac{\pi}{4}\right)$, so the maximum value of $x + y$ is $\sqrt{2}$. Thus,

	the maximum value of $2(x+y)^3$ is $2\left(2^{\frac{3}{2}}\right) = 2 \times 2\sqrt{2} = \mathbf{4\sqrt{2}}$.
23.	The integrand is the three-way Product Rule applied to $F(x) = (x-1)e^x \cos x$. Thus, the integral is equal to $F\left(\frac{\pi}{2}\right) - F(1) = 0 - 0 = \mathbf{0}$.
24.	The integrand is the Quotient Rule applied to $G(x) = \frac{x^2+3}{\sqrt{x}}$. Thus, the integral is equal to $G(9) - G(1) = \mathbf{24}$.
25.	We have $A^2 - B^2 + C^2 - D = 55^2 - \left(4\sqrt{2}\right)^2 + 0^2 - 24 = \mathbf{2969}$.
26.	Notice that vectors **a**, **b**, and **c** are mutually orthogonal to each other. Let $\mathbf{d} = [-6, -17, 6]$. We have $c_1 = \frac{\mathbf{d \cdot a}}{\mathbf{a \cdot a}} = \frac{12}{2} = 6$, $c_2 = \frac{\mathbf{d \cdot b}}{\mathbf{b \cdot b}} = \frac{-68}{34} = -2$, and $c_3 = \frac{\mathbf{d \cdot c}}{\mathbf{c \cdot c}} = \frac{51}{17} = 3$, so $c_1 c_2 c_3 = \mathbf{-36}$.
27.	If n is an integer, we have the identities $\sin(2\pi x) = \cos\left(\frac{\pi}{2} - 2\pi x + 2\pi n\right) = \cos\left(\frac{3\pi}{2} + 2\pi x + 2\pi n\right) = \cos(3\pi x)$, leading to the equations $3\pi x = \frac{\pi}{2} - 2\pi x + 2\pi n$ and $3\pi x = \frac{3\pi}{2} + 2\pi x + 2\pi n$. The first equation has solutions in the interval of .10, .50, 1.90, 1.30, and 1.70. The second equation has a single solution in the interval of 1.5. The sum of all these x-values is **6**.
28.	For integer n, the area bounded by the graph of Greatest Integer function and the x-axis on the interval $[n, n+2]$ consists of two rectangles of dimensions $1 \times (n+1)$ and $1 \times n$. Thus, $\int_n^{n+2} f(x)\ dx = n + (n+1) = 2n+1$, making $\sum_{n=0}^{9}(2n+1) = \mathbf{100}$.
29.	Use the Disc Method for the x-axis rotation and use the Shell Method for the y-axis rotation. This approach yields the equation $\pi \int_0^1 (kx^2)^2\, dx = 2\pi \int_0^1 x(kx^2)\, dx$,

	having solution $k = 5/2$. Thus, $8k = \mathbf{20}$.
30.	We have $\sqrt{A^2} + \sqrt{B + D + 10} + \sqrt{C} = \lvert -36 \rvert + \sqrt{6 + 20 + 10} + \sqrt{100} = \mathbf{52}$.
31.	The coordinates of triangle POQ are $(0,0)$, $(5,0)$, and (x, y), where $x^2 + y^2 = 36$. The centroid of POQ is the average of the coordinates, or $\left(\frac{x+5}{3}, \frac{y}{3}\right)$. Suppose $\left(\frac{x+5}{3}, \frac{y}{3}\right) = (a, b)$ so that $\frac{x+5}{3} = a$ and $\frac{y}{3} = b$. Solving each equation for x and y, squaring both sides, and adding the equations, we arrive at $(3a - 5)^2 + (3b)^2 = 36$, or $\left(a - \frac{5}{3}\right)^2 + b^2 = 4$, an equation of a circle of radius 2. The desired area is $\mathbf{4\pi}$.
32.	We want the angle opposite the side with length 6. Let this angle equal θ. By the Law of Cosines, $\cos\theta = \frac{7^2+8^2-6^2}{2\times7\times8} = \frac{11}{16}$. Thus, $m + n = 11 + 16 = \mathbf{27}$.
33.	The integral is equal to $\arctan\infty - \arctan 0 = \boldsymbol{\frac{\pi}{2}}$.
34.	Let $y = \sqrt{\frac{1-x}{x}}$, so that $x = \frac{1}{1+y^2}$. We have $\int_0^1 y\,dx = \int_0^\infty x\,dy = \int_0^\infty \frac{1}{1+y^2}\,dy = \boldsymbol{\frac{\pi}{2}}$, per the solution on problem #33.
35.	Since $C = D$, the desired quantity equals $\mathbf{0}$.
36.	The equation is of the form $M = PDP^{-1}$, where D is a diagonal matrix. Therefore, we have $M^{10} = (PDP^{-1})^{10} = PD^{10}P^{-1}$. Notice that $D^{10} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}^{10} = \begin{bmatrix} (-1)^{10} & 0 \\ 0 & 1^{10} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ is the identity matrix I. Thus, $M^{10} = PD^{10}P^{-1} = PIP^{-1} = PP^{-1} = I$. The sum of the elements of the 2×2 identity matrix is $\mathbf{2}$.
37.	Note that $\frac{x}{100\pi} - 1$ is less than -1 whenenver $x < 0$ and greater than 1 when

	$x > 200\pi$. Thus, the two graphs will have intersection points on the interval $x \in [0, 200\pi]$, in which the graph of $y = \sin x$ will exhibit 100 full cycles. We subtract 1 from this total to account for the double-counting of intersection points in the middle of the interval. The answer is **199**.
38.	Notice that $f(0) = \int_0^{\pi} \sin t\, dt = 2$ and $f(1) = \int_0^{\pi} t \sin t\, dt = \pi \approx 3.14159$. Since f is a continuous function, by the Intermediate Value Theorem, it will attain all values in between 2 and π, inclusive, on the interval $x \in [0, 1]$. All the elements in the set are contained in this interval; therefore the probability is **1**.
39.	Choose the point on the graph of $y = x^2$ whose tangent line is parallel to the line segment AB. The slope of AB is $\frac{-10-0}{0-2} = 5$, making $2c = 5$, or $c = \mathbf{5/2}$.
40.	We have $2AD + B + C = 2(2)\left(\frac{5}{2}\right) + 199 + 1 = \mathbf{210}$.
41.	 Starting with quadrilateral $ABCD$, draw auxiliary line segments to obtain the diagram above, where AF and DE are perpendicular to EG, which contains the points C, B, and F. Triangle ABF is a 45-45-90 triangle, so $AF = BF = \frac{\sqrt{6}}{\sqrt{2}} = \sqrt{3}$. Triangle DCE is a 30-60-90 triangle, so $EC = 3$ and $DE = 3\sqrt{3}$. Triangles AFG and

	DGE are similar. Therefore, $\frac{AF}{DE} = \frac{FG}{EG}$, or $\frac{\sqrt{3}}{3\sqrt{3}} = \frac{FG}{3+5-\sqrt{3}+\sqrt{3}+FG} = \frac{FG}{8+FG}$, or $FG = 4$. Moreover, $AG = \sqrt{AF^2 + FG^2} = \sqrt{\sqrt{3}^2 + 4^2} = \sqrt{19}$. Based on the earlier equation, triangles AFG and DGE are in a 3-to-1 linear ratio, so $AD = 2 \times AG = \mathbf{2\sqrt{19}}$.
42.	Perhaps the fastest way to do this problem is to know in advance that $\cos\frac{\pi}{5} = \frac{1+\sqrt{5}}{4}$; hence the minimal polynomial will also have $\frac{1-\sqrt{5}}{4}$ as a root. The two roots have a sum of $\frac{1+\sqrt{5}+1-\sqrt{5}}{4} = \frac{1}{2}$ and product of $\left(\frac{1+\sqrt{5}}{4}\right)\left(\frac{1-\sqrt{5}}{4}\right) = \frac{1-5}{16} = \frac{-1}{4}$, leading to a minimal polynomial of $P(x) = x^2 - \frac{1}{2}x - \frac{1}{4}$ or after making all the coefficients integers, $P(x) = 4x^2 - 2x - 1$. Hence, $P(-1) = 4(-1)^2 - 2(-1) - 1 = 4 + 2 - 1 = \mathbf{5}$.
43.	By the Triple-Angle Formula, $f(x) = \frac{2}{3^6}\cos(3x)$. With every application of the Chain Rule, a 3 from the inside of the cosine function “comes out,” multiplicatively. Therefore, it will take at least six differentiations to whittle down the 3^6 in the denominator. Multiplicative constant coefficients aside, the derivatives of the trigonometric functions cycle with a period of 4. Therefore, we have $\frac{d^6 f}{dx^6} = -2\cos(3x)$. But this function evaluated at 0 does not produce a positive integer. If we differentiate two more times we obtain $\frac{d^8 f}{dx^8} = 18\cos(3x)$, and this works. The answer is $n^2 = \mathbf{64}$.
44.	Let the degree of $P(x)$ equal n. Expanding the Quotient Rule to the left side of the equation yields $\frac{P'(x)(x^4+1)-P(x)(4x^3)}{(x^4+1)^2}$. Therefore, $P'(x)(x^4+1) - P(x)(4x^3) = 3x^4 - 1$. Notice that the left side of this equation has degree $\max(n-1+4, n+3) = n+3$ and the right side has degree 4. Therefore, $n = 1$ and $P(x)$ is a linear function, say, $P(x) = mx + b$. If we let $x = 0$ in the equation $m(x^4+1) - (mx+b)(4x^3) = 3x^4 - 1$, we obtain $m = -1$. Using this value as well as $x = 1$ in the

	equation, we obtain $b = 0$. Thus, $P(15) = \mathbf{-15}$.
45.	We have $A^2 + B^2 + (C - D + 1)^2 = \left(2\sqrt{19}\right)^2 + 5^2 + (64 - -15 + 1)^2 = \mathbf{6501}$.
46.	The numbers being plugged into the function are the first five positive perfect numbers. Recall that even perfect numbers have the form $g(x) = 2^{x-1}(2^x - 1)$, where $2^x - 1$ is prime; by inspection, the five smallest positive values of x which makes this true are 2, 3, 5, 7, and 13. We have $f\big(g(x)\big) = \log_2\left(1 + \sqrt{8(2^{x-1}(2^x - 1)) + 1}\right) - 2 = x - 1$. Thus, the answer is $(2-1) + (3-1) + (5-1) + (7-1) + (13-1) = \mathbf{25}$.
47.	Suppose $\csc x = \cot x$. This leads to $\cos x = 1$ and $\sin x = 0$, hence $\csc x$ would be undefined and a triangle cannot be formed. Now suppose $\sec x = \csc x$. This leads to $\cos x = \sin x = 1/\sqrt{2}$, making a triangle with side lengths $\sqrt{2}$, $\sqrt{2}$, and 1. In particular, $\csc x = \sqrt{2}$. For the third case, $\sec x = \cot x$, we get the quadratic equation $\sin x = \cos^2 x = 1 - \sin^2 x$, which has the positive solution $\sin x = \frac{2}{1+\sqrt{5}}$, or $\csc x = \mathbf{\frac{1+\sqrt{5}}{2}}$. This is the largest possible value of $\csc x$.
48.	The function is a constant, with derivative $\mathbf{0}$.
49.	The function is a constant, having integral of $3\pi^2(3 - 0) = \boldsymbol{9\pi^2}$.
50.	We have $A + BC + \cos\sqrt{D} = 25 + B(0) + \cos 3\pi = 25 - 1 = \mathbf{24}$.