1. **(C)** The common difference is $4 - 1 = \mathbf{3}$.
2. **(D)** Note that $\frac{2013\pi}{3} = 671\pi$ is coterminal to π. Therefore, $a_{2013} = \cos\pi + i\sin\pi = \mathbf{-1}$.
3. **(B)** Notice that the series is simply the Maclaurin series for cosine evaluated at $x = \sqrt{2}$. Thus, the answer is $\mathbf{\cos\sqrt{2}}$.
4. **(D)** The first term is $3x$ and the common difference is $(6x+1) - 3x = 3x + 1$, making the 51st term equal to $3x + (51-1)(3x+1) = \mathbf{153x + 50}$.
5. **(D)** The common difference of the sequence is equivalent to the slope of the line passing through (1,1) and (5,60), or $\frac{60-1}{5-1} = \mathbf{\frac{59}{4}}$.
6. **(B)** There is an exponential in the denominator, which eventually dominates the polynomial in the numerator. The limit is **0**.
7. **(E)** The common ratio is $r = \frac{a_2}{a_1} = \sin x$. By the formula for the sum of an infinite geometric series, we have $\frac{a_1}{1-r} = \frac{\sin x}{1-\sin x} = \frac{1}{3}$, or $\sin x = \frac{1}{4}$. **None** of the angles in the answer choices yield this value for sine.
8. **(D)** Using the standard formula, the answer is $\frac{(25)(25+1)(2\times25+1)}{6} = \mathbf{5525}$.
9. **(A)** The exponential term in the numerator of the sequence will eventually dominate the polynomial term in the denominator. So the power series only converges at the center, making a radius of **0**.
10. **(E)** A value smaller than 10 is attainable. For example, if all the numbers are equal to 3, the sum of all the numbers is 21 and the average of the squares is $3^2 = \mathbf{9}$.
11. **(A)** If the first term is a and the common ratio is r, we have $ar = 1 + i$ and $ar^4 = 2 + 2i$, making $ar^4 = a\left(\frac{1+i}{a}\right)^4 = -\frac{4}{a^3} = 2 + 2i$, or $a^3 + \frac{4}{2+2i} = 0$. This polynomial is a cubic with no quadratic term, making the sum of all possible values of a equal to **0**.
12. **(E)** Choice A diverges by the Integral Test since $\int_2^\infty \frac{dn}{n\ln n^2} = \frac{1}{2}\ln\ln\infty - \frac{1}{2}\ln\ln 2 = \infty$. Choice B and C both diverge by the nth-term Test since $\lim\limits_{k\to\infty}\frac{(-2)^k k^k}{3k+7} \neq 0$ and $\lim\limits_{i\to\infty}\frac{5i-4}{2i+1} = \frac{5}{2} \neq 0$. Choice D diverges by Limit Comparison since the series is essentially harmonic. **None** of the series converges.

13. **(A)** Using the formula $S_n = \frac{n}{2}(a_1 + a_n)$, we have $1090 = \frac{20}{2}(a_1 + 102)$. Solving yields $a_1 = \mathbf{7}$.

14. **(A)** In a geometric sequence, the middle term is the geometric mean of its neighbors, resulting in the equation $(2x)^2 = (x-1)(5x+3)$. The solution to this equation is $x \in \{-1, 3\}$. Only **3** is a possible value since -1 will cause negative-valued terms.

15. **(A)** We have $\lim_{n\to\infty} \sum_{k=1}^{n} \frac{(k-1)^2}{n^3} = \lim_{n\to\infty} \frac{1}{n^3}(1^2 + 2^2 + \cdots + (n-1)^2)$. Using the formula for the sum of squares, we have $\lim_{n\to\infty} \frac{1}{n^3}(1^2 + 2^2 + \cdots + (n-1)^2) = \lim_{n\to\infty} \frac{(n-1)((n-1)+1)(2(n-1)+1)}{6n^3} = \mathbf{\frac{1}{3}}$.

16. **(C)** Suppose the geometric sequence has first term a and common ratio r. The sum of the roots is $a + ar + ar^2 = -\frac{-14}{24} = \frac{7}{12}$. The product of the roots is $a(ar)(ar^2) = a^3r^3 = -\left(\frac{3}{24}\right) = -\frac{1}{8}$. Thus, $ar = -\frac{1}{2}$, the second term. The first equation can therefore be rewritten as $\frac{-\frac{1}{2}}{r} - \frac{1}{2} - \frac{1}{2}r = 7/12$, which has solution set $r \in \{-\frac{3}{2}, -\frac{2}{3}\}$. We choose $r = -3/2$ for simplicity's sake, and so the geometric sequence is $\frac{1}{3}, -\frac{1}{2}, \frac{3}{4}$. The value of the sum of the roots taken two at a time is $\frac{k}{24} = \left(\frac{1}{3}\right)\left(\frac{3}{4}\right) + \left(\frac{1}{3}\right)\left(-\frac{1}{2}\right) + \left(-\frac{1}{2}\right)\left(\frac{3}{4}\right) = -\frac{7}{24}$. Thus, $k = -7$ and $|k| = \mathbf{7}$.

17. **(D)** Suppose the roots are a, $a+d$, and $a+2d$; consequently, the sum of the roots is $3a + 3d$. We can obtain the same sum using the coefficients of the polynomial: $-\frac{A}{1} = -A$. Thus, $-A = 3a + 3d$ or $A = -3(a+d)$. Thus, A is a multiple of 3.

18. **(A)** We actually want the cubic term of $(3x-1)^{10}$ as the integral operator would increase the exponent by 1. By the Binomial Theorem, the cubic term is $\binom{10}{3}(3x)^3(-1)^7 = 120(27x^3)(-1) = -3240x^3$. Thus, we have $\int_0^t -3240x^3\,dx = -810t^4$, making the answer $\mathbf{-810}$.

19. **(B)** Let $S = \frac{2}{3} + \frac{5}{9} + \frac{8}{27} + \cdots$, so that $\frac{S}{3} = \frac{2}{9} + \frac{5}{27} + \frac{8}{81} + \cdots$ and $S - \frac{S}{3} = \frac{2}{3} + \frac{3}{9} + \frac{3}{27} + \cdots$, or $\frac{2S}{3} = \frac{2}{3} + \frac{3/9}{1-1/3} = \frac{7}{6}$. Thus, $S = \mathbf{\frac{7}{4}}$.

20. **(C)** If the first term and common ratio of the original series is a and r, respectively, then we have the equations $\frac{a}{1-r} = 2013$ and $\frac{a^2}{1-r^2} = 33 \times 2013$. Solve these equations simultaneously to get $r = \mathbf{30/31}$.

21. **(C)** Let the radius of convergence of the series equal R. If the series converges, then the sequence a_n tends to 0. Thus, it is bounded by some number, say C, after some point. So we have $\sum |a_n x^n| < \sum C|x^n| = C\sum |x^n|$. The series on the rightmost side converges when $|x| < 1$. Thus, $R \geq 1$. However, if $R > 1$, then the series would converge absolutely for all $|x| < R$, in particular, when $x = 1$. Therefore, R must equal **1**.

22. **(E)** The formula for the sum of the first n positive perfect cubes is $\left(\frac{n(n+1)}{2}\right)^2$, which is a perfect square. Perfect squares have an odd number of positive integral factors. All the answer choices are even.

23. **(A)** We have $\sum_{n=1}^{100}(5n+1) - \sum_{n=1}^{100} 5n + 1 = \sum_{n=1}^{100} 5n + \sum_{n=1}^{100} 1 - \sum_{n=1}^{100} 5n + 1 = \left(\sum_{n=1}^{100} 1\right) + 1 = 100 + 1 = \mathbf{101}$.

24. **(E)** We first try to find a pattern in the powers of M. We have $M^2 = \begin{bmatrix} 1 & 0 \\ 3 & 4 \end{bmatrix}$, $M^3 = \begin{bmatrix} 1 & 0 \\ 7 & 8 \end{bmatrix}$, and in general, $M^n = \begin{bmatrix} 1 & 0 \\ 2^n - 1 & 2^n \end{bmatrix}$. Thus, $e^M = \sum_{n=0}^{\infty} \frac{1}{n!} M^n =$

$$\sum_{n=0}^{\infty} \frac{1}{n!} \begin{bmatrix} 1 & 0 \\ 2^n - 1 & 2^n \end{bmatrix} = \sum_{n=0}^{\infty} \begin{bmatrix} \frac{1}{n!} & 0 \\ \frac{2^n}{n!} - \frac{1}{n!} & \frac{2^n}{n!} \end{bmatrix} = \begin{bmatrix} \sum \frac{1}{n!} & 0 \\ \sum \frac{2^n}{n!} - \frac{1}{n!} & \sum \frac{2^n}{n!} \end{bmatrix} = \begin{bmatrix} \boldsymbol{e} & \mathbf{0} \\ \boldsymbol{e^2 - e} & \boldsymbol{e^2} \end{bmatrix}.$$

25. **(D)** In the theory of Generating Functions, it's the coefficient of the polynomial that determines the number of ways for a certain outcome that is related to a certain exponent. In the case of rolling dice, one roll corresponds to $x + x^2 + x^3 + x^4 + x^5 + x^6$, so 2013 rolls would yield an exponent of 2013. The number of ways to roll a sum of 500 is the coefficient of the x^{500}th term of $(x + x^2 + x^3 + x^4 + x^5 + x^6)^{2013}$.

26. **(C)** Multiply top and bottom of $\frac{a_{n-1}}{\overline{a_{n-1}}}$ to obtain $a_n = \frac{a_{n-1}^2}{|a_{n-1}|^2}$. Suppose $a_1 = x$. Then $a_2 = \frac{x^2}{|x|^2} = \frac{x^2}{1^2} = x^2$. Similarly, $a_3 = \frac{(x^2)^2}{|x^2|^2} = \frac{x^4}{1^2} = x^4$. We see that the pattern is $a_n = x^{2^{n-1}}$. So $a_{2013} = 1 = x^{2^{2012}}$, or $x^{2^{2012}} - 1 = 0$. By the Fundamental Theorem of Algebra, this polynomial has $\mathbf{2^{2012}}$ solutions in complex numbers.

27. **(B)** Notice that $f(x) = 2x + 3 + 2013\sum_{n=0}^{\infty}\frac{(-1)^n 2^n x^n}{n!} = 2x + 3 + 2013e^{-2x}$; this allows us to calculate the various derivatives much easier. Specifically, we have $f'(x) = 2 - 4026e^{-2x}$ and $f''(x) = 8052e^{-2x}$, so that $2f''(5) + 3f'(5) - 2f(5) = 16104e^{-10} + 3(2 - 4026e^{-10}) - 2(13 + 2013e^{-10}) = \mathbf{-20}$.

28. **(C)** We tackle the problem recursively, breaking up the problem into various cases. Let a_n equal the number of n-letter words where there is an even amount of As and Bs. Let b_n equal the number of n-letter words where there is an even amount of As and an odd number of Bs or an odd amount of As and an even amount of Bs. Finally, let c_n equal the number of n-letter words where there is an odd amount of As and Bs. Notice that since these sequences cover all possible cases of n-letter words, $a_n + b_n + c_n = 4^n$. We then have $a_{n+1} = 2a_n + b_n$ and $b_{n+1} = 2a_n + 2b_n + 2c_n = 2 \times 4^n$, and combining these results yields $a_{n+1} = 2a_n + 2 \times 4^{n-1}$. We know that $a_1 = 2$ (the words are: C and D), and working through the sequence, we find that $a_4 = \mathbf{72}$.

29. **(C)** Let $x_1 = 2^a$ be the first term and the common ratio $r = 2^d$ so that $\sum_{n=1}^{10}\log_2(x_n) = 10a + 45d = 500$, or $2a + 9d = 100$. The ordered pairs (a, d) that solve this equation are (5, 10), (14, 8), (23, 6), (32, 4), and (41,2). To help narrow things down, we turn to the expression $\log_2(\sum_{n=1}^{10} x_n)$. Since we're dealing with a geometric series, $\sum_{n=1}^{10} x_n = \frac{2^a((2^d)^{10}-1)}{2^d-1} \approx \frac{2^a(2^{10d})}{2^d} = 2^{a+9d}$, so that $\log_2(\sum_{n=1}^{10} x_n) \approx a + 9d$. The inequality $90 < a + 9d < 100$ only holds when $(a, d) = (5, 10)$, making $\log_2(x_{20}) = a + 19d = 5 + 19(10) = \mathbf{195}$.

30. **(B)** Use Integration by Parts with $u = (x^2 - 1)^n$ and $dv = dx$, resulting in $a_n = \int_{-1}^{1} (x^2 - 1)^n \, dx = x(x^2 - 1)^n |_{-1}^{1} - \int_{-1}^{1} 2x^2 n (x^2 - 1)^{n-1} \, dx$. Evaluating the part of this equation without the integral sign yields 0, so we have $a_n = -\int_{-1}^{1} 2x^2 n (x^2 - 1)^{n-1} \, dx = -2n \int_{-1}^{1} x^2 (x^2 - 1)^{n-1} \, dx$. Using the fact that $x^2 = (x^2 - 1) + 1$ and the distributive property of integrals, we have $a_n = -2n \int_{-1}^{1} \big((x^2 - 1) + 1\big)(x^2 - 1)^{n-1} \, dx = -2n \int_{-1}^{1} ((x^2 - 1)^n + (x^2 - 1)^{n-1}) \, dx = -2n \int_{-1}^{1} (x^2 - 1)^n \, dx - 2n \int_{-1}^{1} (x^2 - 1)^{n-1} \, dx = -2na_n - a_{n-1}$. Thus, $\frac{a_n}{a_{n-1}} = \frac{-2n}{2n+1}$. Taking limits to positive infinity results in an answer of $\mathbf{-1}$.