Practice Round Mu State Bowl
Mu Alpha Theta National Convention 2013

P1. Solve for x: $3x - 10 = 20$

P2. Find the value of $\sec^4 \frac{5\pi}{6}$ as a common fraction.

P3. Let $f(x) = \ln 2013$. Evaluate: $2013f'(2013)$

P4. Let $g(x) = 5$. Evaluate: $\int_0^{10} g(x)\ dx$

P5. Let A, B, C, and D be the answers to questions P1, P2, P3, and P4, respectively.
Evaluate: $A^C + \left(D \div \sqrt{9B + 16/B}\right)$

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Round #1 Mu State Bowl
Mu Alpha Theta National Convention 2013

1. Find x as a common fraction: $4+\sqrt{10-x}=6+\sqrt{4-x}$

2. Find the amplitude of the graph $y=2\cos x-2\sqrt{3}\sin x$.

3. Evaluate: $\lim_{x\to 0}\frac{1-\cos(2013x)}{x}$

4. Evaluate: $\lim_{x\to 0}\frac{16-16\cos x^2}{x\sin x^3}$

5. Let A, B, C, and D be the answers to problems 1, 2, 3, and 4, respectively.
Evaluate: $AB+CD$

Round #1 Mu State Bowl
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Evaluate: $AB+CD$

Round #2 Mu State Bowl
Mu Alpha Theta National Convention 2013

6. For integer n, let $\tau(n)$ equal the number of positive divisors of n. How many integers $N \in (0,200)$ satisfy the congruence $\tau(N) \equiv 1 \pmod 2$?

7. If x is a real number, find the number of solutions to $x + \sin x + e^x = 2$.

8. If $f(h) = \frac{(10+h)^2-100}{h}$, find $f(10) + f'(10)$.

9. Evaluate: $\lim_{h\to 0} \frac{\ln(2+h)^{1024} - 1024 \ln 2}{h}$

10. Let A, B, C, and D be the answers to problems 6, 7, 8, and 9, respectively.
Find the determinant of $\begin{bmatrix} B & A \\ C & D \end{bmatrix}$.

Round #2 Mu State Bowl
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Round #3 Mu State Bowl
Mu Alpha Theta National Convention 2013

11. Find, as a common fraction, the sum of all *real numbers* x such that $2x^3 + x^2 - 4 = 8x$.

12. Find the sum of the solutions to $\sin^2(5\theta) + \sin(2\theta) + \cos^2(5\theta) = 1$, where $\theta \in (\pi, 5\pi]$.

13. For $f(x) = \arctan x$ (as always, subject to the traditional restrictions on domain and range), let $L = \lim_{h \to 0} \frac{f(1+2h) - 2f(1+h) + f(1)}{h^2}$. Find $100L$.

14. Let θ be a positive acute angle such that $\sin\theta = 3\cos\theta$. If $T(x) = \sin(2x)$, find $25T'(\theta)$.

15. Let A, B, C, and D be the answers to problems 11, 12, 13, and 14, respectively.
Evaluate: $C\cos(AB) + D$

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Round #4 Mu State Bowl
Mu Alpha Theta National Convention 2013

16. How many integers x satisfy $\left||x|-7\right| \leq 8$?

17. The line with equation $2x - ky = 2013$ makes a $30°$ angle with the positive x-axis. Find k^4.

18. Evaluate: $\int_0^{\pi} x \sin\frac{x}{2}\, dx$

19. Evaluate: $\int_1^{e} 16x^3 \ln x\, dx$

20. Let A, B, C, and D be the answers to problems 16, 17, 18, and 19, respectively.
Evaluate: $2\ln\left(\frac{A+D-2}{3} - \frac{B-11C}{10}\right)$

Round #4 Mu State Bowl
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Round #5 Mu State Bowl
Mu Alpha Theta National Convention 2013

21. Find the area of a quadrilateral with side lengths of 39, 52, 25, and 60 in that order.

22. A cube has volume of $\cos^3 x$ (where $0 < x < \frac{\pi}{2}$) and surface area of $36/17$. If $\sin^2 x = m/n$, where m and n are positive relatively prime integers, find $m + n$.

23. Let C represent the locus of points in the plane equidistant from the graphs of $x^2 + y^2 = 1$ and $y = -3$. Find the slope of the line tangent to C at $(8,6)$.

24. A random variable X has a probability density function $f(x)$ such that $f(x) = 8x/\pi^2$ when $0 < x < \pi/2$ and $f(x) = 0$ for all other values of x. What is the expected value of $3\pi^2 \sin X$?

25. Let A, B, C, and D be the answers to problems 21, 22, 23, and 24, respectively.
Evaluate: $\frac{A}{B} + \frac{D}{C}$

Round #5 Mu State Bowl
Mu Alpha Theta National Convention 2013

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Evaluate: $\frac{A}{B} + \frac{D}{C}$

Round #6 Mu State Bowl
Mu Alpha Theta National Convention 2013

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26. Let M be a 4×4 matrix such that $M \times \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} b \\ c/2 \\ 3d \\ a/4 \end{bmatrix}$ for all real numbers a, b, c, and d. Find the sum of the elements of $3M^{-1}$.

27. The domain of $f(x) = \sin^6 x + \cos^6 x$ is all real numbers x. The range of f is the interval $I = [a, b]$. Find the midpoint of I.

28. Evaluate: $\int_{3}^{-3} \left(\frac{\sin x}{1+x^{20}} + x^2 \right) dx$

29. Let $f(x) = 1 + x + x^7$ and g be the inverse of f. Find $1024g'(3)$.

30. Let A, B, C, and D be the answers to problems 26, 27, 28, and 29, respectively.
Evaluate: $A + BD + C$

Round #6 Mu State Bowl
Mu Alpha Theta National Convention 2013

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Evaluate: $A + BD + C$

Round #7 Mu State Bowl
Mu Alpha Theta National Convention 2013

31. Find $P(100)$, where $P(x)$ is a polynomial with real coefficients and $P(x^2) + 2x^2 + 10x = 2xP(x+1) + 3$ for all real x.

32. A triangle inscribed in the unit circle has angles measuring α, β, and γ. The perimeter of the triangle is 5. Evaluate: $\sin\alpha + \sin\beta + \sin\gamma$

33. If S is the set of distinct critical values of $f(x) = (x-1)^2(x+1)^5$, let c be the median of the elements of S. If $\cos\theta = c$ and $|\cos(2\theta)| = \frac{m}{n}$, where m and n are relatively prime positive integers, find $m + n$.

34. Find the maximum value of $f(a,b) = 10(b^2 - a^2) - 16(b-a) - \frac{4}{3}(b^3 - a^3)$, where $0 < a < b$.

35. Let A, B, C, and D be the answers to problems 31, 32, 33, and 34, respectively.
Evaluate: $-A + BC + D$

Round #7 Mu State Bowl
Mu Alpha Theta National Convention 2013

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35. Let A, B, C, and D be the answers to problems 31, 32, 33, and 34, respectively.
Evaluate: $-A + BC + D$

Round #8 Mu State Bowl
Mu Alpha Theta National Convention 2013

36. In triangle ABC with centroid P, let D and E be the foot of the medians to sides BC and AC, respectively. If AP is perpendicular to BE, $|AD| = 6$, and $|BE| = 9$, find the area of ABC.

37. Find the number of times the polar graph $r = 2^{\frac{2\theta}{\pi}}$ intersects the line segment whose endpoints are the Cartesian coordinates $(\sqrt{2}, \sqrt{2})$ and $(64\sqrt{2}, 64\sqrt{2})$.

38. If $I = \int_1^{25} \frac{1}{x+\sqrt{x}}\, dx$, find e^I.

39. If $I = \int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}\, dx$, find I.

40. Let A, B, C, and D be the answers to problems 36, 37, 38, and 39, respectively.
Evaluate: $ABC \tan D$

Round #8 Mu State Bowl
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39. If $I = \int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}\, dx$, find I.

40. Let A, B, C, and D be the answers to problems 36, 37, 38, and 39, respectively.
Evaluate: $ABC \tan D$

Round #9 Mu State Bowl
Mu Alpha Theta National Convention 2013

41. Define $\Pi(S)$ as the product of the elements of a set S. Let $S_1, S_2, S_3, \dots, S_{15}$ be the nonempty subsets of $S = \{1, 2, 3, 4\}$. Evaluate: $\sum_{n=1}^{15} \left(\Pi(S_n)\right)^{-1}$

42. Find, in degrees, the measure of the smallest angle in a right triangle with legs of length a and b and hypotenuse of length $2\sqrt{ab}$, where a and b are positive numbers.

43. If $y = \sin x$ and $F(x) = \sin x + \sum_{n=1}^{2013} \frac{d^n y}{dx^n}$, find $F(\pi)$.

44. Let $f(x) = \frac{\cos(5x)+\cos(3x)}{\sin(5x)-\sin(3x)}$. Evaluate: $f'(\pi/4)$

45. Let A, B, C, and D be the answers to problems 41, 42, 43, and 44, respectively.
Evaluate: $ABCD$

Round #9 Mu State Bowl
Mu Alpha Theta National Convention 2013

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45. Let A, B, C, and D be the answers to problems 41, 42, 43, and 44, respectively.
Evaluate: $ABCD$

Round #10 Mu State Bowl
Mu Alpha Theta National Convention 2013

46. Find the sum of all positive integers n such that $\frac{2210}{(3n+5)(2n+3)}$ is an integer.

47. Find the smallest positive angle x (in radians) satisfying the equation
$$\left(\sin\left(\frac{2x}{3}\right)\cos\left(\frac{4x}{3}\right)+\cos\left(\frac{2x}{3}\right)\sin\left(\frac{4x}{3}\right)\right)\left(\cos\left(\frac{16x}{5}\right)\cos\left(\frac{6x}{5}\right)+\sin\left(\frac{16x}{5}\right)\sin\left(\frac{6x}{5}\right)\right)=\frac{1}{4}.$$

48. Suppose f and g are functions that $f'(x)=g'(x)$ for all x. If $f(5)-g(5)=5$, evaluate:
$$\int_{-10}^{10} f(x)\ dx-\int_{-10}^{10} g(x)\ dx$$

49. If $f(x)=e^{x}(12\sin(3x)+5\cos(3x))$, evaluate: $f''(0)-6f'(0)+9f(0)$

50. Let A, B, C, and D be the answers to problems 46, 47, 48, and 49, respectively.
Evaluate: $A+4\cos^2(BC)-D$

Round #10 Mu State Bowl
Mu Alpha Theta National Convention 2013

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47. Find the smallest positive angle x (in radians) satisfying the equation
$$\left(\sin\left(\frac{2x}{3}\right)\cos\left(\frac{4x}{3}\right)+\cos\left(\frac{2x}{3}\right)\sin\left(\frac{4x}{3}\right)\right)\left(\cos\left(\frac{16x}{5}\right)\cos\left(\frac{6x}{5}\right)+\sin\left(\frac{16x}{5}\right)\sin\left(\frac{6x}{5}\right)\right)=\frac{1}{4}.$$

48. Suppose f and g are functions that $f'(x)=g'(x)$ for all x. If $f(5)-g(5)=5$, evaluate:
$$\int_{-10}^{10} f(x)\ dx-\int_{-10}^{10} g(x)\ dx$$

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