P1. Solve for $x$: $3x - 10 = 20$

P2. Find the value of $\sec^4 \frac{5\pi}{6}$ as a common fraction.

P3. Let $f(x) = \ln 2013$. Evaluate: $2013f'(2013)$

P4. Let $g(x) = 5$. Evaluate: $\int_{0}^{10} g(x) \, dx$

P5. Let $A$, $B$, $C$, and $D$ be the answers to questions P1, P2, P3, and P4, respectively. Evaluate: $A^C + \left(D \div \sqrt{9B + 16/B}\right)$
1. Find $x$ as a common fraction: $4 + \sqrt{10 - x} = 6 + \sqrt{4 - x}$

2. Find the amplitude of the graph $y = 2 \cos x - 2\sqrt{3} \sin x$.

3. Evaluate: $\lim_{x \to 0} \frac{1 - \cos(2013x)}{x}$

4. Evaluate: $\lim_{x \to 0} \frac{16 - 16\cos x^2}{x \sin x^3}$

5. Let $A, B, C,$ and $D$ be the answers to problems 1, 2, 3, and 4, respectively. Evaluate: $AB + CD$
6. For integer \( n \), let \( \tau(n) \) equal the number of positive divisors of \( n \). How many integers \( N \in (0,200) \) satisfy the congruence \( \tau(N) \equiv 1 \pmod{2} \)?

7. If \( x \) is a real number, find the number of solutions to \( x + \sin x + e^x = 2 \).

8. If \( f(h) = \frac{(10+h)^2-100}{h} \), find \( f(10) + f'(10) \).

9. Evaluate: \( \lim_{h \to 0} \frac{\ln(2+h)^{1024}-1024 \ln 2}{h} \).

10. Let \( A, B, C, \) and \( D \) be the answers to problems 6, 7, 8, and 9, respectively. Find the determinant of \( \begin{bmatrix} B & A \\ C & D \end{bmatrix} \).
11. Find, as a common fraction, the sum of all real numbers $x$ such that $2x^3 + x^2 - 4 = 8x$.

12. Find the sum of the solutions to $\sin^2(5\theta) + \sin(2\theta) + \cos^2(5\theta) = 1$, where $\theta \in (\pi, 5\pi]$.

13. For $f(x) = \arctan x$ (as always, subject to the traditional restrictions on domain and range), let $L = \lim_{h \to 0} \frac{f(1+2h) - 2f(1+h) + f(1)}{h^2}$. Find $100L$.

14. Let $\theta$ be a positive acute angle such that $\sin \theta = 3 \cos \theta$. If $T(x) = \sin(2x)$, find $25T'(\theta)$.

15. Let $A, B, C,$ and $D$ be the answers to problems 11, 12, 13, and 14, respectively. Evaluate: $C \cos(AB) + D$
16. How many integers $x$ satisfy $\big| |x| - 7 \big| \leq 8$?

17. The line with equation $2x - ky = 2013$ makes a $30^\circ$ angle with the positive $x$-axis. Find $k^4$.

18. Evaluate: $\int_0^\pi x \sin \frac{x}{2} \, dx$

19. Evaluate: $\int_1^e 16x^3 \ln x \, dx$

20. Let $A$, $B$, $C$, and $D$ be the answers to problems 16, 17, 18, and 19, respectively. Evaluate: $2 \ln \left( \frac{A + D - 2}{3} - \frac{B - 11C}{10} \right)$
21. Find the area of a quadrilateral with side lengths of 39, 52, 25, and 60 in that order.

22. A cube has volume of \( \cos^3 x \) (where \( 0 < x < \frac{\pi}{2} \)) and surface area of \( \frac{36}{17} \). If \( \sin^2 x = \frac{m}{n} \), where \( m \) and \( n \) are positive relatively prime integers, find \( m + n \).

23. Let \( C \) represent the locus of points in the plane equidistant from the graphs of \( x^2 + y^2 = 1 \) and \( y = -3 \). Find the slope of the line tangent to \( C \) at (8,6).

24. A random variable \( X \) has a probability density function \( f(x) \) such that \( f(x) = \frac{8x}{\pi^2} \) when \( 0 < x < \frac{\pi}{2} \) and \( f(x) = 0 \) for all other values of \( x \). What is the expected value of \( 3\pi^2 \sin X \)?

25. Let \( A \), \( B \), \( C \), and \( D \) be the answers to problems 21, 22, 23, and 24, respectively.
   Evaluate: \( \frac{A}{B} + \frac{D}{C} \)
26. Let $M$ be a $4 \times 4$ matrix such that $M \times \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} b \\ c/2 \\ 3d \\ a/4 \end{bmatrix}$ for all real numbers $a, b, c, \text{ and } d$. Find the sum of the elements of $3M^{-1}$.

27. The domain of $f(x) = \sin^6 x + \cos^6 x$ is all real numbers $x$. The range of $f$ is the interval $I = [a, b]$. Find the midpoint of $I$.

28. Evaluate: $\int_{-3}^{3} \left( \frac{\sin x}{1+x^2} + x^2 \right) \ dx$

29. Let $f(x) = 1 + x + x^7$ and $g$ be the inverse of $f$. Find $1024g'(3)$.

30. Let $A, B, C, \text{ and } D$ be the answers to problems 26, 27, 28, and 29, respectively. Evaluate: $A + BD + C$
31. Find \( P(100) \), where \( P(x) \) is a polynomial with real coefficients and 
\( P(x^2) + 2x^2 + 10x = 2xP(x + 1) + 3 \) for all real \( x \).

32. A triangle inscribed in the unit circle has angles measuring \( \alpha, \beta, \) and \( \gamma \). The perimeter of the triangle is 5. Evaluate: \( \sin \alpha + \sin \beta + \sin \gamma \)

33. If \( S \) is the set of distinct critical values of \( f(x) = (x - 1)^2(x + 1)^5 \), let \( c \) be the median of the elements of \( S \). If \( \cos \theta = c \) and \( |\cos(2\theta)| = \frac{m}{n} \), where \( m \) and \( n \) are relatively prime positive integers, find \( m + n \).

34. Find the maximum value of \( f(a, b) = 10(b^2 - a^2) - 16(b - a) - \frac{4}{3}(b^3 - a^3) \), where \( 0 < a < b \).

35. Let \( A, B, C, \) and \( D \) be the answers to problems 31, 32, 33, and 34, respectively.
Evaluate: \( -A + BC + D \)
36. In triangle \(ABC\) with centroid \(P\), let \(D\) and \(E\) be the foot of the medians to sides \(BC\) and \(AC\), respectively. If \(AP\) is perpendicular to \(BE\), \(|AD| = 6\), and \(|BE| = 9\), find the area of \(ABC\).

37. Find the number of times the polar graph \(r = 2^\theta\) intersects the line segment whose endpoints are the Cartesian coordinates \((\sqrt{2}, \sqrt{2})\) and \((64\sqrt{2}, 64\sqrt{2})\).

38. If \(I = \int_1^{25} \frac{1}{x + \sqrt{x}} \, dx\), find \(e^I\).

39. If \(I = \int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x + \cos x}} \, dx\), find \(I\).

40. Let \(A\), \(B\), \(C\), and \(D\) be the answers to problems 36, 37, 38, and 39, respectively. Evaluate: \(ABC \tan D\)
41. Define $\Pi(S)$ as the product of the elements of a set $S$. Let $S_1, S_2, S_3, \ldots, S_{15}$ be the nonempty subsets of $S = \{1, 2, 3, 4\}$. Evaluate: $\sum_{n=1}^{15} (\Pi(S_n))^{-1}$

42. Find, in degrees, the measure of the smallest angle in a right triangle with legs of length $a$ and $b$ and hypotenuse of length $2\sqrt{ab}$, where $a$ and $b$ are positive numbers.

43. If $y = \sin x$ and $F(x) = \sin x + \sum_{n=1}^{2013} \frac{d^n y}{dx^n}$ find $F(\pi)$.

44. Let $f(x) = \frac{\cos(5x) + \cos(3x)}{\sin(5x) - \sin(3x)}$. Evaluate: $f'(\pi/4)$

45. Let $A, B, C,$ and $D$ be the answers to problems 41, 42, 43, and 44, respectively. Evaluate: $ABCD$
46. Find the sum of all positive integers $n$ such that $\frac{2210}{(3n+5)(2n+3)}$ is an integer.

47. Find the smallest positive angle $x$ (in radians) satisfying the equation

$$
\left( \sin \left( \frac{2x}{3} \right) \cos \left( \frac{4x}{3} \right) + \cos \left( \frac{2x}{3} \right) \sin \left( \frac{4x}{3} \right) \right) \left( \cos \left( \frac{16x}{5} \right) \cos \left( \frac{6x}{5} \right) + \sin \left( \frac{16x}{5} \right) \sin \left( \frac{6x}{5} \right) \right) = \frac{1}{4}.
$$

48. Suppose $f$ and $g$ are functions that $f'(x) = g'(x)$ for all $x$. If $f(5) - g(5) = 5$, evaluate:

$$
\int_{-10}^{10} f(x) \, dx - \int_{-10}^{10} g(x) \, dx
$$

49. If $f(x) = e^x (12 \sin(3x) + 5 \cos(3x))$, evaluate: $f''(0) - 6f'(0) + 9f(0)$

50. Let $A, B, C, D$ be the answers to problems 46, 47, 48, and 49, respectively. Evaluate: $A + 4 \cos^2 (BC) - D$