Question	Solution
P1.	We have $x = \frac{20+10}{3} = \mathbf{10}$.
P2.	We have $\sec^4 \frac{5\pi}{6} = \left(-\frac{2}{\sqrt{3}}\right)^4 = \mathbf{\frac{16}{9}}$.
P3.	Since ln 2013 is a constant, the answer is $\mathbf{0}$.
P4.	The answer is $5(10-0) = \mathbf{50}$.
P5.	We have $A^C + \left(D \div \sqrt{9B + 16/B}\right) = 10^0 + \left(50 \div \sqrt{16+9}\right) = \mathbf{11}$.
1.	Rewrite as $\sqrt{10-x} - \sqrt{4-x} = 2$, then square both sides and simplify to obtain $5 - x = \sqrt{(10-x)(4-x)}$. Square both sides again and simplify and get $25 - 10x + x^2 = 40 - 14x + x^2$, or $4x = 15$, thus $x = \mathbf{15/4}$.
2.	The desired amplitude is $\sqrt{2^2 + \left(-2\sqrt{3}\right)^2} = \mathbf{4}$.
3.	By L'Hopital's Rule, $\lim_{x\to 0} \frac{1-\cos(2013x)}{x} = \lim_{x\to 0} \frac{2013\sin(2013x)}{1} = \mathbf{0}$.
4.	For this problem, a Power Series approximation might be more efficient than multiple applications of L'Hopital's Rule. We have $\lim_{x\to 0} \frac{16-16\cos x^2}{x \sin x^3} = \frac{16-16\left(1-\frac{x^4}{2!}+\frac{x^8}{4!}-\cdots\right)}{x\left(x^3-\frac{x^9}{3!}+\cdots\right)} = \frac{\frac{16}{2!}-\frac{16x^4}{4!}+\cdots}{1-\frac{x^6}{3!}+\cdots} = \frac{\frac{16}{2!}-0+0-\cdots}{1-0+0-\cdots} = \frac{16}{2!} = \mathbf{8}$.
5.	We have $AB + CD = \left(\frac{15}{4}\right)(4) + (0)(8) = \mathbf{15}$.
6.	Perfect squares have an odd number of positive divisors, hence it is for those values that the Tau Function will be congruent to 1 in modulo 2. The set of

	positive integer perfect squares less than 200 is $\{1^2, 2^2, \dots, 14^2\}$, having **14** elements.
7.	The graph of $y = f(x) = x + \sin x + e^x$ is strictly increasing. Since $f(0) = 1$ and $f(5) > 2$, we know there is one solution on the interval $x \in (0, 5)$. Because f is strictly increasing, there is exactly **1** solution.
8.	We have $f(h) = \frac{(10+h)^2-100}{h} = \frac{100+20h+h^2-100}{h} = 20 + h$, so $\frac{df}{dh} = 1$ for all h, making $f(10) + f'(10) = 30 + 1 = \mathbf{31}$.
9.	The limit represents the derivative of $f(x) = 1024 \ln x$ evaluated at $x = 2$. The answer is $f'(2) = \frac{1024}{2} = \mathbf{512}$.
10.	The determinant is equal to $BD - AC = (1)(512) - (14)(31) = \mathbf{78}$.
11.	The polynomial factors as $(2x+1)(x^2-4) = 0$, so the sum of the roots is $-\frac{\mathbf{1}}{\mathbf{2}}$. Note that this is the same value obtained via the "$-b/a$" trick, only because there are no imaginary roots.
12.	The equation simplifies to $\sin(2\theta) = 0$, or $\theta = \frac{\pi n}{2}$ for integer n. Since we have the restriction $\pi < \frac{\pi n}{2} \le 5\pi$, we must have $2 < n \le 10$. The sum of all solutions is $\sum_{n=3}^{10} \frac{\pi n}{2} = \frac{\pi}{2}(55 - 3) = \mathbf{26\pi}$.
13.	By L'Hopital's Rule, $L = \lim_{h\to 0} \frac{f(1+2h)-2f(1+h)+f(1)}{h^2} = \lim_{h\to 0} \frac{2f'(1+2h)-2f'(1+h)}{2h} = \lim_{h\to 0} \frac{f'(1+2h)-f'(1+h)}{h} = \lim_{h\to 0} \frac{2f''(1+2h)-f''(1+h)}{1} = 2f''(1) - f''(1) = f''(1)$. For $f(x) = \arctan x$, we have $f''(x) = -\frac{2x}{(x^2+1)^2}$, and so $L = f''(1) = -\frac{1}{2}$, making $100L = \mathbf{-50}$.

14.	If $\sin\theta = 3\cos\theta$, then $\cos\theta = \frac{1}{\sqrt{10}}$ and $\sin\theta = \frac{3}{\sqrt{10}}$. If $T(x) = \sin(2x)$, then $T'(x) = 2\cos(2x) = 2\cos^2 x - 2\sin^2 x$, so $T'(\theta) = 2\left(\left(\frac{1}{\sqrt{10}}\right)^2 - \left(\frac{3}{\sqrt{10}}\right)^2\right) = -\frac{8}{5}$. Therefore, $25T'(\theta) = \mathbf{-40}$.
15.	We have $C\cos(AB) + D = -50\cos(-13\pi) + -40 = 50 - 40 = \mathbf{10}$.
16.	If $\left\|\,\|x\| - 7\right\| \leq 8$, then $-8 \leq \|x\| - 7 \leq 8$, or $\|x\| \leq 15$, or $-15 \leq x \leq 15$ since the absolute value of any number must be at least 0. There are **31** integers in this interval.
17.	The slope of the line needs to equal $\tan 30° = 1/\sqrt{3}$. From the given equation, the slope of the line is $2/k$. Thus, $k = 2\sqrt{3}$, or $k^4 = \mathbf{144}$.
18.	Using Integration by Parts, we have $\int_0^\pi x\sin\frac{x}{2}\,dx = 4\sin\left(\frac{x}{2}\right) - 2x\cos\left(\frac{x}{2}\right)\Big\|_0^\pi = \mathbf{4}$.
19.	Using Integration by Parts, we have $\int_1^e 16x^3\ln x\;dx = 4x^4\ln x - x^4\|_1^e = \mathbf{3e^4 + 1}$.
20.	We have $2\ln\left(\frac{A+D-2}{3} - \frac{B-11C}{10}\right) = 2\ln\left(\frac{31+3e^4+1-2}{3} - \frac{144-11(4)}{10}\right) = 2\ln e^4 = \mathbf{8}$.
21.	Notice that $39^2 + 52^2 = 25^2 + 60^2$. Thus, the quadrilateral in question consists of two right triangles glued together at their hypotenuse. The area is therefore equal to $\frac{(39)(52)+(25)(60)}{2} = \mathbf{1764}$.
22.	The side length of the cube is $\cos x$. We have $6(\cos x)^2 = 36/17$, or $\cos^2 x = \frac{6}{17}$, so $\sin^2 x = 1 - \frac{6}{17} = \frac{11}{17} = \frac{m}{n}$, making $m + n = \mathbf{28}$.
23.	Let (x, y) represent a point in C in the neighborhood of $(8, 6)$. The description of the locus yields the equation $\sqrt{x^2 + y^2} - 1 = y + 3$, or $x^2 = 8y + 16$. Implicit

	differentiation yields $2x = 8\frac{dy}{dx}$, so the desired slope is $\frac{dy}{dx} = \frac{2x}{8} = \frac{2(8)}{8} = \mathbf{2}$.
24.	The expected value is given by $\int_0^{\pi/2} \left(\frac{8x}{\pi^2}\right)(3\pi^2 \sin x)\, dx = \mathbf{24}$. Use Integration by Parts to evaluate the integral.
25.	We have $\frac{A}{B} + \frac{D}{C} = \frac{1764}{28} + \frac{24}{2} = \mathbf{75}$.
26.	Observe that M is a sort of permutation-scaling matrix, where the first element goes to the fourth position and gets scaled by $\frac{1}{4}$, the second element goes to the first position without scaling, etc. Using this reasoning, we can go backwards and deduce that $M^{-1} \times \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 4d \\ a \\ 2b \\ \frac{c}{3} \end{bmatrix}$, making $M^{-1} = \begin{bmatrix} 0 & 0 & 0 & 4 \\ 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & \frac{1}{3} & 0 \end{bmatrix}$, so the sum of the elements of $3M^{-1}$ is $3\left(4 + 1 + 2 + \frac{1}{3}\right) = \mathbf{22}$.
27.	The given function can be written as $f(x) = 1 - \frac{3}{4}\sin^2 x$, having maximum value of 1 and minimum value of $\frac{1}{4}$. The midpoint of I is $\mathbf{\frac{5}{8}}$.
28.	The first term of the integrand is an odd function, so its definite integral on an interval symmetric to the origin is equal to 0. The answer is $\int_3^{-3} x^2\ dx = \mathbf{-18}$.
29.	We have $f'(x) = 1 + 7x^6$. By inspection, $f(1) = 3$ and thus, $g(3) = 1$. We have $1024g'(3) = \frac{1024}{f'(1)} = \frac{1024}{8} = \mathbf{128}$.
30.	We have $A + BD + C = 22 + \left(\frac{5}{8}\right)(128) + -18 = \mathbf{84}$.
31.	Suppose P has degree n. The left-hand side of the equation has degree $2n$ while the right-hand side has degree of $n + 1$. Therefore, $n = 1$ and P is a linear

	function, say $P(x) = mx + b$. Substitute this into the equation to obtain $mx^2 + b + 2x^2 + 10x = 2x(m(x+1) + b) + 3$, and combine like-coefficients to get $(m+2)x^2 + 10x + b = 2mx^2 + (2b + 2m)x + 3$. Setting corresponding coefficients to each other yields $m = 2$ and $b = 3$, so $P(x) = 2x + 3$ and $P(100) = 2(100) + 3 = \mathbf{203}$.
32.	From the Extended Law of Sines, $\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c} = \frac{1}{2R}$. Adding up these three equations yields $\sin\alpha + \sin\beta + \sin\gamma = \frac{a+b+c}{2R} = \frac{p}{2R} = \frac{5}{2(1)} = \mathbf{\frac{5}{2}}$. (Note: Inscribed triangles in the unit circle with a perimeter of 5 *is* possible. For example, pick one vertex to be at (1,0), the second at (0,1), and the third to be near the point $\left(\cos\frac{5\pi}{4}, \sin\frac{5\pi}{4}\right)$. Such a triangle will have perimeter greater than 5. By continuity, a perimeter of exactly 5 is attainable.)
33.	By the Product Rule, $f'(x) = (x-1)(x+1)^4(7x-3)$, yielding the critical point set of $S = \{-1, \frac{3}{7}, 1\}$. Therefore, $\cos\theta = c = 3/7$ and $\cos(2\theta) = 2\left(\frac{3}{7}\right)^2 - 1 = -\frac{31}{49}$, making $m = 31$ and $n = 49$. The answer is $31 + 49 = \mathbf{80}$.
34.	Note that $f(a,b)$ can be written as $f(a,b) = \int_a^b (20x - 16 - 4x^2)\,dx$, where $0 < a < b$. The integrand has zeroes at $x \in \{1, 4\}$, so set $a = 1$ and $b = 4$ to obtain the maximum value for f. We have $f(1,4) = \mathbf{18}$.
35.	We have $-A + BC + D = -203 + \left(\frac{5}{2}\right)(80) + 18 = \mathbf{15}$.
36.	Let P equal the intersection of the medians BE and AD. Point P divides the medians into a 2:1 ratio, so AP = 4 and EP = 3. Triangle APE is a right triangle with area 6, which happens to be one-sixth the area of ABC. The answer is **36.**

37.	The slope of the segment connecting the endpoints is 1; therefore, θ must be coterminal to $\frac{\pi}{4}$, or $\theta = \frac{\pi}{4} + 2\pi n$ for integer n. The r-values for each endpoint are $(\sqrt{2})\sqrt{2} = 2$ and $(64\sqrt{2})\sqrt{2} = 128$. Thus, we have $2^1 \le 2^{\frac{2\theta}{\pi}} \le 2^7$, or $\frac{\pi}{2} \le \theta \le \frac{7\pi}{2}$. Therefore, $\frac{\pi}{2} \le \frac{\pi}{4} + 2\pi n \le \frac{7\pi}{2}$, leading to a singular answer of $n = 1$. There is only **1** intersection point.
38.	Rewrite the integral as $2\int_1^{25} \frac{1}{2\sqrt{x}(1+\sqrt{x})}\, dx$ and let $u = 1 + \sqrt{x}$, so that the integral transforms to $2\int_2^6 \frac{1}{u}\, du = 2\ln\frac{6}{2} = 2\ln 3 = \ln 9$. If $I = \ln 9$, then $e^I = \mathbf{9}$.
39.	On the interval $x \in [0, \pi/2]$, $\sin x$ and $\cos x$ attain the same values, only in reverse order. Therefore, $I = \int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}\, dx = \int_0^{\pi/2} \frac{\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}}\, dx$, and adding these two equations together yields $2I = \int_0^{\pi/2} \frac{\sqrt{\sin x}+\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}}\, dx = \int_0^{\pi/2} 1\ dx = \frac{\pi}{2}$, making $I = \boldsymbol{\frac{\pi}{4}}$.
40.	We have $ABC \tan D = (36)(1)(9)\tan\frac{\pi}{4} = \mathbf{324}$.
41.	If $S = \{1\}$, then $\sum_{n=1}^{1} \frac{1}{\Pi(S_n)} = 1$. If $S = \{1, 2\}$, then $\sum_{n=1}^{3} \frac{1}{\Pi(S_n)} = 2$. In general, it can be proven by induction that if $S = \{1, 2, 3, \dots, n\}$, then $\sum_{n=1}^{2^n-1} \frac{1}{\Pi(S_n)} = n$, so for this problem the answer is **4**.
42.	Let θ denote the angle opposite the side with length a. We have $\sin(2\theta) = 2\sin\theta\cos\theta = 2\left(\frac{a}{2\sqrt{ab}}\right)\left(\frac{b}{2\sqrt{ab}}\right) = \frac{1}{2}$. Thus, $2\theta = 30°$ or $\theta = \mathbf{15°}$.
43.	The terms in the sum $\sum_{n=1}^{2013} \frac{d^n y}{dx^n}$ repeat with a period of 4, and the sum of those repeating four terms is $\cos x - \sin x - \cos x + \sin x = 0$. Since $2013 = 503(4) + 1$, $F(x) = \sin x + 503(0) + \cos x = \sin x + \cos x$, making $F(\pi) = \mathbf{-1}$.

44.	First, use the Sum-to-Product Identities to simplify the function before differentiating. We have $f(x)=\frac{\cos(5x)+\cos(3x)}{\sin(5x)-\sin(3x)}=\frac{2\cos\frac{5x+3x}{2}\cos\frac{5x-3x}{2}}{2\cos\frac{5x+3x}{2}\sin\left(\frac{5x-3x}{2}\right)}=\cot x$, so that $f'(x)=-\csc^2 x$. Thus, $f'\left(\frac{\pi}{4}\right)=\mathbf{-2}$.
45.	We have $ABCD=(4)(15)(-1)(-2)=\mathbf{120}$.
46.	Since $2(3n+5)-3(2n+3)=1$, $3n+5$ and $2n+3$ are relatively prime. Also, since $2210=2\times5\times13\times17$, we have $\frac{2210}{(3n+5)(2n+3)}=\frac{2\times5\times13\times17}{(3n+5)(2n+3)}=5\frac{(2\times13)(17)}{(3n+5)(2n+3)}$. Setting $3n+5=26$ and $2n+3=17$ yields $n=\mathbf{7}$. Turns out this is the only valid value for n.
47.	By the Sum-and-Difference Formulae, the left-hand-side of the equation simplifies to $\sin\frac{2x+4x}{3}\cos\frac{16x-6x}{5}=\sin(2x)\cos(2x)$, or $\frac{1}{2}\sin(4x)$ by the Double-Angle Formula for sine. Thus, the equation is $\frac{1}{2}\sin(4x)=\frac{1}{4}$, and $4x=\frac{\pi}{6}$, making $x=\boldsymbol{\pi/24}$.
48.	If $f'(x)=g'(x)$, then $f'(x)-g'(x)=0$, and after integrating both sides, $f(x)-g(x)=C$ for all real x. The given information indicates that $C=5$, so the desired integral has value $5\big(10-(-10)\big)=\mathbf{100}$.
49.	Since $12\sin(3x)+5\cos(3x)=13\sin(3x+\phi)$, where $\tan\phi=\frac{5}{12}$, we have $f(x)=13e^x\sin(3x+\phi)$ and $f(0)=5$. Differentiating, we get $f'(x)=f(x)+39e^x\cos(3x+\phi)$, so $f'(0)=f(0)+39\cos\phi=5+36=41$. Differentiating again, we get $f''(x)=2f'(x)-f(x)-117e^x\sin(3x+\phi)$, making $f''(0)=2(41)-5-117\left(\frac{5}{13}\right)=32$. Combining these results yields $32-6(41)+9(5)=\mathbf{-169}$.

50.	We have $A + 4\cos^2(BC) - D = 7 + 4\cos^2\frac{100\pi}{24} - (-169) = \mathbf{179}$.