1. If Brandon is last in line, there are 4! ways for the others to line up. If Brandon is second to last in line, Danielle has 3 choices in line and the other 3 can be arranged in 3! ways. This pattern continues for Brandon 3rd in line and 2nd in line (obviously he cannot be first in line). Summing up the cases we obtain, 4! + (3*3!) + (2 * 3!) + (1 * 3!) = **60**

2. As long as Link does not grab a regular bomb from his bag he will survive. **7/12**

3. Tom will earn $7.50 a total of 22 times in 24 hours, while Jerry will earn $4 a total of 44 times in 24 hours. The difference between their earnings is **$11**

4. The famous birthday problem states that all *n* birthdays are different with probability $\frac{n!\binom{365}{n}}{365^n}$. Take the complement of this to find the probability that at least two of the members have the same birthday. This probability has a value greater than 50% when *n* is 23, thus they need **20** more people in the room.

5. A complete bipartite graph, K_{mn} has (mn) edges and (m + n) vertices. Thus, 28*11 = **308**

6. To be semi-Eulerian, the graph must be connected and contain an open Eulerian trail. This rules out option III which is not connected. Option V can also be ruled out since there can only be exactly two vertices of odd degree for a graph to contain an open Eulerian trail (a theorem in graph theory). Thus **III** and **V** cannot be semi-Eulerian.

7. Same restrictions as in problem #6.

8. The Stirling number of the second kind $S(p,k)$ counts the number of partitions of a set of p elements into k nonempty indistinguishable boxes. Thus, we desire $S(6,3)$ = **90**

9. The Bell number B_p counts the number of partitions of a set of p elements into nonempty indistinguishable boxes. This is the sum of the Stirling numbers of the second kind $S(p,k)$ for all values of k. Note the value of k is restricted to 6 or fewer since we require the boxes to be nonempty. $B_6 = \mathbf{203}$

10. Let the steady state vector be $[x \quad y \quad z]$. Set $[x \quad y \quad z]\begin{bmatrix} 0.4 & 0.4 & 0.2 \\ 0.4 & 0.4 & 0.2 \\ 0.2 & 0.2 & 0.6 \end{bmatrix} = [x \quad y \quad z]$ and solve for x, y, and z. We find $x = y = z = {}^1/_3$.

11. Think of the circular permutations formula. There are $n!$ different ways we can pick a cycle of the vertices, with n different places that we can start. Hence, we count $n!$ / n =(n-1)! cycles. Since the graph is directed, we do not divide by 2 as in the circular permutations formula. Thus there are **(n-1)!** cycles

12. The seventh Catalan number is 429, and the chromatic number of a tree with more than 1 vertex is 2. 429 + 2 = **431**.

13. Plug the values into the given formula. Lambda is the average (6) and k is the desired value (8).

14. The transition matrix is constructed like an adjacency matrix. Rows count from nodes 1-3, top to bottom, and columns count from nodes 1-3, left to right. The entries are the weights, or probabilities, on each edge.

15. Perform the multiplication $\begin{bmatrix} 0.2 & 0.2 & 0.6 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0.8 & 0 & 0.2 \\ 0.4 & 0 & 0.6 \end{bmatrix}$ to find the probability vector after one repetition. Use this new probability vector with the transition matrix to find the probability vector after two repetitions. Repeat the multiplication process once more to obtain the probability vector after three repetitions.

16. The number of derangements of order 7 is 1854. There are 7! ways for the children to pick their shoes. Thus, 1854/5040 = **103/280**

17. Using the definition of a binomial coefficient, generalized to negative values, we find -7nCr3 = -84 and (3/2)nCr6 = 7/1024. The product is **-147/256**

18. Euler solved this problem in 1736, finding that the graph representation of the bridges was not Eulerian.

19. (0.8 * 2/30) / ((.8 * 2/30)(.1*28/30) = **4/11**

20. (.1) / ((1/10 * 1) + (9/10 * ½^5)) = **32/41**

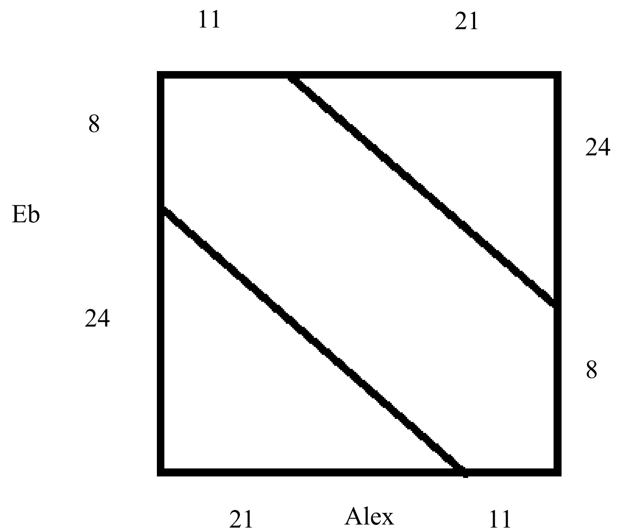
21. The Horton (96) graph is cubic since each vertex has degree 3. It is bipartite since there are no odd cycles. Its girth, length of the shortest cycle, is 6. Clearly it is not planar since edges are crossing one another in its construction. This famous graph is a counterexample to the Tutte conjecture that every cubic 3-connected bipartite graph is Hamiltonian. Thus, the graph is not Hamiltonian. To be Eulerian, the graph must contain an Eulerian cycle. This happens 'iff' every vertex has even degree (a theorem in graph theory). Since all vertices have odd degree, the Horton graph is not Eulerian.

22. The strategy would be for a player who sees six hats of the same color to guess the opposite color, and if he sees different colors, then he should pass. This strategy will result in a win for the partygoers in all cases except when the random hat distribution is all the same color. The general form for the probability of a win with 2 hat colors and $2^k - 1$ partygoers is given by $1 - \frac{2^{2^k - k - 1}}{2^{2^k - 1}}$. Substituting k = 3 in, we obtain 1 - 1/8 = **7/8**

23. Apply the Gale-Shapley algorithm to find one stable matching, with men proposing to women. Apply it again with women proposing to men. Respectively, this will find options II and III. I is not a stable matching since Angie prefers Joe over Henry and Joe prefers Angie over Cindy. IV is not a stable matching since Joe prefers Angie over Billie and Angie prefers Joe over Henry.

24. This is an application of the inclusion exclusion principle. First, we have 9! / 2 = 181440 permutations of all of the letters. Then we proceed to count permutations where the words HOW, ARE, and YOU appear. In other words, we count permutations of the letters HOW, A, R, E, Y, O, U (do this 3 times since we have three words). This amounts to 3*7! = 15120. Now we do the same where two words appear consecutively, or the permutations of the letters HOW, ARE, Y, O, U (again 3 sets of these since we have 3 words and we are picking two to be complete). This amounts to 3 * 5! = 360. Lastly, we count permutations where all three words appear, or the permutations of the letters HOW, ARE, YOU. This amounts to 3! = 6. Altogether we have 181440 – 15120 + 360 – 6 = 166674 permutations where none of the words are spelled out consecutively. Thus, the probability is given by 166674/181440 = **27779/30240**

25. Consider a geometric solution by constructing a 32 x 32 square as follows



65/128 is equivalent to 520/1024 which means that the area we subtracted from the 32 minute x 32 minute square was 504. Since Alex waits 11 minutes, his leg of the two triangles we subtract off from the square measured 32-11 = 21. 504/21 = 24 for Eb's leg of the triangles. Hence, Eb waited 32-24 = **8** minutes for Alex to arrive.

26. Since we have distinguishable objects, we apply the multiset combination formula with a slight change. In the top we have (n + k -1) where n represents the total candy bars and k represents the number of kids. To account for the repeated objects, in the bottom we have 1! * 2! * 2! * 2!, for each of the candy bar numbers in much the same way as a word permutation problem when accounting for repetition. We also have a 3! in the denominator which can be thought of as the partitioning bars to form 4 distinct groups of candy bars. Thus, 10! / 48 = 75600. I think this allows for the kids to receive 0 objects, which is against the premise I laid out in the problem. I need advice on this solution.

27. Maxwell's path forms a rectangle 15 units (in the x-direction) by 8 units (in the y-direction). Using a theorem in combinatorics, we find the number of interiors crossed by computing 15 + 8 – GCD(15,8) = **22**

28. The total lattice paths to work is given by 17 nCr 8 = 24310. There are 7 nCr 4 paths to the restaurant and from there another 10 nCr 4 paths to work. Thus, there are (7 nCr 4) * (10 nCr 4) = 7350 paths to work that will result in him buying a burger. 7350 / 24310 = **735/2431**

29. Sylvester's formula for non-representable integers is given by (A-1)(B-1)/2, where A and B represent the integers 11 and 17. Thus, there are 80 amounts that the two cannot make. 80/160 = **½**

30. We break this down into cases by enumerating the combination of the top three individuals and permuting them.

Case 1: B and G are both in top 3 with one other person

$\binom{2}{2} * \binom{6}{1} * 3! = 36$

and the corresponding probability that the names were read correctly would be ½ since the presenter would only have to guess once (between two names).

Case 2: one of either B or G plus 2 others with the same initial

$\binom{2}{1} * \binom{3}{1} * \binom{2}{2} * 3! = 36$

and the corresponding probability that the names were read correctly would be ½ since the presenter would only have to guess once (between two names).

Case 3: one of either B or G plus 2 others from different repeating initials

$\binom{2}{1} * \left(\binom{3}{2} * \binom{2}{1}\binom{2}{1}\right) * 3! = 144$

and the corresponding probability that the names were read correctly would be ¼ since the presenter must guess twice

Case 4: two from the same initial plus 1 from one repeating initial

$\left(\binom{3}{1} * \binom{2}{2}\right) * \left(\binom{2}{1} * \binom{2}{1}\right) * 3! = 72$

and the corresponding probability that the names were read correctly would be ¼ since the presenter must guess twice

Case 5: one from each repeating initial

$\binom{3}{3} * \binom{2}{1} * \binom{2}{1} * \binom{2}{1} * 3! = 48$

and the corresponding probability that the names were read correctly would be 1/8 since the presenter must guess three times

In total, (36 * ½) + (36 * ½) + (144 * ¼) + (72 * ¼) + (48 * 1/8) = 96. Thus, 96/336 = **2/7**