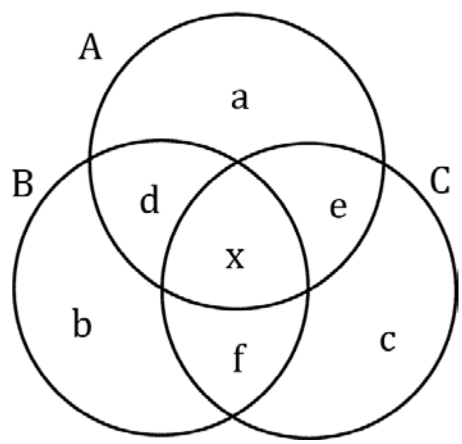
1. **(B)** The union of sets A and B is the set whose elements belong to at least one of A or B. Thus, $A \cup B = \{-2, -1, 0, 1, 2, 5\}$.
2. **(A)** The intersection of sets A and B is the set whose elements belong to both A and B. Thus, $A \cap B = \{0, 1\}$.
3. **(C)** The complement of A will contain 0 as an element, making the product of all elements in the complement of A equal to 0.
4. **(D)** The sum of all the elements of A is $\frac{13}{2}(0 + 12) = 78$. The sum of all the primes contained in A is $2 + 3 + 5 + 7 + 11 = 28$. The answer is $78 - 28 = 50$.
5. **(D)** Since S has 6 elements, the PowerSet has $2^6 = 64$ elements.
6. **(B)** The contrapositive involves reversing the order of premise and conclusion and negating them. Thus, the contrapositive is $\neg Q \rightarrow P$.
7. **(A)** The inverse involves negating both the premise and conclusion, but not reversing their order.
8. **(C)** Since $\forall$ means "for all," $\exists$ means "there exists," and $\wedge$ means "and," choice C is the best answer.
9. **(D)** Use the process of elimination. Choice A is incorrect because it says "There exists an x." Choice B is incorrect because it says "For all y." Choice C is incorrect because $x^2 > 0$ does not imply that x is positive. By inspection, Choice D is the correct answer.
10. **(C)** Choice A, B, and D all contain a symmetry that keeps the sets the same when switching the elements. The correct answer is C.
11. **(C)** The first bullet point is true. Clearly if $A = B$, then $A - B = B - A = \emptyset$. Now, suppose that $A \neq B$. Then there exists $a \in A$ such that $a \notin B$; by definition, $a \in A - B$. But since $A - B = B - A$, we must have $a \in B - A$, or $a \in B$ and $a \notin A$. Contradiction. The second bullet point is false; use the counterexample $A = \{1, 2, 3\}$, $B = \{2\}$, and $C = \{3\}$. The third bullet point is true; it is a distributive law for unions and intersections. Finally, the fourth bullet point is false; the union is a superset of its member sets, not a subset.
12. **(A)** The elements of set Y are just the first 50 smallest positive even integers, which has sum of $50^2 + 50 = 2550$.

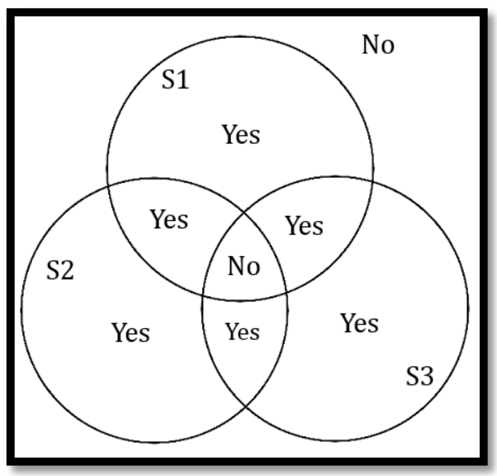
13. **(A)** Choice A is Russel's Paradox. Choice B is the Banach-Tarski Paradox. Choice C is Skolem's Paradox. Choice D is Zeno's Paradox.
14. **(B)** All are equivalent to the Axiom of Choice.
15. **(B)** For finite sets, we have $|A \times B| = |A||B| = (4)(3) = 12$.
16. **(A)** For each element x in the domain, there are 3 possible values for $f(x)$. There are 4 elements in the domain. Thus, there are $3 \times 3 \times 3 \times 3 = 81$ possible functions.
17. **(A)** The common difference of the sequence is $-58 - -65 = 7$. Cardinality is simply the number of terms in the sequence, which is $\frac{628-(-65)}{7} + 1 = 100$.
18. **(B)** It's easy to get lost in a jungle of curly braces, so we proceed carefully. Note that X is a set with a single element, that is, the empty set. Thus, the PowerSet of X will be a set with $2^1 = 2$ elements, or $Y = \{\emptyset, X\}$. Set Y has 2 elements, therefore, the PowerSet of Y—let's call it Z—will have $2^2 = 4$ elements. By construction, we have $Z = \{\emptyset, \{\emptyset\}, \{X\}, Y\} = \{\emptyset, X, \{X\}, \{\emptyset, X\}\}$. All the elements in the problem are in Z.
19. **(A)** Consider the Venn Diagram below



By the Principle of Inclusion-Exclusion, we have $|A \cup B \cup C| = 120 = |A| + |B| + |C| - |A \cap B| - |B \cap C| - |A \cap C| + |A \cap B \cap C|$, or $120 = 50 + 70 + 60 - (d + x) - (f + x) - (e + x) + x$. Simplifying this yields $d + e + f + 2x = 60$. To maximize $|A \cap B \cap C| = x$, simply let $d = e = f = 0$, making $x = 30$.

20. **(D)** If $11x - 6 \leq x^2 + 18$, then the interval is empty or doesn't make any sense. If $11x - 6 > x^2 + 18$, then the interval will be nonempty and have the same cardinality as $\mathbb{R}$. We have $11x - 6 > x^2 + 18$ when $3 < x < 8$; the sum of all integers in this interval is $4 + 5 + 6 + 7 = 22$.
21. **(C)** This problem is inspired by the iterative construction of the Natural Numbers. Due to the potential curly brace overload, we proceed carefully, like before: $S_1 = S_0 \cup \{S_0\}$, or $S_1 = \emptyset \cup \{\emptyset\} = \{\emptyset\}$. We have $S_2 = S_1 \cup \{S_1\} = \{\emptyset\} \cup \left\{\{\emptyset\}\right\} = \{\emptyset, \{\emptyset\}\}$. Finally, we have $S_3 = S_2 \cup \{S_2\}$, or
$S_3 = \left\{\emptyset, \{\emptyset\}\right\} \cup \left\{\left\{\emptyset, \{\emptyset\}\right\}\right\} = \{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}\}$.
22. **(A)** An ordering relation is a linear order if any two elements are comparable using that relation. For the divisibility relation $\leq_d$, this is false because, for example, the statements $7 \leq_d 13$ and $13 \leq_d 7$ are both false. Thus, Item I is false. A partial order is an order relation that is reflexive, antisymmetric, and transitive. The divisibility relation is clearly reflexive (any integer is divisible by itself). To check for antisymmetry, note that if a is divisible by b, then $a = kb$ for some positive integer k. If b is divisible by a, then $b = k'a$ for some positive integer k'. Thus we have $a = k(k'a) = kk'a$, or $kk' = 1$. Clearly k and k' both equal 1. Thus, the divisibility relation is antisymmetric. Similar reasoning shows that divisibility is transitive, and thus it satisfies the criteria for a partial order. Item II is true. An equivalence relation is a relation that is reflexive, symmetric, and transitive. The divisibility relation is not symmetric (e.g., 10 is divisible by 2, but not the other way around), so divisibility is not an equivalence relation. Item III is false. A well-ordering is a linear order where every nonempty subset has a smallest element. Because divisibility is not a linear order to begin with, it is also not a well-order. Item IV is false.
23. **(D)** The correct statement is choice D. Choice A is the Axiom of Regularity, Choice B is made up, and Choice C is the Axiom of Infinity.

24. **(B)** Item I is a topology, typically called the *Trivial Topology*. Likewise, II is a topology, typically called the *Discrete Topology*. Item III is not a topology because $\{a\} \cup \{b\} = \{a, b\} \notin T$. Item IV is a topology. Item V is not a topology because $\{b\} \cup \{c\} = \{b, c\} \notin T$. Item VI is a topology. Finally, item VII is a topology. There are 5 topological spaces in the problem.
25. **(B)** By definition, independence from the axioms of ZF means that a statement, or its negation, cannot be proved using the axioms of ZF.
26. **(A)** Consider the Venn Diagram below:

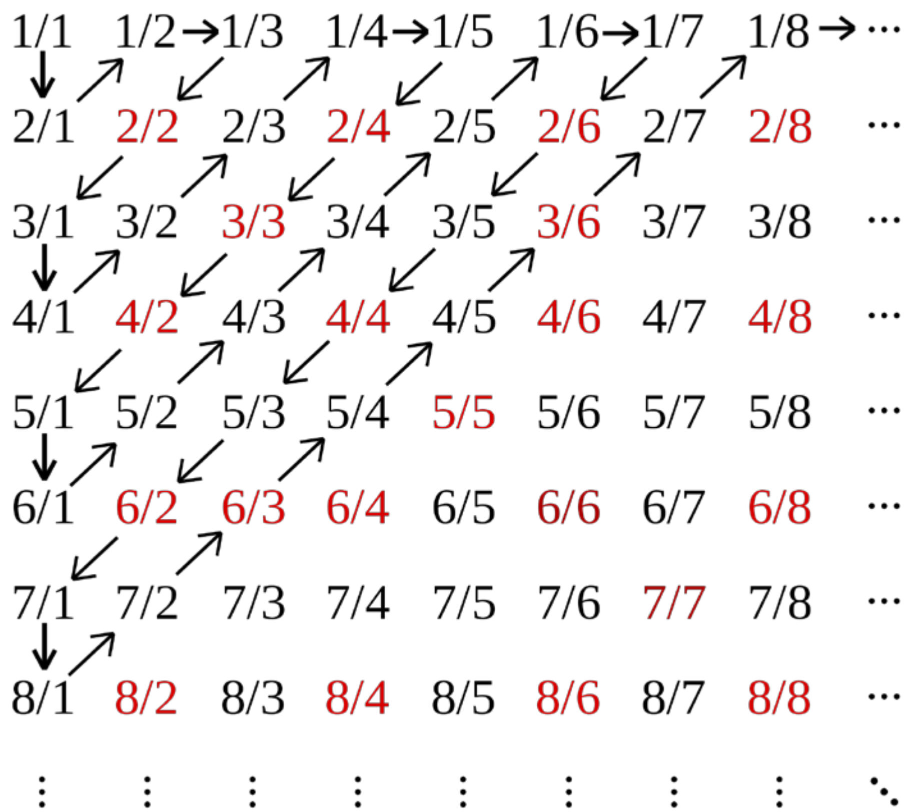


Imagine putting the integers from 1 to 5 (inclusive) into this Venn Diagram, with the circles representing S_1, S_2, and S_3. Putting the numbers into the regions marked "Yes" will yield $S_1 \cup S_2 \cup S_3 = S$ and $S_1 \cap S_2 \cap S_3 = \emptyset$. There are six "Yes" regions and five integers to allocate, so the answer is $6^5 = 7776$.

27. **(C)** The set of real numbers is in a "higher level" of infinite than the set of rational numbers, and no finite amount of unions, intersections, nor Cartesian products will change that. Thus, the only possible answer choice is C. We double-check to make sure that $S = \mathbb{R}^3$ indeed works. If we assume the Axiom of Choice, then we have $|S \times S| + 2|S| + |\mathbb{Q}| = |\mathbb{R}^3 \times \mathbb{R}^3| + |\mathbb{R}^3| + |\mathbb{R}^3| + |\mathbb{Q}| = |\mathbb{R}^3| + |\mathbb{R}^3| + |\mathbb{R}^3| = |\mathbb{R}^3| = |\mathbb{R}|$.

28. **(D)** Let $X = \{1, 2, 3, 4, 5\}$ and $Y = \{6, 7, 8, 9, 10\}$. For any subset $A \subset X$, we can "cycle" through its elements by successively increasing each element by 1. For example, if $A = \{1, 2\}$, this process would bring about the sets {1, 2}, {2, 3}, {3, 4}, {4, 5}, and {5, 1} (once an element reaches 5, it drops back to 1). Notice three things: one, this process produces five distinct sets. Second, the sum of the elements of each sets, when taken in modulo 5, form all the possible remainders upon division by 5 exactly once. Third, all the proper subsets of X are partitioned via this process; any subset $A \subset X$ belongs to exactly one group brought about by this process. Now consider a five-element subset S of $X \cup Y$. Clearly X and Y have the sum of their elements divisible by 5; let's exclude these possibilities for now. The elements of S can be divided into two groups: those that come from X, and those that come from Y. Suppose the sum of the elements of S that come from Y are congruent to r in modulo 5. To obtain a five-element set that satisfies the criteria in the problem, all we need to do is take the elements of S that come from X, assign it to the partition obtained from the process described earlier, and pick the elements from the same family whose element-sum is congruent to $-r$ in modulo 5. Exactly one of those sets will work, so one-fifth of the sets will have a sum divisible by 5. There are $C(10,5) = 252$ five-element subsets of $X \cup Y$, 250 of those not equal to X nor Y, and $\frac{1}{5}(250) = 50$ of those have an element-sum divisible by 5. Adding back the 2 sets we subtracted yields a total count of 52 sets.

29. **(B)** It's not difficult to extend the pattern:



Based on the above, the 25th element is 4/5.

30. **(A)** Careful! Ordinal arithmetic is not commutative! We have $(1+\alpha)\cdot 2+\alpha+2\cdot\alpha=(1+\alpha)+(1+\alpha)+\alpha+2\cdot\alpha=(\alpha)+(\alpha)+\alpha+\alpha=\alpha\cdot 4$.