

Note: For all questions, answer "(E) NOTA" means none of the above answers is correct.  
**Unless otherwise specified, all angles are measured in degrees.**

- The three angles of a triangle have measures given by  $3x - 5$ ,  $x + 30$ , and  $3x + 15$ , for some  $x$ . What is the measure of the smallest angle?  
(A) 20            (B) 50            (C) 55            (D) 75            (E) NOTA
- Triangle  $ABC$  is inscribed in a circle of radius 18. Arc  $\widehat{AB}$  has length  $6\pi$  and  $\widehat{BC}$  has length  $12\pi$ . Find  $m\angle B$  in radians.  
(A)  $\frac{\pi}{4}$             (B)  $\frac{\pi}{2}$             (C)  $\frac{2\pi}{3}$             (D)  $18\pi$             (E) NOTA
- Let  $m$  be the measure of an interior angle of a regular polygon that tessellates. Find the sum of all possible values of  $m$ .  
(A) 90            (B) 150            (C) 270            (D) 360            (E) NOTA
- The measures of two angles of a triangle, when expressed in Roman Numbers, are LXXXI and LVII. What is the measure of the third angle? Express your answer in Roman Numbers.  
(A) XXXII            (B) XLII            (C) LXII            (D) LIII            (E) NOTA
- Let  $ABC$  be a triangle with  $|\overline{AB}| = |\overline{AC}|$ , and let  $D$  be the midpoint of side  $\overline{BC}$ . Which of the following terms can be used to describe  $\overline{AD}$ ?  
1) Angle Bisector  
2) Median  
3) Altitude  
(A) 1 only            (B) 1 and 2 only            (C) 1 and 3 only            (D) 1, 2, and 3            (E) NOTA

For problems 6 through 8, a circle has two chords  $\overline{AB}$  and  $\overline{CD}$  that intersect at point  $P$ . Arc  $\widehat{AD}$  measures  $95^\circ$  and  $\widehat{BC}$  measures  $65^\circ$ .

- Find the measure of  $\angle APC$ .  
(A) 60            (B) 80            (C) 100            (D) 120            (E) NOTA
- Find the measure of  $\angle ABD$ .  
(A) 47.5            (B) 60            (C) 65            (D) 80            (E) NOTA

8. Find the difference between the measures of  $\angle BAC$  and  $\angle BDC$ .
- (A) 5                      (B) 10                      (C) 15                      (D) 20                      (E) NOTA
9. Let  $\triangle ABC$  and  $\triangle DEF$  be similar right triangles. If  $|\overline{AB}| = |\overline{BC}| = 1$ , find  $m\angle F$ .
- (A) 30                      (B) 45                      (C) 50                      (D) 60                      (E) NOTA
10. Let  $A$  be the measure of the base angle of an isosceles triangle with a vertex angle of  $100^\circ$ ,  $B$  be the measure of each interior angle of a regular pentagon, and  $C$  be the sum of the measures of complementary angles. Which of the following correctly portrays the relationship between  $A$ ,  $B$ , and  $C$ ?
- (A)  $A < B < C$     (B)  $B < A < C$     (C)  $A < C < B$     (D)  $C < B < A$     (E) NOTA
11. A circle has a radius of 10 meters, and a central angle measures out an arc of length  $\pi$ . What is the measure of this angle?
- (A) 18                      (B) 30                      (C) 36                      (D) 45                      (E) NOTA
12. The measures of the interior angles of a quadrilateral form a geometric sequence with common ratio of 2. Find the measure of the largest angle in this polygon.
- (A) 184                      (B) 192                      (C) 200                      (D) 216                      (E) NOTA
13. Which of the following is a name for an eleven-sided polygon?
- (A) Heptagon    (B) Unadecagon    (C) Odecagon    (D) Hendecagon    (E) NOTA
14. Circle  $O$  has a chord  $\overline{AB}$  and a diameter  $\overline{CD}$  that intersect at point  $Q$ .  $\overline{CD}$  bisects  $\overline{AB}$ . Find the measure of  $\angle BQC$ .
- (A) 60                      (B) 75                      (C) 90                      (D) 105                      (E) NOTA
15. Triangle  $\triangle DEF$  is an equilateral triangle. Line  $\ell_1$  intersects sides  $\overline{AC}$  and  $\overline{BC}$ , and a parallel line  $\ell_2$  intersects sides  $\overline{AB}$  and  $\overline{BC}$ . If the smaller of the two angles between  $\ell_1$  and  $\overline{AC}$  is  $40^\circ$ , what is the larger of the two angles between  $\ell_2$  and  $\overline{AB}$ ?
- (A) 120                      (B) 140                      (C) 150                      (D) 160                      (E) NOTA

16. A circle is circumscribed about an equiangular triangle. If the area of the circle is  $x$ , what is the perimeter of the triangle (in terms of  $x$ )?

- (A)  $\sqrt{\frac{27x}{\sqrt{3}}}$       (B)  $3\sqrt{\frac{3x}{\pi}}$       (C)  $\sqrt{\frac{4x}{\sqrt{3}}}$       (D)  $\sqrt{\frac{3x}{\pi}}$       (E) NOTA

For problems 17 and 18, consider a triangle  $\triangle ABC$  where  $m\angle A = 100^\circ$  and  $m\angle B = 50^\circ$ . Point  $D$  exists such that  $\triangle BCD \sim \triangle ABC$ , where these two triangles do not overlap.

17. If there is a circle centered at  $D$  with a radius smaller than any of the sides of the triangles, what percent of the circle's area lies inside  $\triangle BCD$ ?

- (A)  $8\frac{1}{3}\%$       (B)  $13\frac{8}{9}\%$       (C)  $25\%$       (D)  $27\frac{7}{9}\%$       (E) NOTA

18. Suppose there is a circle centered at  $B$  with a radius smaller than any of the sides of the triangles. Let  $m$  be the percent of the circle's area that lies inside of  $\triangle ABC$ , and let  $n$  be the percent of the circle's area that lies inside of  $\triangle BCD$ . Find  $\frac{m}{n}$ .

- (A)  $\frac{3}{5}$       (B)  $\frac{1}{2}$       (C)  $\frac{3}{10}$       (D) 1      (E) NOTA

19. A special billiards table is shaped like an equilateral triangle, with pockets (labeled A, B, and C) at the corners. A ball is located at the midpoint of the wall between pockets B and C. Let  $\theta$  be the angle at which the ball must be hit to bounce off wall AB and then sink into pocket C, where  $\theta$  is measured with respect to the normal to wall BC. What is  $\tan \theta$ ?

- (A)  $\frac{\sqrt{3}}{2}$       (B)  $\frac{2\sqrt{3}}{3}$       (C)  $\frac{1}{2}$       (D)  $\frac{\sqrt{2}}{3}$       (E) NOTA

20. Let  $C$  and  $D$  be the centers of two circles, not necessarily with the same radius, that overlap and intersect at points  $X$  and  $Y$ . Let  $Z$  be the point at which  $\overline{CD}$  and  $\overline{XY}$  intersect. Find the measure of  $\angle XZC$ .

- (A) 100      (B) 80      (C) 90      (D) 120      (E) NOTA

21. If  $A = (1,0,0)$ ,  $B = (1,1,1)$ , and  $C = (0,0,1)$ , what is  $m\angle ABC$ ?

- (A) 40      (B) 45      (C) 50      (D) 55      (E) NOTA

22. In Spherical Geometry, which of the following can be the value of the sum of the interior angles of a triangle?
- 1)  $90^\circ$
  - 2)  $180^\circ$
  - 3)  $270^\circ$
- (A) 1 only      (B) 2 only      (C) 1 and 2 only      (D) 2 and 3 only      (E) NOTA
23. Let  $ABCD$  be a right trapezoid with bases  $|\overline{AB}| = 2$  and  $|\overline{CD}| = 3$ . Angle  $B$  is a right angle, and diagonal  $\overline{AC}$  bisects angle  $C$ . What is the length of the diagonal  $\overline{BD}$ ?
- (A)  $\sqrt{7}$       (B) 3      (C)  $\sqrt{11}$       (D)  $\sqrt{13}$       (E) NOTA
24. Let  $\triangle ABC$  be inscribed in a circle and  $D$  be another point on the circle. Which of the following is/are possible?
- 1)  $\triangle ABC \sim \triangle ADC$
  - 2)  $\triangle ABC \sim \triangle ACD$
  - 3)  $\triangle ABC \sim \triangle CDA$
- (A) 1 only      (B) 1 and 2 only      (C) 1 and 3 only      (D) 1, 2, and 3      (E) NOTA
25. In circle  $O$ , points  $D$ ,  $E$ , and  $F$  are on the circle,  $|\overline{DE}| = |\overline{EF}|$ , and  $m\angle DOF = 84$ . Find  $m\angle EDF$ .
- (A) 20      (B) 21      (C) 23      (D) 25      (E) NOTA
26. For any polygon, we define a *diagonal* to be a line segment connecting two non-adjacent vertices. Let  $P$  be a vertex on a regular decagon, and let  $\theta$  be the angle between two randomly-selected diagonals that pass through  $P$ . Find the difference between the maximum and minimum possible values of  $\theta$ .
- (A) 45      (B) 60      (C) 90      (D) 120      (E) NOTA
27. Points  $A$  and  $B$  move along the Cartesian plane parametrically:  $A = (2t, 3 - t)$ ,  $B = (t + 2, 1 - t)$ . Point  $C$  is fixed at  $(4, -4)$ . Find the sum of all values of  $t$  for which  $A$ ,  $B$ , and  $C$  will *not* form a triangle.
- (A) 0      (B) 2      (C) 5      (D) 9      (E) NOTA

28. Let  $\ell$  and  $m$  be two lines that intersect at point  $O$ . Lines  $x_1$  and  $x_2$  are parallel lines that intersect  $\ell$  at points  $A$  and  $B$ , and intersect  $m$  at points  $A'$  and  $B'$ , respectively. Lines  $y_1$  and  $y_2$  are also parallel lines. Line  $y_1$  passes through  $B$  and intersects  $m$  at point  $C'$ , and  $y_2$  passes through  $A'$  and intersects  $\ell$  at point  $C$ . Which of the following sets of triangles are not similar?

- (A)  $\triangle OBC'$  and  $\triangle OCA'$                       (B)  $\triangle OAA'$  and  $\triangle OBB'$   
(C)  $\triangle OAB'$  and  $\triangle OCC'$                       (D)  $\triangle OAC'$  and  $\triangle OCB'$                       (E) NOTA

29. Given any triangle  $\triangle DEF$ , let  $O$  be a circle passing through  $D$  and  $F$  such that  $\overline{DE}$  is tangent to  $O$ . In the circle, draw a chord through  $D$  that is parallel to  $\overline{EF}$  – the new point of intersection between this chord and the circle is point  $P$ . Draw the line connecting  $E$  to  $P$  – this will intersect circle  $O$  at  $P$  and a new point – call it  $Q$ . Which of the following angles is not equal in measure to the others?

- (A)  $\angle QDE$               (B)  $\angle QFD$               (C)  $\angle QPD$               (D)  $\angle QEF$               (E) NOTA

30. Let  $ABCDE$  be a regular polygon and  $CDFG$  be a square inside of the pentagon. Find the difference between the measures of  $\angle AGF$  and  $\angle BAG$ .

- (A) 25              (B) 30              (C) 32              (D) 36              (E) NOTA