

Question	Solution
0.	Multiply both sides of the equation by 3 to obtain $3(3x + 4) = 9x + 12 = 3(6) = \mathbf{18}$ .
1.	There are 2013 terms in the series. Since addition and subtraction is associative, the series is equal to $(1 - 2) + (3 - 4) + \dots + (2011 - 2012) + 2013$ , or $-1 - 1 - 1 - \dots + 2013$ . There are $\frac{2012}{2} = 1006$ copies of $-1$ , making the sum $-1006 + 2013 = \mathbf{1007}$ .
2.	Since $\frac{.25+.652}{2} = .451$ , the answer is $4 + 5 + 1 = \mathbf{10}$ .
3.	Divide both sides of the equation by $e^2$ to obtain $e^{2x} - e^x = 20$ , or after some factoring, $(e^x - 5)(e^x + 4) = 0$ . This means that $e^x = 5$ or $e^x = -4$ . Since $x$ is real, the exponential cannot be negative, making the sum equal to $\mathbf{5}$ .
4.	The exterior angle of the nonagon is $360^\circ/9 = 40^\circ$ . Notice that since triangle GHI is isosceles, angles GIH and HGI are congruent, and by the Exterior Angle Theorem, add up to $40^\circ$ . Therefore, the measure of angle GIH is $40^\circ/2 = \mathbf{20^\circ}$ .
5.	The sum of the roots of $f$ is 14. Since $a$ and $b$ are prime, the only possible values are 3 and 11. Thus, $a^3 + b^3 = 3^3 + 11^3 = 27 + 1331 = \mathbf{1358}$ .
6.	Straightforward calculation. The determinant is $(46)(76) - (2)(-34) = \mathbf{3564}$ .
7.	Experiment with multiples of Pythagorean Triples, and see that the desired triangle is a multiple of the 5-12-13 triple, specifically, the 20-48-52 triangle. The perimeter is $\mathbf{120}$ .
8.	In order for a number to be divisible by 4, the last two digits have to be a multiple of 4. There are $(5)(4) = 20$ possibilities for the last two digits, two of which—48 and 84—are multiples of 4. The probability is $2/20 = \mathbf{1/10}$ .

9.	<p>Since <math>2500^n = 2^{2n} \times 5^{4n}</math>, we are only interested in the exponents of 2 and 5 in the prime factorization of 2013!. The exponent of 5 will be significantly less than the exponent of 2 in the factorization, so that will be the “limiting factor” in our search for the largest <math>n</math>. We find the exponent of 5 in the factorization as follows:</p> $\left\lfloor \frac{2013}{5} \right\rfloor + \left\lfloor \frac{2013}{25} \right\rfloor + \left\lfloor \frac{2013}{125} \right\rfloor + \dots = 501$ <p>So <math>4n \leq 501</math>, making <b>125</b> the largest value for <math>n</math>.</p>
10.	<p>Write out the product, use change-of-base, and observe that the terms cancel:</p> $\frac{\ln 3}{\ln 2} \times \frac{\ln 4}{\ln 3} \times \dots \times \frac{\ln n}{\ln(n-1)} \times \frac{\ln(n+1)}{\ln n} = \frac{\ln(n+1)}{\ln 2} = \log_2(n+1)$ <p>If <math>\log_2(n+1) = 2013</math>, then <math>n = 2^{2013} - 1</math>, which, when converted to binary, is simply a string of 2013 1s. Thus, <math>D(n) = \mathbf{2013}</math>.</p>
11.	<p>Take natural logs of both sides to obtain <math>(x^2 + x - 12) \ln 2 = (x^2 - 5x + 6) \ln 3</math>, or after factoring, <math>(x + 4)(x - 3) \ln 2 = (x - 3)(x - 2) \ln 3</math>. By inspection, <math>x = \mathbf{3}</math> is a solution, and really, the only integer solution because after cancellation, the resulting equation is <math>(x + 4) \ln 2 = (x - 2) \ln 3</math>, which does not have solutions if <math>x</math> is an integer.</p>
12.	<p>Let <math>x = 1</math> in the functional equation. We have <math>f(f(1)) = f(3) - 3</math>, or <math>f(4) = f(3) - 3</math>, so that <math>f(3) = f(4) + 3 = 3 + 3 = 6</math>. Next, let <math>x = 4</math> in the functional equation to obtain <math>f(f(4)) = f(6) - 3</math>, or <math>f(3) = f(6) - 3</math>, so that <math>f(6) = f(3) + 3 = 6 + 3 = 9</math>. Finally, let <math>x = 3</math> in the functional equation to obtain <math>f(f(3)) = f(5) - 3</math>, or <math>f(6) = f(5) - 3</math>, so that <math>f(5) = f(6) + 3 = 9 + 3 = \mathbf{12}</math>.</p>