Note: For all questions, answer "(E) NOTA" means none of the above answers is correct.

The following conventions will be used throughout this test: Of course, \( i = \sqrt{-1} \). For a complex number \( z \), \(|z|\) denotes the absolute value (sometimes called the modulus or magnitude) of \( z \), \( \text{Re}(z) \) and \( \text{Im}(z) \) are the Real and Imaginary parts of \( z \), respectively. Moreover, \( \bar{z} \) will denote the conjugate of \( z \).

1. Evaluate: \((-5 + 2i) - (12 - 4i)\)
   
   (A) \(-17 - 2i\)  (B) \(-17 + 6i\)  (C) \(17 - 2i\)  (D) \(17 + 6i\)  (E) NOTA

2. Solve for \( x \): \(|24 - 7i| = x|20 + 21i|\)
   
   (A) \(\frac{25}{29}\)  (B) \(\frac{27}{23}\)  (C) \(\frac{23}{27}\)  (D) \(\frac{29}{25}\)  (E) NOTA

3. Simplify: \(-\frac{5\sqrt{18}}{\sqrt{45}}\)
   
   (A) \(-i\sqrt{10}\)  (B) \(-i\sqrt{15}\)  (C) \(i\sqrt{10}\)  (D) \(i\sqrt{15}\)  (E) NOTA

4. Solve for \( z = a + bi \) where \( a \) and \( b \) are real, \( a = b - 2 \), and \( z^2(z - 2i) = 2z \).
   
   (A) \(0\)  (B) \(-1 + i\)  (C) \(2i\)  (D) \(1 + i\)  (E) NOTA

5. Evaluate: \((i\sqrt{3} + 1)^3\)
   
   (A) \(8i\)  (B) \(-8i\)  (C) \(8\)  (D) \(-8\)  (E) NOTA

6. Find the distance between the points \(4 - 10i\) and \(3i + 7\) in the complex plane.
   
   (A) \(\sqrt{290}\)  (B) \(\sqrt{58}\)  (C) \(\sqrt{178}\)  (D) \(\sqrt{10}\)  (E) NOTA

7. Simplify: \(\frac{s+5i}{3-4i} + \frac{20}{3i+4}\)
   
   (A) \(1 + i\)  (B) \(1 + 2i\)  (C) \(3 - i\)  (D) \(-3 + 2i\)  (E) NOTA

8. When plotted in the complex plane, the numbers \(3 - 7i\), \(10\), and \(i - 4\) form the vertices of a triangle with area equal to \(m/n\), where \(m\) and \(n\) are relatively prime positive integers. Find the value of \(m + n\).
   
   (A) \(8\)  (B) \(43\)  (C) \(78\)  (D) \(113\)  (E) NOTA
9. If \( z = 9 - 12i \), calculate: \( \frac{z}{\sqrt{z^2}} \)

(A) \( \frac{3+4i}{5} \)  
(B) \( \frac{3-4i}{5} \)  
(C) \( \frac{-3+4i}{5} \)  
(D) \( \frac{-3-4i}{5} \)  
(E) NOTA

10. If \( \text{Im}(z) = 6 \), \( |z| = 15 \), and \( z \) lies in the second quadrant of the complex plane, what is the value of \( \text{Re}(z) \)?

(A) \(-3\sqrt{21}\)  
(B) 9  
(C) \(3\sqrt{21}\)  
(D) -9  
(E) NOTA

11. Which of the following is/are necessarily true?

I) If \( z_1 \) and \( z_2 \) lie in the first quadrant of the complex plane, then \( z_1 + z_2 \) is also in the first quadrant of the complex plane.

II) If \( z_1 \) and \( z_2 \) lie in the first quadrant of the complex plane, then \( z_1 z_2 \) is also in the first quadrant of the complex plane.

III) If \( z_1 \) and \( z_2 \) lie in the first quadrant of the complex plane, then \( z_1/z_2 \) is also in the first or fourth quadrant of the complex plane.

(A) I and II  
(B) I  
(C) II  
(D) III  
(E) NOTA

12. Calculate the reciprocal of \( 1 + 4i \).

(A) \( \frac{1-4i}{17} \)  
(B) \( \frac{4+i}{4} \)  
(C) \( \frac{4-i}{4} \)  
(D) \( \frac{1+4i}{17} \)  
(E) NOTA

13. What is the imaginary part of \( (2 + 2i)^4 \)?

(A) 16  
(B) -16  
(C) 0  
(D) 8  
(E) NOTA

14. A Gaussian Integer is a complex number of the form \( z = a + bi \), where \( a \) and \( b \) are integers. Consider the Gaussian Integer \( z = m + 3ni \). Which of the following is not a possible value for \( |z|^2 \)?

(A) 2010  
(B) 2011  
(C) 2012  
(D) 2013  
(E) NOTA

15. Compute the sum of the absolute values of the solutions \( z \) in the equation \( z - 5 = \sqrt{-10z} + 4z\sqrt{-5} \).

(A) 10\sqrt{5}  
(B) \sqrt{5}  
(C) 5\sqrt{5}  
(D) 25\sqrt{5}  
(E) NOTA

16. Evaluate: \( \left( \frac{7-3i}{1+i} \right) \left( \frac{6-5i}{4-10i} \right) \)

(A) \( \frac{21-3i}{2} \)  
(B) \( \frac{21+3i}{2} \)  
(C) \( \frac{6-5i}{2} \)  
(D) \( \frac{6+5i}{2} \)  
(E) NOTA
17. Find the sum of the geometric series: \(1 + \frac{i}{3} - \frac{1}{9} - \frac{i}{27} + \cdots\) (Assume the series converges.)

(A) \(\frac{3}{2}\)  (B) \(\frac{9+3i}{10}\)  (C) \(\frac{9-3i}{10}\)  (D) \(\frac{-9-3i}{10}\)  (E) NOTA

18. Compute the product of the absolute values of the solutions \(z\) in the equation \(z^2 + 2|z|^2 = 2\).

(A) \(\frac{1}{4}\)  (B) \(\frac{2}{3}\)  (C) \(1\)  (D) \(\frac{4}{3}\)  (E) NOTA

19. The polynomial \(P(x) = x^4 + ax^3 + bx^2 + cx + d\) has real-number coefficients and \(P(1 + i) = P(3i) = 0\). Find the value of \(P(2)\).

(A) \(26\)  (B) \(10\)  (C) \(50\)  (D) \(130\)  (E) NOTA

20. Evaluate: \(\left(\sqrt{6} + i\sqrt{2}\right)^8\)

(A) \(-2048 - 2048i\sqrt{3}\)  (B) \(1296 + 16i\)  (C) \(1312\)  (D) \(-16 - 16i\sqrt{3}\)  (E) NOTA

21. Solve the equation for real \(x\) and \(y\): \(3(3 - 2i) - (1 - 8i) = x(1 + i) - y(i - 1)\). Express your answer as an ordered pair \((x, y)\).

(A) \((-3, 11)\)  (B) \((-3, -11)\)  (C) \((5, -3)\)  (D) \((5, 3)\)  (E) NOTA

22. Let \(z\) be a complex number with absolute value equal to 1. Let \(N = \frac{z^{-1}}{z+1}\). Find the ratio of \(\text{Re}(N)\) to \(\text{Im}(N)\).

(A) \(2\)  (B) \(\frac{1}{2}\)  (C) \(1\)  (D) \(0\)  (E) NOTA

23. Calculate the sum of the complex roots \(z\) of \((z + z^2)^2 = 15(3z^2 + 2)(z^2 + 2z)\).

(A) \(-2i\)  (B) \(-2\)  (C) \(2i\)  (D) \(2\)  (E) NOTA

24. Let \(x, y,\) and \(z\) be the three positive prime factors of 2013 where \(0 < x < y < z\). Calculate \(i^{2x} + i^y + i^z\).

(A) \(1\)  (B) \(-1 + 2i\)  (C) \(1 - 2i\)  (D) \(-1\)  (E) NOTA

25. Let \(k = 2013^{3001}\) and \(n = 37^k\). Simplify: \(i^n\)

(A) \(1\)  (B) \(-i\)  (C) \(-1\)  (D) \(i\)  (E) NOTA
26. For what real values of $c$ will $f(z) = 3z^2 + 5z + c^2$ have no real roots?

(A) $|c| > \frac{\sqrt{3}}{6}$
(B) $|c| > \frac{5\sqrt{3}}{6}$
(C) $|c| > \frac{\sqrt{3}}{3}$
(D) $|c| > \frac{5}{2}$
(E) NOTA

27. If $\frac{2-\sqrt{3}+5i+2i\sqrt{3}}{2+5i} = z^2$, which of the following is a possible value for $z$?

(A) $\sqrt{3} + i$
(B) $1 + i\sqrt{3}$
(C) $\frac{\sqrt{6}}{2} + \frac{i\sqrt{2}}{2}$
(D) $\frac{\sqrt{2}}{2} + \frac{i\sqrt{6}}{2}$
(E) NOTA

28. If $x$ and $y$ are integers, $z = x + yi$, and $z^3 = 18 + 26i$, find the value of $x^5 + y^5$.

(A) 3126
(B) 992
(C) 244
(D) 33
(E) NOTA

29. Given the matrix $A = \begin{pmatrix} 3 & -2 \\ 4 & -1 \end{pmatrix}$, determine all possible values of $\lambda$ such that the determinant of the matrix $A - \lambda I$ is equal to 0. Recall that $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, the $2 \times 2$ identity matrix.

(A) $1 \pm 2i$
(B) $2 \pm 3i$
(C) $3 \pm 4i$
(D) $4 \pm 5i$
(E) NOTA

30. Evaluate: $\left(1 + i\sqrt{3}\right)^4 \left(2 - 2i\sqrt{3}\right)^5$

(A) $512 + 512i\sqrt{3}$
(B) $256 - 256i\sqrt{3}$
(C) $512 - 512i\sqrt{3}$
(D) $256 + 256i\sqrt{3}$
(E) NOTA