Note: For all questions, answer "(E) NOTA" means none of the above answers is correct.

The following conventions will be used throughout this test: Of course, $i = \sqrt{-1}$. For a complex number z, $|z|$ denotes the absolute value (sometimes called the modulus or magnitude) of z, $\mathrm{Re}(z)$ and $\mathrm{Im}(z)$ are the Real and Imaginary parts of z, respectively. Moreover, $\bar{z}$ will denote the conjugate of z.

1. Evaluate: $(-5+2i)-(12-4i)$

   (A) $-17-2i$ (B) $-17+6i$ (C) $17-2i$ (D) $17+6i$ (E) NOTA

2. Solve for x: $|24-7i| = x|20+21i|$

   (A) $\frac{25}{29}$ (B) $\frac{27}{23}$ (C) $\frac{23}{27}$ (D) $\frac{29}{25}$ (E) NOTA

3. Simplify: $\frac{-5\sqrt{18}}{\sqrt{-45}}$

   (A) $-i\sqrt{10}$ (B) $-i\sqrt{15}$ (C) $i\sqrt{10}$ (D) $i\sqrt{15}$ (E) NOTA

4. Solve for $z = a+bi$ where a and b are real, $a = b-2$, and $z^2(z-2i) = 2z$.

   (A) 0 (B) $-1+i$ (C) $2i$ (D)$1+i$ (E) NOTA

5. Evaluate: $(i\sqrt{3}+1)^3$

   (A) $8i$ (B) $-8i$ (C) 8 (D) -8 (E) NOTA

6. Find the distance between the points $4-10i$ and $3i+7$ in the complex plane.

   (A) $\sqrt{290}$ (B) $\sqrt{58}$ (C) $\sqrt{178}$ (D) $\sqrt{10}$ (E) NOTA

7. Simplify: $\frac{5+5i}{3-4i} + \frac{20}{3i+4}$

   (A) $1+i$ (B) $1+2i$ (C) $3-i$ (D) $-3+2i$ (E) NOTA

8. When plotted in the complex plane, the numbers $3-7i$, 10, and $i-4$ form the vertices of a triangle with area equal to m/n, where m and n are relatively prime positive integers. Find the value of $m+n$.

   (A) 8 (B) 43 (C) 78 (D) 113 (E) NOTA

9. If $z = 9 - 12i$, calculate: $\frac{z}{\sqrt{\bar{z}z}}$

(A) $\frac{3+4i}{5}$ (B) $\frac{3-4i}{5}$ (C) $\frac{-3+4i}{5}$ (D) $\frac{-3-4i}{5}$ (E) NOTA

10. If $\mathrm{Im}(z) = 6$, $|z| = 15$, and z lies in the second quadrant of the complex plane, what is the value of $\mathrm{Re}(z)$?

(A) $-3\sqrt{21}$ (B) 9 (C) $3\sqrt{21}$ (D) -9 (E) NOTA

11. Which of the following is/are necessarily true?
I) If z_1 and z_2 lie in the first quadrant of the complex plane, then $z_1 + z_2$ is also in the first quadrant of the complex plane.
II) If z_1 and z_2 lie in the first quadrant of the complex plane, then $z_1 z_2$ is also in the first quadrant of the complex plane.
III) If z_1 and z_2 lie in the first quadrant of the complex plane, then z_1/z_2 is also in the first or fourth quadrant of the complex plane.

(A) I and II (B) I (C) II (D) III (E) NOTA

12. Calculate the reciprocal of $1 + 4i$.

(A) $\frac{1-4i}{17}$ (B) $\frac{4+i}{4}$ (C) $\frac{4-i}{4}$ (D) $\frac{1+4i}{17}$ (E) NOTA

13. What is the imaginary part of $(2 + 2i)^4$?

(A) 16 (B) -16 (C) 0 (D) 8 (E) NOTA

14. A *Gaussian Integer* is a complex number of the form $z = a + bi$, where a and b are *integers*. Consider the Gaussian Integer $z = m + 3ni$. Which of the following is *not* a possible value for $|z|^2$?

(A) 2010 (B) 2011 (C) 2012 (D) 2013 (E) NOTA

15. Compute the sum of the absolute values of the solutions z in the equation
$z - 5 = \sqrt{-10z + 4z\sqrt{-5}}$.

(A) $10\sqrt{5}$ (B) $\sqrt{5}$ (C) $5\sqrt{5}$ (D) $25\sqrt{5}$ (E) NOTA

16. Evaluate: $\left(\frac{7-3i}{1+i}\right)\left(\frac{6-5i}{4-10i}\right)$

(A) $\frac{21-3i}{2}$ (B) $\frac{21+3i}{2}$ (C) $\frac{6-5i}{2}$ (D) $\frac{6+5i}{2}$ (E) NOTA

17. Find the sum of the geometric series: $1+\frac{i}{3}-\frac{1}{9}-\frac{i}{27}+\cdots$ (Assume the series converges.)

(A) $\frac{3}{2}$ (B) $\frac{9+3i}{10}$ (C) $\frac{9-3i}{10}$ (D) $\frac{-9-3i}{10}$ (E) NOTA

18. Compute the product of the absolute values of the solutions z in the equation $z^2+2|z|^2=2$.

(A) 1/4 (B) 2/3 (C) 1 (D) 4/3 (E) NOTA

19. The polynomial $P(x)=x^4+ax^3+bx^2+cx+d$ has real-number coefficients and $P(1+i)=P(3i)=0$. Find the value of $P(2)$.

(A) 26 (B) 10 (C) 50 (D) 130 (E) NOTA

20. Evaluate: $\left(\sqrt{6}+i\sqrt{2}\right)^8$

(A) $-2048-2048i\sqrt{3}$ (B) $1296+16i$ (C) 1312
(D) $-16-16i\sqrt{3}$ (E) NOTA

21. Solve the equation for real x and y: $3(3-2i)-(1-8i)=x(1+i)-y(i-1)$. Express your answer as an ordered pair (x,y).

(A) $(-3,11)$ (B) $(-3,-11)$ (C) $(5,-3)$ (D) $(5,3)$ (E) NOTA

22. Let z be a complex number with absolute value equal to 1. Let $N=\frac{z-1}{z+1}$. Find the ratio of $\mathrm{Re}(N)$ to $\mathrm{Im}(N)$.

(A) 2 (B) 1/2 (C) 1 (D) 0 (E) NOTA

23. Calculate the sum of the complex roots z of $(z+z^2)^2=15(3z^2+2)(z^2+2z)$.

(A) $-2i$ (B) -2 (C) $2i$ (D) 2 (E) NOTA

24. Let x, y, and z be the three positive prime factors of 2013 where $0<x<y<z$. Calculate $i^{2x}+i^y+i^z$.

(A) 1 (B) $-1+2i$ (C) $1-2i$ (D) -1 (E) NOTA

25. Let $k=2013^{9001}$ and $n=37^k$. Simplify: i^n

(A) 1 (B) $-i$ (C) -1 (D) i (E) NOTA

26. For what real values of c will $f(z) = 3z^2 + 5z + c^2$ have no real roots?

(A) $|c| > \frac{\sqrt{3}}{6}$ (B) $|c| > \frac{5\sqrt{3}}{6}$ (C) $|c| > \frac{5\sqrt{3}}{3}$ (D) $|c| > \frac{5}{2}$ (E) NOTA

27. If $\frac{2-5\sqrt{3}+5i+2i\sqrt{3}}{2+5i} = z^2$, which of the following is a possible value for z?

(A) $\sqrt{3} + i$ (B) $1 + i\sqrt{3}$ (C) $\frac{\sqrt{6}}{2} + \frac{i\sqrt{2}}{2}$ (D) $\frac{\sqrt{2}}{2} + \frac{i\sqrt{6}}{2}$ (E) NOTA

28. If x and y are integers, $z = x + yi$, and $z^3 = 18 + 26i$, find the value of $x^5 + y^5$.

(A) 3126 (B) 992 (C) 244 (D) 33 (E) NOTA

29. Given the matrix $A = \begin{pmatrix} 3 & -2 \\ 4 & -1 \end{pmatrix}$, determine all possible values of λ such that the determinant of the matrix $A - \lambda I$ is equal to 0. Recall that $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, the 2×2 Identity Matrix.

(A) $1 \pm 2i$ (B) $2 \pm 3i$ (C) $3 \pm 4i$ (D) $4 \pm 5i$ (E) NOTA

30. Evaluate: $\left(1 + i\sqrt{3}\right)^4 (2 - 2i\sqrt{3})^5$

(A) $512 + 512i\sqrt{3}$ (B) $256 - 256i\sqrt{3}$
(C) $512 - 512i\sqrt{3}$ (D) $256 + 256i\sqrt{3}$ (E) NOTA