Gemini
Theta
Test #931

1. In the Name blank, print the names of both members of the Gemini Team; in the Subject blank, print the name of the test; in the Date blank, print your school name (no abbreviations). If your Gemini Team consists of members from different schools, write down the names of both schools.
2. In the EXAM NO. grid, write the 3-digit Test # on this test cover and bubble.
3. In the I.D. NUMBER grid, write down the 6-digit ID# of one person in your Gemini Team, left-justified, and bubble. Check that each column has only one number darkened.
4. Scoring for this test is 5 times the number correct + the number omitted.
5. You may not sit adjacent to anyone from your school.
6. TURN OFF ALL CELL PHONES OR OTHER PORTABLE ELECTRONIC DEVICES NOW.
7. No calculators may be used on this test.
8. Any inappropriate behavior or any form of cheating will lead to a ban of the student and/or school from future national conventions, disqualification of the student and/or school from this convention, at the discretion of the Mu Alpha Theta Governing Council.
9. If a student believes a test item is defective, select "E) NOTA" and file a Dispute Form explaining why.
10. If a problem has multiple correct answers, any of those answers will be counted as correct. Do not select "E) NOTA" in that instance.
11. Unless a question asks for an approximation or a rounded answer, give the exact answer.

For all questions, answer choice "E. NOTA" means none of the above answers is correct.

1. The saltwater pool at a San Diego hotel has 18,000 gallons of saltwater that is 7% salt. How many gallons of water must be added such that the pool contains the 4.5% salt?

A. 10,000 B. 12,000 C. 8,000 D. 9,000 E. NOTA

2. There is a 75% chance Sara and Brad go surfing today. If they do, they'll randomly go to one of their four favorite San Diego beaches: La Jolla Beach, Mission Beach, Pacific Beach and Windansea Beach. If Sara and Brad do go surfing, they will visit these beaches with equal likelihood. Their friend Jake went surfing earlier today at Mission Beach and Ocean Beach, accidently leaving his sunglasses at one of the locations (with equal likeliness of leaving them at either beach). He asked Sara and Brad to pick them up if they go to the beach where he left them. Sara and Brad are expert sunglass lookers, and will definitely find the sunglasses, provided they are in the right beach. What are the chances Sara and Brad find Jake's sunglasses for him?

A. $\frac{1}{8}$ B. $\frac{1}{4}$ C. $\frac{3}{8}$ D. $\frac{3}{32}$ E. NOTA

3. Find the coefficient of the x^3y^3term in the expansion of $(3x - 2y)^6$ with like-terms combined.

A. 4320 B. -4032 C. -4320 D. 4032 E. NOTA

4. What is the sum of all roots of $f(x) = 5x^3 + 4x^2 - 2x^4 + 3x - 1$?

A. -0.8 B. 0.4 C. 0.8 D. 2.5 E. NOTA

5. Find the value of the product SAN, ignoring units, if
S = the number of diagonals of a heptagon,
A = the area of circle $y^2 + x^2 - 6y + 6x + 2 = 0$,
N = the abscissa of the x-intercept of the line perpendicular to x + 2y = 3 and passing though the point (3, 2).

A. 448π B. 7168π C. 896π D. -896π E. NOTA

6. Find the value of the sum D+I+E+G+O, ignoring units, if
D = units digit of 5^{8^3},
I = the ordinate of the y-intercept of the line parallel to x + 2y = 3 and passing though the point (3, 2),
E = the units digit of $3^{58} + 8^{35}$,
G = the circumference of circle $y^2 + x^2 - 6y + 6x + 2 = 0$,
O = the sum of the angles of a heptagon.

A. 909.5 + 16π B. 909 + 8π C. 902 + 8π D. 902 + 16π E. NOTA

7. Three buses of 70 math team students each are going to the San Diego Zoo. Fifty-two students want to see the giraffes, 29 students want to see the snakes and 32 students want to see the lions. Nineteen students want to see the lions and the giraffes, but only 15 students want to see the lions and the snakes. Eight students want to see the lions, giraffes, and snakes. Finally, two students want to see only the snakes. How many students do not want to see any of the animals?

A. 3 B. 4 C. 6 D. 7 E. NOTA

8. Governor Jerry Brown was spotted running in Balboa Park. According to news reports, he started at the visitor's center and ran for 15 minutes before reaching the carousal 2 miles away. He then ran 1.2 miles to the miniature railroad at a rate of 6 miles per hour. Finally, he ended his run at a 6.8 miles per hour rate at the photographic arts building 1.7 miles away. What rate did Jerry Brown average on his run?

A. 6.4 miles/hour B. 7.0 miles/hour C. 7.2 miles/hour D. 6.9 miles/hour
E. NOTA

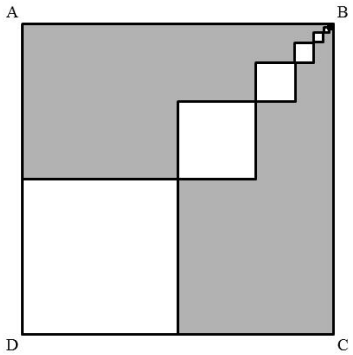
9. Evaluate: $\begin{vmatrix} 2 & 7 & -2 \\ 6 & -11 & 6 \\ -2 & 7 & 2 \end{vmatrix}$

A. 366 B. -336 C. -366 D. 336 E. NOTA

10. If f(2) = 1, f(-5) = -3, f(-3) = 2, f(-7) = -5, f(1) = 0, f(0) = -7, then what is f(f(f(f(-3))))?

A. -5 B. -7 C. 0 D. 1 E. NOTA

11) Square ABCD below has a perimeter of 36. It is filled with an infinite series of squares, each half the width of the previous square. What is the area of the shaded region?



A. 54 B. 27 C. 18 D. 12 E. NOTA

12) Assuming $\pi \approx \frac{22}{7}$, which of the following is closest to the volume of the figure formed by rotating the line 3x – 7y = -21 around the x-axis between the lines x = 0 and x = 7?

A. 528 B. 4620 C. 5280 D. 462 E. NOTA

13) Suppose that $f\left(\sqrt[3]{x} - 5\right) = \frac{3+2i}{-\sqrt{x}+3i}$, where $i = \sqrt{-1}$. Find $f(-1)$.

A. $\frac{18-3i}{73}$ B. $\frac{18+3i}{-73}$ C. $\frac{18+25i}{73}$ D. $\frac{18+25i}{-73}$ E. NOTA

14) Evaluate: $\sum_{n=1}^{\infty} \frac{3n}{4^{n+1}}$

A. 3 B. $\frac{1}{3}$ C. $\frac{1}{4}$ D. ∞ E. NOTA

15) The current salaries of the 12 Padres players are directly related to their batting average, which is the average number of hits the player gets per game, from the previous season. Six players bat each game in a specific order called the lineup. Huston Street's current salary is $7.5M and batting average was 12 hits per game last season. Carlos Quentin's current salary is $7M. He batted an average of 10.8 hits for the first 140 games of last season.

G = the number of hits Carlos Quentin made in the last 25 games of last season
O = the number of possible Padre lineups

What is the value of G/O?

A. 2310 B. 3 C. 2.75 D. 1980 E. NOTA

16) What is the shortest distance between the graphs $2x^2 + 2y^2 + 16x - 12y + 42 = 0$ and $-3x^2 - 3y^2 + 6x - 54y - 198 = 0$?

A. 149 B. 13 C. 7 D. 163 E. NOTA

17) What is the area of a trapezoid with a height of 145_6 and bases measuring 24_5 and 120_4? Express your answer in base 7.

A. 2323_8 B. 4646_8 C. 6826_7 D. 3413_7 E. NOTA

18) The California Quail is California's state bird and mainly eats berries. Three California Quails can eat eight berries in 40 seconds. Assuming the birds eat at the same rate, how many seconds does it take five California Quails to eat seven berries?

A. 35 B. 21 C. $58.\overline{3}$ D. $58.\overline{6}$ E. NOTA

19) For $x \neq 2.25$, give the sum of the real solutions to the equation $\frac{1}{2\sqrt{x}+3} + \frac{1}{\sqrt{x}-3} = \frac{-3\sqrt{x}}{2\sqrt{x}+3}$.

A. 4.25 B. 6.25 C. 4 D. 2 E. NOTA

20) Students must have graduated high school to have attended the University of California San Diego (UCSD). Having a UCSD class ring is sufficient evidence for having attended the UCSD. If a person receives UCSD alumni letters the person must have attended UCSD. Which of the following is true?

A. If a person attended UCSD, the person has a UCSD class ring.
B. If a person attended UCSD, the person receives UCSD alumni letters.
C. If a person did not attend UCSD, the person did not graduate high school.
D. If a person did not attend UCSD, the person does not have a UCSD class ring.
E. NOTA

21) Triangle ABC has vertices A(-5,20), B(9, -18), C(-10,25). The coordinates of the centroid of triangle ABC is (d,e). Find $d^2 + \sqrt{e}$.

A. 7 B. $4 + \sqrt{3}$ C. 13 D. $3 + \frac{3}{2}\sqrt{6}$ E. NOTA

22) In a seven-element data set of integers from 51 to 150 inclusive, the median is 83, the range is 62 and the mode is 104. What is the smallest possible value of the arithmetic mean?

A. 63 B. 72 C. 65 D. 80 E. NOTA

23) Seaport Village is home of the best shopping in San Diego. One of the best shops, Crazy Shirts, is having its annual sale on sportswear. The store is handing out 100 coupons for 30% off, 50 coupons for 35% off and 25 coupons for 50% off your total purchase as you walk in the store today only. Women's shirts were originally priced at $88 and men's shirts were originally priced at $54. You happen to have a $12 gift card from your last visit that can be applied to your final purchase amount after all mark downs are made. You wake up early and are the first person at the store when it opens. You plan on buying two men's shirts and a woman's shirts. How much cash should you expect to spend?

A. $116.80 B. $55.20 C. $128.80 D. $67.20 E. NOTA

24) How many positive four-digit palindromes are divisible by 3?

A. 24 B. 27 C. 30 D. 36 E. NOTA

25) Suppose that a sequence is defined by $a_0 = 2$, $a_1 = 3$, $a_2 = 6$, and for $n \geq 3$:

$$a_n = (a_{n-1})(n+4) - (4n)(a_{n-2}) + 4(n-2)a_{n-3}.$$

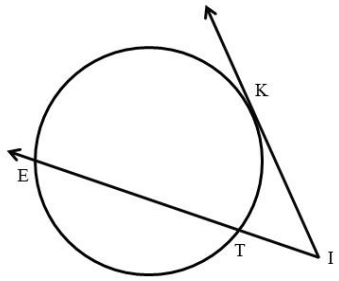
Find the sum of the digits of a_{10}.

A. 32 B. 34 C. 36 D. 38 E. NOTA

26) Let C equal the coefficient of the x^7th term when $(2x+3)^{10}$ is expanded and like-terms combined. Find the sum of the digits of C.

A. 16 B. 18 C. 20 D. 22 E. NOTA

27) Suppose $\overrightarrow{IK}$ is tangent to the circle shown below at point *K*, and $\overrightarrow{IE}$ intersects the circle at points *T* and *E*. The circumference of the circle is 16π, and the length of the minor arc *TK* is 4π. The measure of minor arc *EK* is 140 degrees. What is the measure of angle *KIT*?



A. 20 degrees B. 25 degrees C. 40 degrees D. 50 degrees E. NOTA

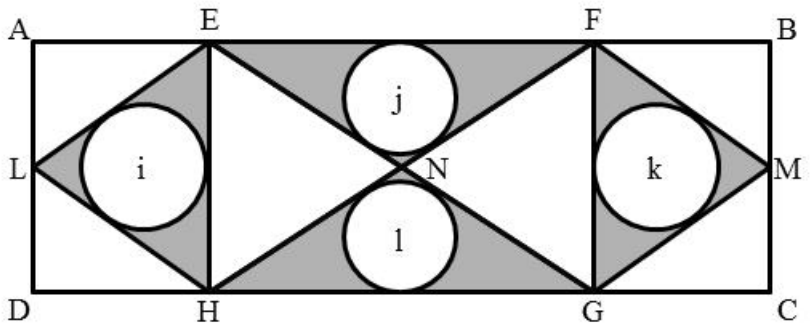
28) Let $\frac{2}{\sqrt[3]{5}-\sqrt[3]{7}} = \frac{\sqrt[3]{M}+\sqrt[3]{A}+\sqrt[3]{T}}{H}$ for integers *M*, *A*, *T*, and *H*. Moreover, $M < A < T < H$, and *T* and *H* are relatively prime. What does *M* + *H* equal?

A. -48 B. 48 C. 24 D. -24 E. NOTA

29) Find the absolute value of the difference between the distance between the vertices and the distance between the foci of the conic section with equation $25x^2 - y^2 - 100x - 6y + 66 = 0$.

A. $2\sqrt{26} - 2$ B. $-2\sqrt{26} - 2$ C. $2\sqrt{26} + 2$ D. $-2\sqrt{26} + 2$ E. NOTA

30) In the figure below, ABCD is a rectangle. Lines AD, EH, FG, and BC are parallel. FG=6. Circle i is inscribed inside isosceles triangle ELH. Circle k is inscribed inside isosceles triangle FMG. Circle j is inscribed inside isosceles triangle ENF. Circle l is inscribed inside isosceles triangle HNG. The length of rectangle EFGH is 8, and the length of AE and GC are 4. What is the area of the shaded region?



A. $96 - \frac{145}{18}\pi$ B. $48 - \frac{145}{18}\pi$ C. $48 - \frac{145}{36}\pi$ D. $96 - \frac{145}{36}\pi$
E. NOTA